Solving the Teukolsky Equation with Spectral Methods

Tiago Valente (Nr: 100370)

MASTER'S INTEGRATED PROJECT IN ENGINEERING PHYSICS

SUPERVISORS:
DR. HANNES RÜTER
DR. DAVID HILDITCH

Motivation

- Hermite-4 method → Implicit and *Time-symmetric*

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  "Energy" conservation

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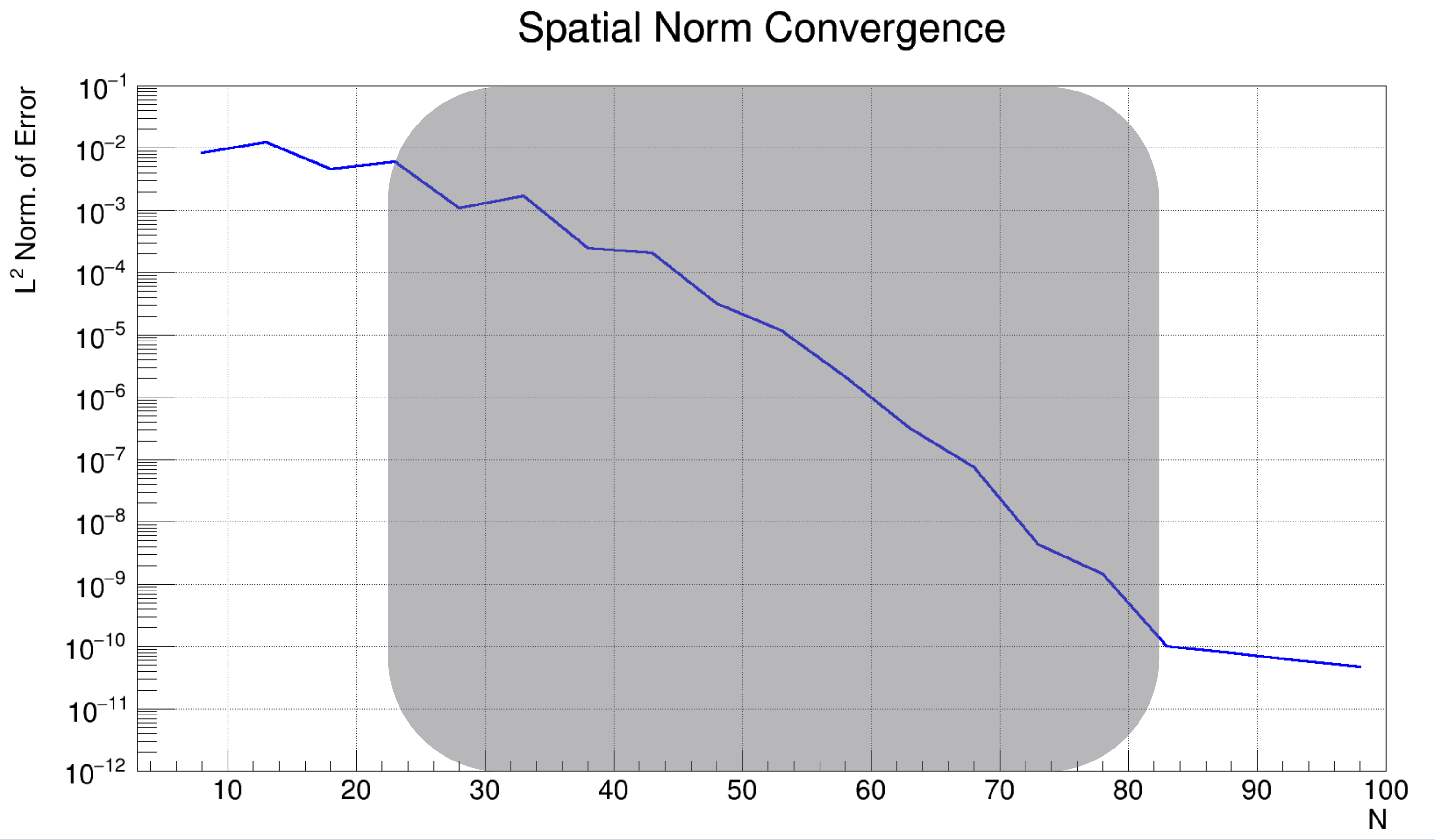
- Convergence tests → Not shown in the article we have been following

Spatial Norm Convergence

$$\varepsilon(N) \sim e^{-N} \Rightarrow \log(\varepsilon(N)) \sim -N$$

Log scale



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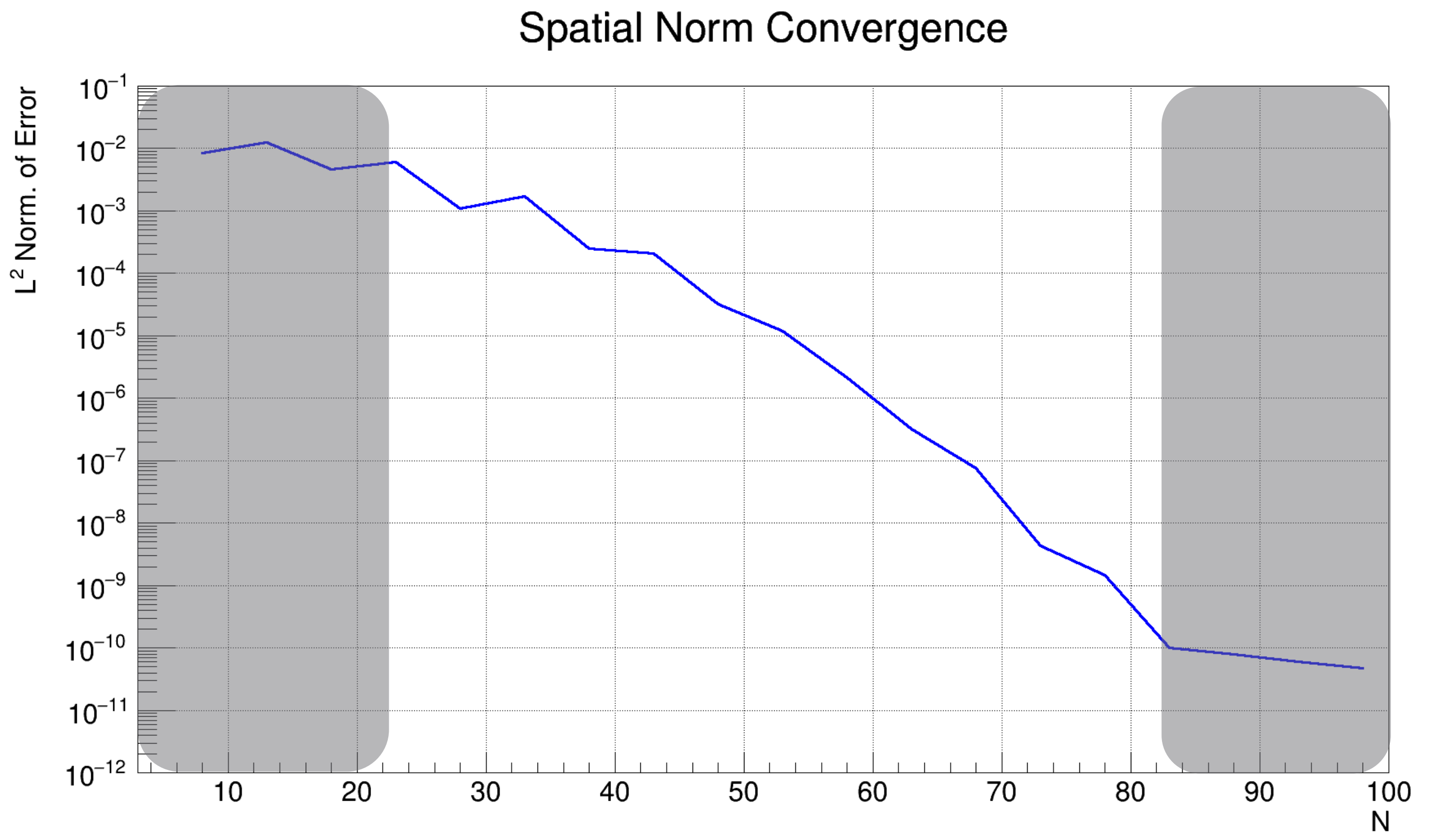
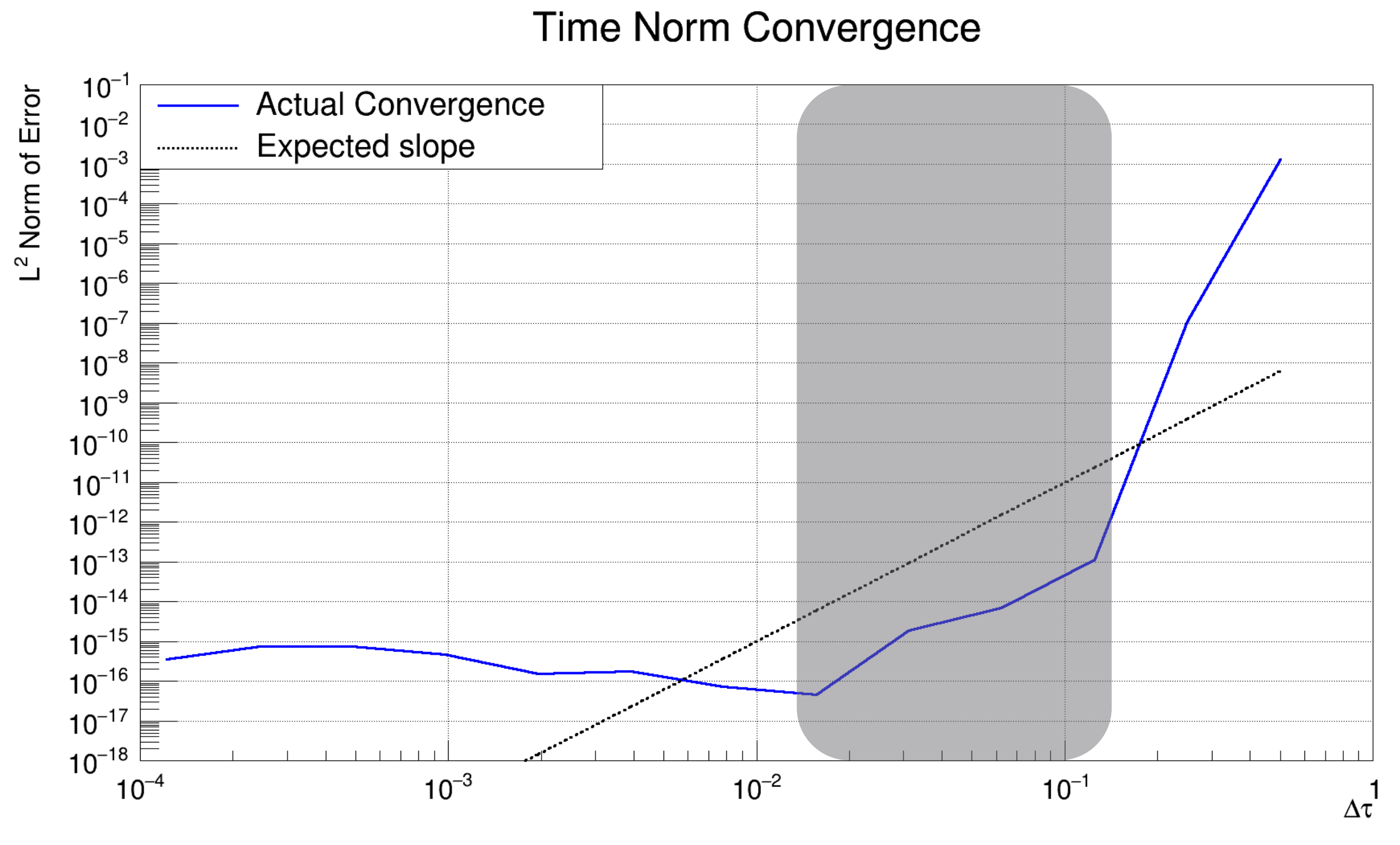


Temporal Norm Convergence

$$\varepsilon(\tau) \sim \mathcal{O}(\Delta\tau^4) \Rightarrow \log\left[\varepsilon(\tau)\right] \sim 4 \log \Delta\tau$$

Log scale



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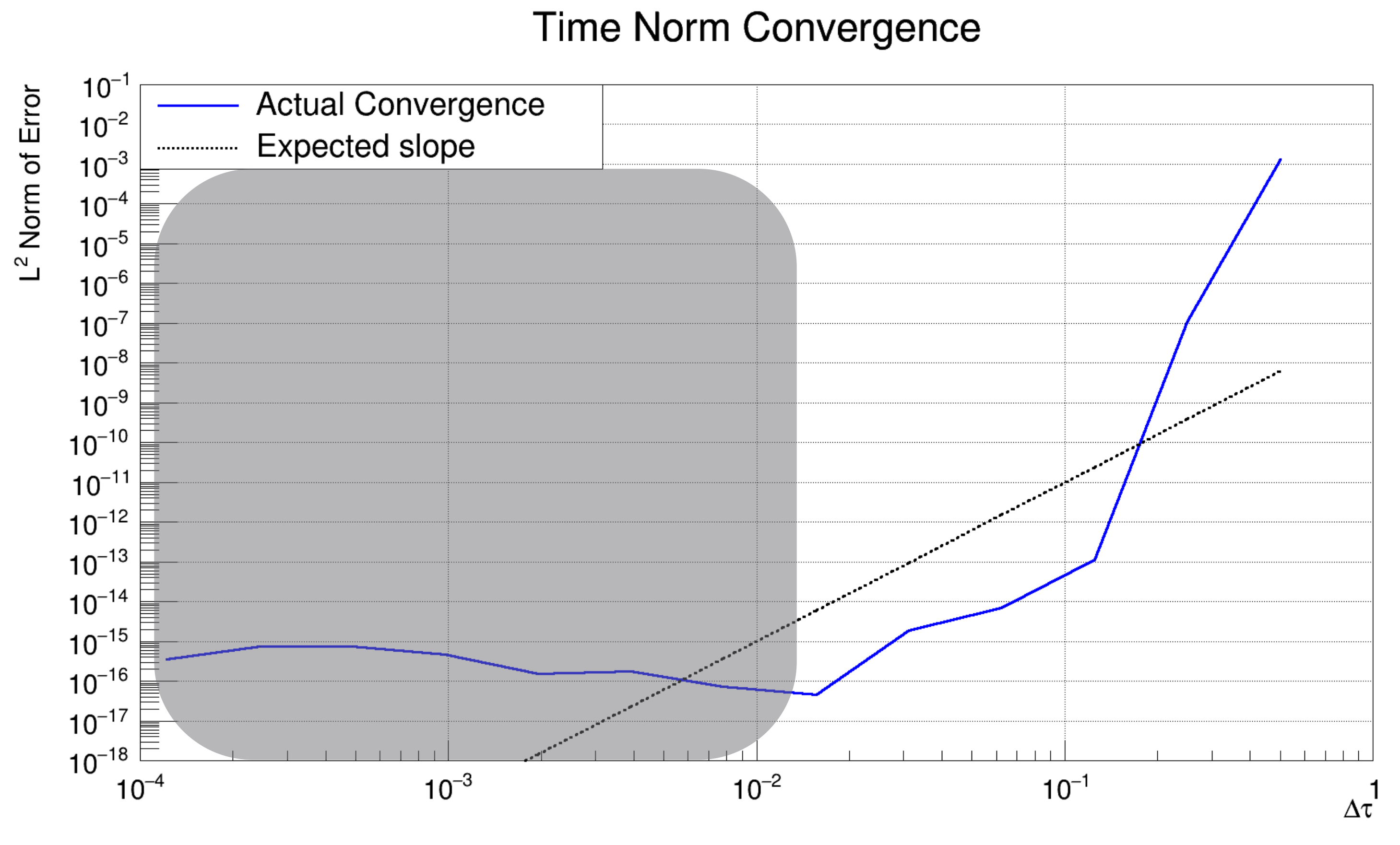

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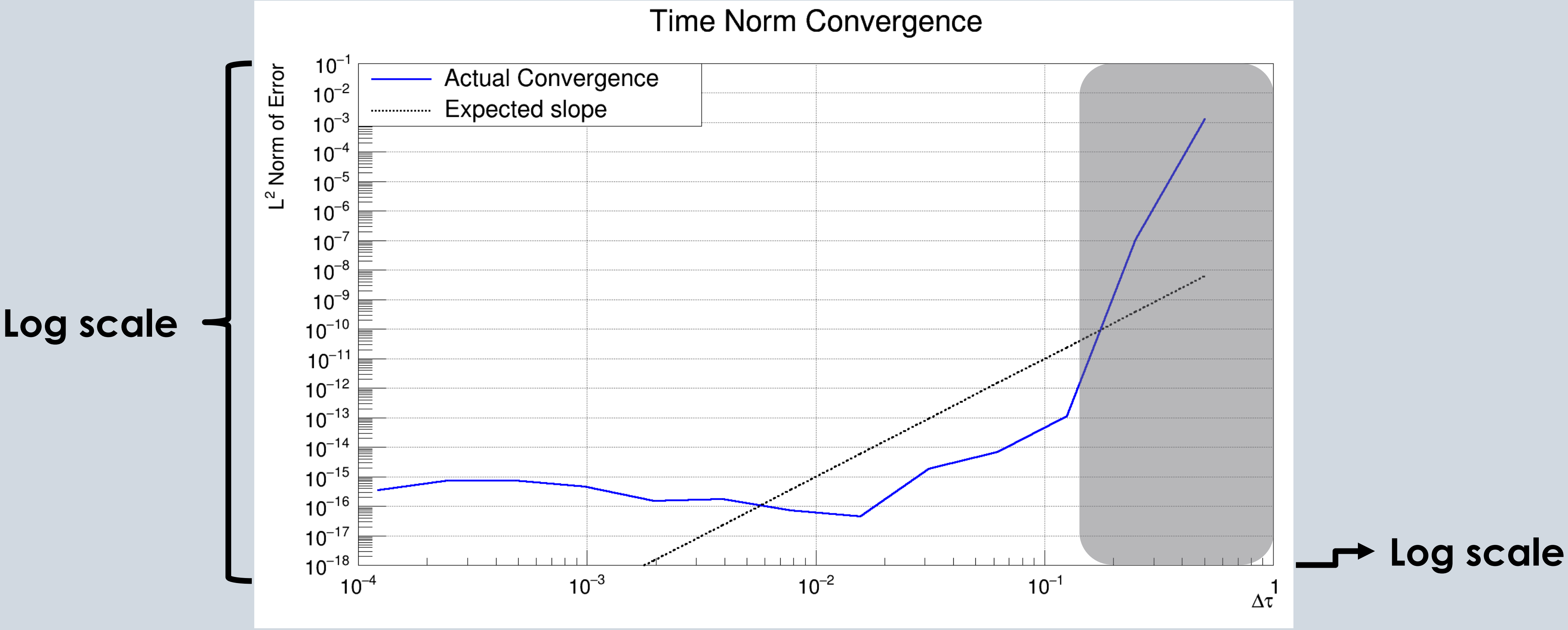
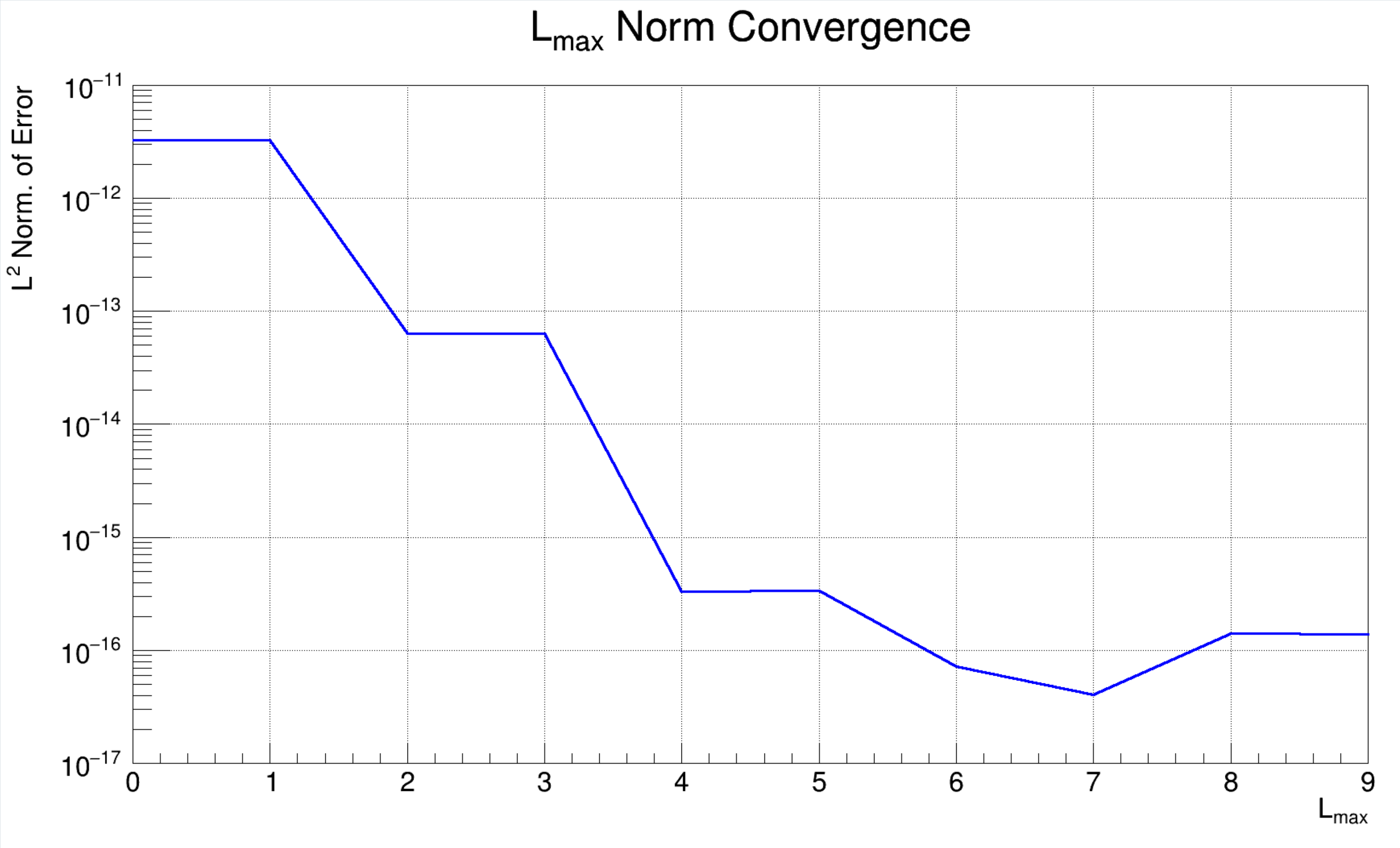


L_{max} Norm Convergence



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Future Work

- Next step → Inhomogeneous Teukolsky Equation → Simulate geodesic orbits

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- Improvements to the code:
  - Fully spectral method
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  - Explore spheroidal harmonics (maybe)