Tidal Evolution and Dynamics: Timescales of Force-Deformation Laws in Celestial Bodies.

Clodoaldo Grotta Ragazzo
Universidade de São Paulo

Collaborators:
L. Ruiz Y. Gevorgyan G. Boué A. Correia
I. Matsuyama V. Oliveira C. Asano

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Summary

1) Tides: why they matter in modern science.
2) Why do tides occur?
3) Common hypotheses in the theory.
4) Deformation “toy model”.
5) Self-gravity and rheology.
6) Fast time scale: dynamical frequency.
7) Tides on terrestrial planets act slowly.
8) Tides on fluid bodies, do they act slowly?
9) Terrestrial bodies: viscous and elastic regimes.
10) Conclusion.

1. Tides: Why They Matter in Modern Science

Recent Scientific Breakthroughs

- **Exoplanets:** Up to January, 29 2025, **5832** exoplanets have been confirmed [1].
- **Ocean Worlds:** Several moons in the Solar System contain subsurface oceans, including:
  - Europa, Ganymede, Callisto (Jupiter)
  - Titan, Enceladus (Saturn)
- These discoveries suggest that environments favorable to life may be more common than previously thought.

[1] https://exoplanetarchive.ipac.caltech.edu/docs/counts_detail.html

The Role of Tides in Planetary and Satellite Evolution

- **Long Timescales:**
  - Circularization of orbits and spin-orbit synchronization.
  - Most major satellites show the same face to their host planets.
- **Short Timescales: Tidal Heating**
  - Converts mechanical energy into heat.
  - Crucial for moons far from the Sun, such as Europa and Enceladus.
  - Sustains subsurface liquid water, possibly supporting extraterrestrial life.
- Estimating tidal energy flux is essential for assessing habitability.

Important Questions

Among the ocean-hosting satellites of Jupiter and Saturn, has any of them maintained this state for billions of years?
If not, for how long have they sustained liquid water due to tidal heating?

2. Why Do Tides Occur?

Explanation

- Tides result from the variation of gravitational force exerted by the Moon (or the Sun) on different parts of the Earth.
- The difference between the local force and the average force at the center of mass deforms the Earth, creating bulges.

Tide-Raising Force

The tide-raising force is the deviating part of the Moon's gravitational field:

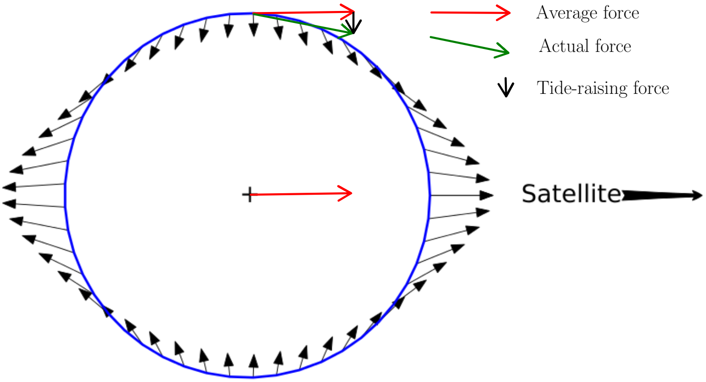


Figure: The tide-raising field is given by $\nabla\Phi^T(\mathbf{x}) = \nabla\Phi(\mathbf{x}) - \overline{\nabla\Phi}$.

3. Common Hypotheses in the Theory

- The body, in its undeformed state, is spherically symmetric (radially stratified).

- Deformations are small: the body remains slightly aspherical all the time.

- The body is composed of incompressible material.

4. Deformation "Toy Model"

$x =$ represents tidal deformation and
$F =$ tide-raising force.

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = F(t)$$

$$\omega_0 := \text{natural frequency} = \sqrt{\frac{\text{rigidity}}{\text{inertia}}}$$

$$\zeta := \text{damping ratio}$$

Harmonic forcing $F(t) = \mathrm{e}^{i\omega}$ and $x(t) = \hat{x}\mathrm{e}^{i\omega}$:

$$\left(\frac{\omega^2}{\omega_0^2} + 2\zeta i\,\frac{\omega}{\omega_0} + 1\right)\hat{x} = \frac{1}{\omega_0^2}$$

If $\omega \ll \omega_0$, then the inertial term $\left(\frac{\omega}{\omega_0}\right)^2$ is negligible.

The dissipative term $2\zeta i\,\frac{\omega}{\omega_0}$ may be a perturbation of the elastic term.

5. Self-Gravity and Rheology

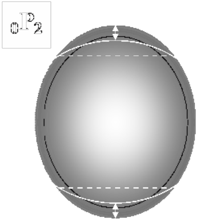
A body is held together by two main forces: self-gravity and rheology, which encompasses elasticity and viscosity across multiple layers.

For a complex body, rigidity and damping rate depend on the forcing frequency ω.

6. ω_0: Threshold Between High and Low Frequencies

$$\omega_0^2 = \frac{4}{5}\frac{Gm}{R^3} \quad \text{(dynamical frequency)}$$

frequency of free-oscillations of a uniform body made of a perfect fluid.



ω_0: Effects of Stratification and Rheology

Effect of density stratification on Earth:

$$T = \frac{2\pi}{\omega_0} \approx 82 \text{ minutes}$$

Observed value (including the effect of rheology):

$$T_{\text{obs}} = 54 \text{ minutes}$$

7. Tides on Terrestrial Planets Act Slowly

Typically, for bodies with a solid layer (e.g., the Earth), $\omega_0 \gg \omega$, and deformation inertia can be neglected.

For the Earth:

$$\frac{2\pi}{\omega} = 12 \text{ hours}, \quad \frac{2\pi}{\omega_0} \approx 1 \text{ hour}.$$

Thus, the ratio between the tidal forcing frequency and the threshold frequency is:

$$\frac{\omega}{\omega_0} \approx \frac{1}{12}.$$

8. Fluid Bodies: Internal-Wave Frequency$< \omega_0$.

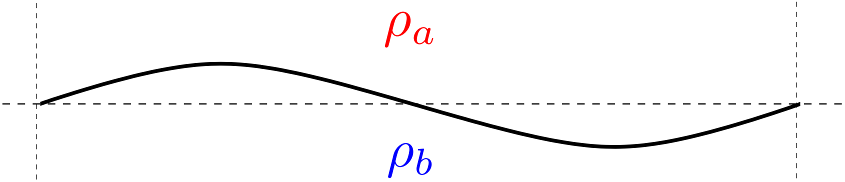
In fluid bodies: internal inertial waves have slower frequencies, then surface waves. See Figure.

In Saturn and Jupiter, these waves may affect rings (Marley, Porco,...) and satellites (Fuller, Lainey,...).

For Saturn: rotation period = 10.7 hours and

$$T_{\text{Surface wave}} = \frac{2\pi}{\omega_0} \approx 3 \text{ hours}$$

Internal Wave Figure



Archimedes Principle $\Rightarrow$

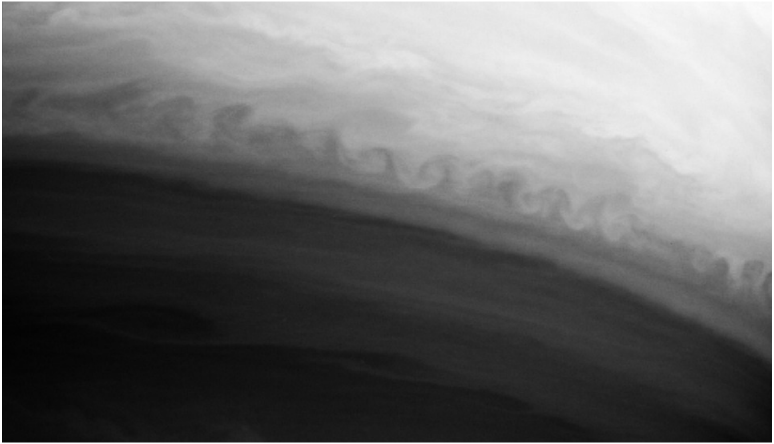
$$\text{Modified gravity} = g\frac{\rho_b - \rho_a}{\rho_b}$$

Inertia of deformation is important in fluid bodies with density stratification

In Jupiter and Saturn:

- Viscosity is small, and the Coriolis force is significant.
- Energy dissipation is primarily driven by Kelvin-Helmholtz instabilities and breaking waves.
- Other factors, such as differential rotation, acoustic modes, and others, also play a role.

Kelvin-Helmholtz instability in Saturn's atmosphere (NASA)



Relaxation Times of Terrestrial Bodies

The rheology of terrestrial bodies can be characterized by a set of relaxation times: $\tau_1, \tau_2, \ldots$.

Range of relevant relaxation times (theoretical estimates):

- For the Moon: from 10 days to 10^7 days or more.
- For the solid Earth: from 2 to 2×10^5 centuries or more.

Terrestrial Body Visco$\times$Elastic Dicotomy

Viscous regime: The relaxation times of the rheology are much shorter than the typical despinning time and deviations from sphericity are caused only by tidal and centrifugal stresses (Earth).

Elastic regime: At least one of the relaxation times of the rheology is much longer than the typical despinning time. The body may have a permanent (fossil) deformation (Moon).

10. Conclusion

- The inertia of deformation and wave propagation play a significant role in tidal interactions of fluid bodies.
- In most cases, deformation inertia can be neglected when a solid layer is present.
- Terrestrial bodies can behave as either viscous or elastic materials, depending on their thermal history.
- Spin and orbital evolution depend on a body's force-deformation response, which varies significantly among different bodies, as well as on its characteristic timescales.
- Force-deformation properties evolve with the body's internal changes, with temperature being a crucial factor.
- We do not have answers to some important questions.