Dressed quark mass function from a one-gluon exchange and linear confining interaction

LABORATÓRIO DE INSTRUMENTAÇÃO
E FÍSICA EXPERIMENTAL DE PARTÍCULAS

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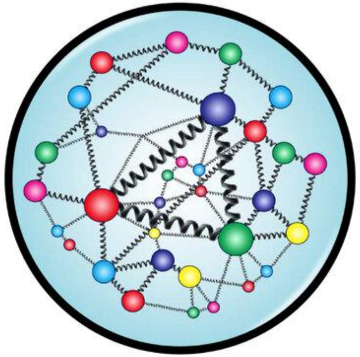
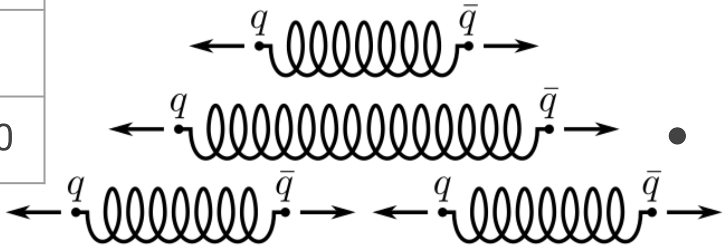
QCD and the problem with quark masses

quark	m (MeV)
u	~ 2-3
d	~ 4-6
c	~ 100
s	~1300
t	~ 4200
b	~ 175200





Proton: uud (Mp≃ 938 MeV/c^2)

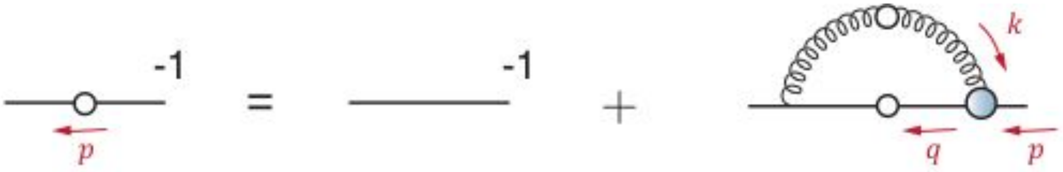
Neutron: ddu (Mn≃ 940 MeV/c^2)

Constituent quark mass: m + 300 MeV

Problems:

- The light quark “running” mass. Where does ~98% of the quark mass come from?
- Cannot isolate a single quark. What is the origin of confinement?

Dyson-Schwinger equation and dressed quark propagator



$\triangleright \Sigma_A(p^2) = \int_0^{\Lambda} dq \int_{-1}^{1} dz \sqrt{1-z^2} \mathcal{F}_A(p^2, q^2, z)$

$\triangleright \Sigma_M(p^2) = \int_0^{\Lambda} dq \int_{-1}^{1} dz \sqrt{1-z^2} \mathcal{F}_M(p^2, q^2, z)$

Where:

$\rightarrow \mathcal{F}_A = \frac{4\pi}{(2\pi)^4} q^3 \sigma_v(q^2) g(k^2) F(p^2, q^2, z)$

$\rightarrow \mathcal{F}_M = \frac{4\pi}{(2\pi)^4} q^3 \sigma_s(q^2) g(k^2)$

- $S(p)^{-1} = S_0(p)^{-1} - i\Sigma(p)$

(dressed quark propagator)

- $S(p) = \frac{1}{A(p^2)} \frac{i(M(p^2) + \not{p})}{p^2 - M(p^2)^2 + i\epsilon}$

$\equiv \sigma_s(p^2) + \not{p}\sigma_v(p^2)$

(full quark propagator)

- $\Sigma(p) = \Sigma_A(p^2) + \not{p}\Sigma_M(p^2)$

(quark self energy)

It will be solved by iteration!

Regularization and renormalization

How to remove divergences in the diagrams? → Regularization and renormalization!

➤ Regularization (isolate divergences):

- Hard momentum cut-off:

$$\int d^4q \longrightarrow \int_\Lambda d^4q$$

- Pauli-Villars

Introduces a scale $\Lambda(\mu)$

Where $m_0 = Z_m m$ depends on cutoff:

$$Z_m(\mu^2, \Lambda^2) = \frac{M(\Lambda^2)}{M(\mu^2)}$$

$$m_0(\Lambda^2) = Z_m(\mu^2, \Lambda^2) m_\mu$$

$$= M(\Lambda^2) \equiv M_\Lambda$$

 Renormalization (remove divergences):

For an arbitrary renorm. point $S(p^2=\mu^2)$:

$A(p^2=\mu^2) \stackrel{!}{=} 1$; $M(p^2=\mu^2) \stackrel{!}{=} m$

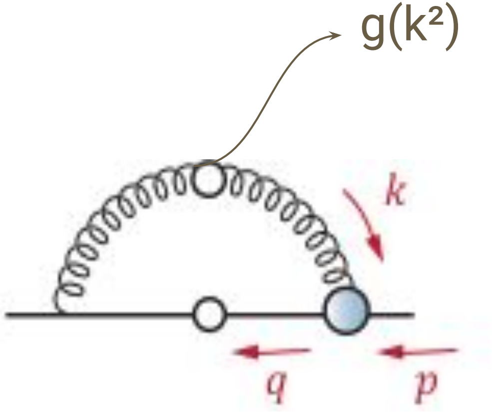
Leads to:

$$A(p^2) = Z_2 + \Sigma_A(\mu^2) \stackrel{!}{=} 1 \Rightarrow \boxed{Z_2 = 1 - \Sigma_(\mu^2)}$$

$$M(p^2) = \frac{Z_2 Z_m m + \Sigma_M(\mu^2)}{A(\mu^2)} \stackrel{!}{=} m$$

$$\Rightarrow \boxed{M_\Lambda = \frac{m - \Sigma_M(\mu^2)}{1 - \Sigma_A(\mu^2)}}$$

Maris-Tandy interaction



On the DSE we have the coupling term g(k²) as:

$$g(k^2) = Z_2^2 \frac{16\pi}{3} \frac{\alpha(k^2)}{k^2}$$

The alpha for the Maris-Tandy interaction:

$$\alpha(k^2) = \alpha_{IR} + \alpha_{UV}$$

$$\rightarrow \alpha_{IR} = \pi\eta^7 x^2 e^{-\eta x^2}$$

$$\rightarrow \alpha_{UV} = \frac{2\pi\gamma_m(1 - e^{-k^2/\Lambda_+})}{ln[e^2 - 1 + (1 + k^2/\Lambda_{QCD}^2)]}$$

Accounts for chiral symmetry breaking and confinement!

Code implementation

M(p^2) and A(p^2) will be solved by solving iteratively $\Sigma_M(p^2)$ and $\Sigma_A(p^2)$. Taking μ=19GeV, Λ=1000GeV, m_μ=0.004GeV,

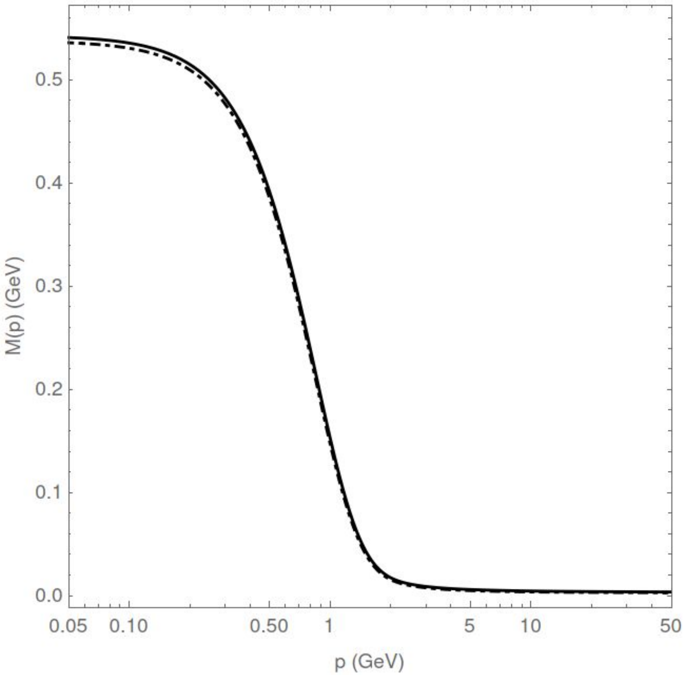
- ➔ 0th iteration: $Z_2^{(0)} = 1GeV, M_\Lambda^{(0)} = 1GeV, M^{(0)}(p^2) = 1, A^{(0)}(p^2) = 1$
- ➔ 1st iteration:
  - ◆ Calculate $\Sigma_M^{(1)}(p^2)$ and $\Sigma_A^{(1)}(p^2)$
  - ◆ Calculate $Z_2^{(1)}(\mu^2), M_\Lambda^{(1)}(\mu^2), M^{(1)}(p^2), A^{(1)}(p^2)$
  - ◆ Calculate $\sigma_v^{(1)}$ and $\sigma_s^{(1)}$ as:

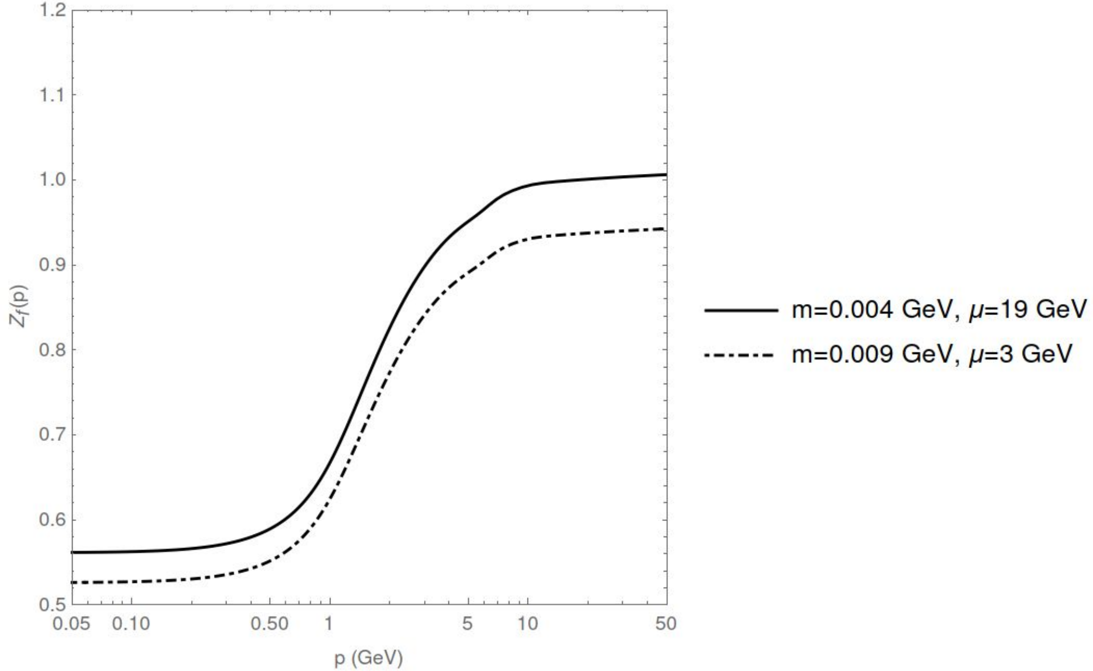
$$\to \sigma_v^{(1)}(p^2) = \frac{1}{A^{(1)}(p^2)} \frac{1}{p^2 + [M^{(1)}(p^2)]^2}$$

$$\to \sigma_s^{(1)}(p^2) = \sigma_v^{(1)} M^{(1)}(p^2)$$

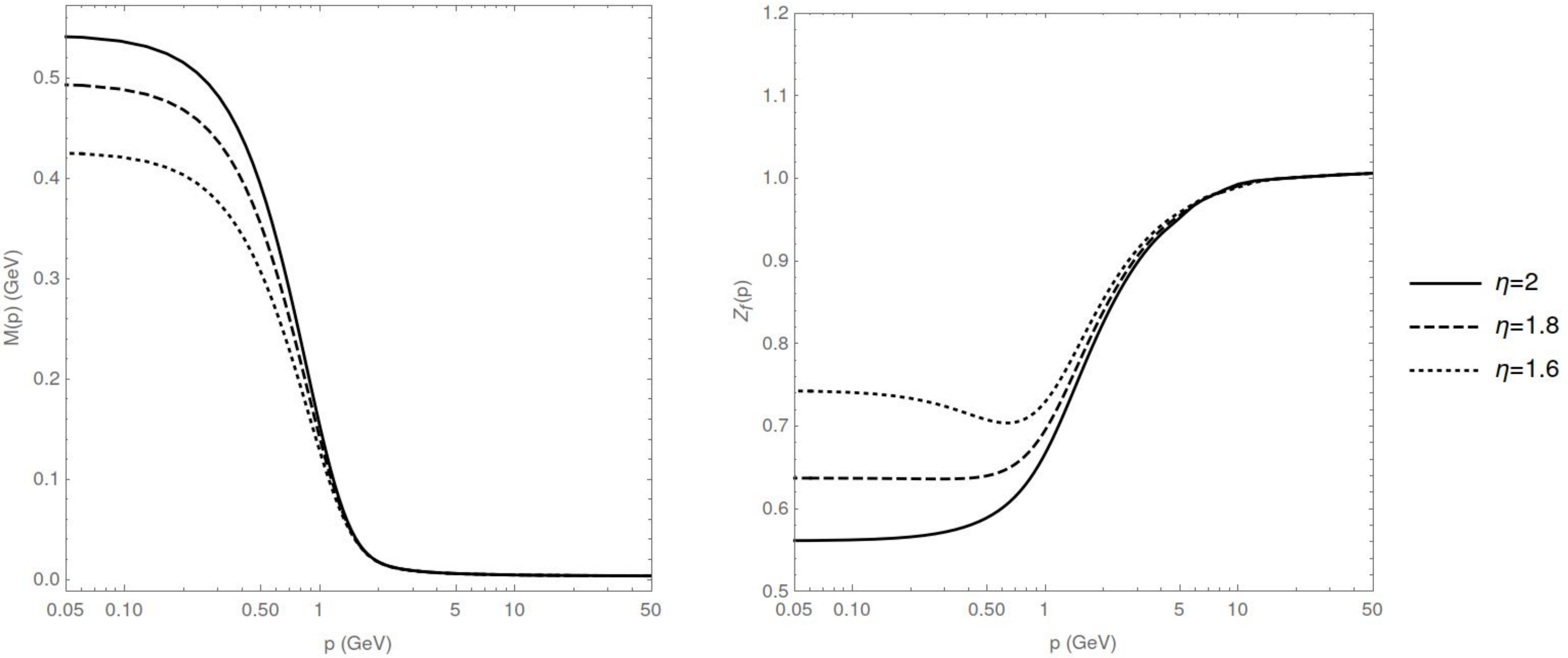
- ➔ 2nd iteration: calculate with $\Sigma_M^{(2)}(p^2)$ and $\Sigma_A^{(2)}(p^2)$ values with $\sigma_s^{(1)}$ and $\sigma_v^{(1)}$. Repeat process!
- ➔ 3rd, 4rd, 5th,.... iteration and so on: do as before !

$M(p^2)$ and $A(p^2)$ function results

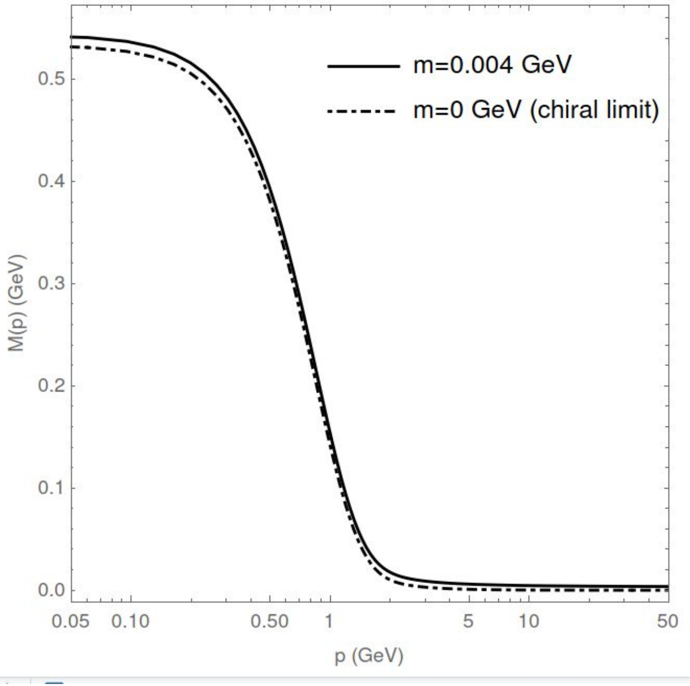




M(p²) and A(p²) function results (for different η values)



$M(p^2)$ on Chiral limit and chiral quark condensate



Chiral quark condensate defined as:

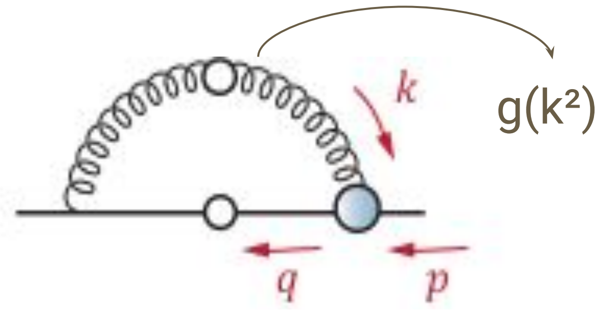
$$< q\bar{q} > = < 0|q\bar{q}|0 >$$

If $< q\bar{q} > \neq 0$ we have spontaneous chiral symmetry breaking.

We obtained the following results:

	η=1.6	η=1.8	η=2
$- < q\bar{q} >$	$(0.2152\ \mathrm{GeV})^3$	$(0.2163\mathrm{GeV})^3$	$(0.2171\mathrm{GeV})^3$

OGE + linear confining interaction used in CST



We get the kernel on Landau gauge:

$$\nu(k) = -[\frac{1}{4}\sum_a \lambda_a \otimes \lambda_a]\gamma^\mu \otimes \gamma^\nu [V_{L,\epsilon}(k) + V_G(k) + V_C(k)][g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}]$$

The non-relativistic potential(Cornell potential):

$$V(r) = [\frac{1}{4}\sum_a \lambda_a \otimes \lambda_a][\sigma r - \frac{\alpha_s}{r} + C]$$

Putting it into the DSE we get the interaction term $g(k^2)$:

Introducing screening on the confinement term:

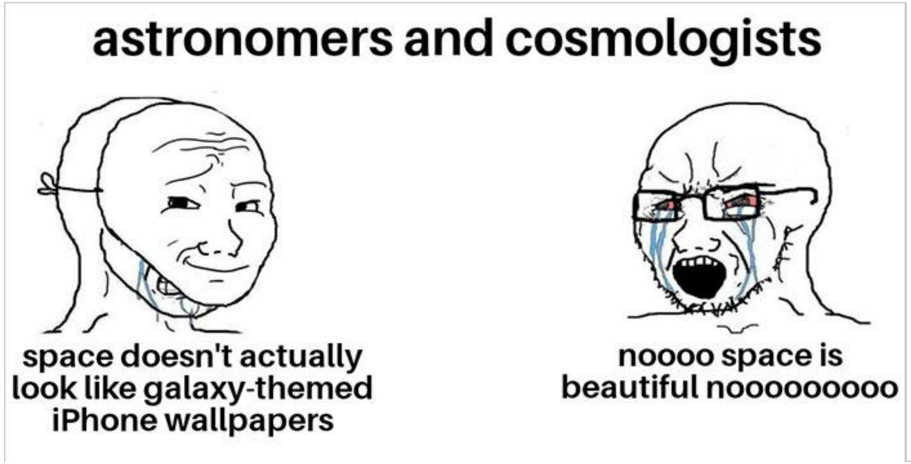
$$V_{L,\epsilon}(r) = -\frac{\sigma}{\epsilon}(e^{-\epsilon r} - 1)$$

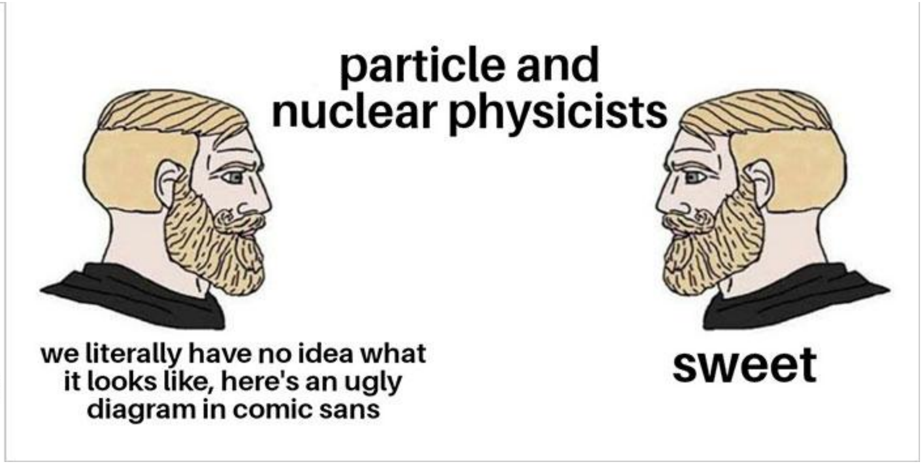
$$g(k^2) = Z_2^2 \frac{16\pi}{3}[\frac{\alpha_s}{k^2} + 2\sigma[\frac{1}{(k^2+\epsilon^2)^2} - \frac{(2\pi)^4}{\epsilon}\delta^4(k)] - 4\pi^3 C\delta^4(k)]$$

Final Remarks and Conclusion

- We focused on key points of non-perturbative QCD
- Obtained as predicted the $M(p^2)$ and $A(p^2)$ for different parameter in the Maris-Tandy interaction.
- Calculated the chiral quark condensates
- Next: we want to use a one-gluon exchange + linear confinement kernel to obtain a mass function to be used in meson CST calculations.

Thank you!


astronomers and cosmologists
space doesn't actually look like galaxy-themed iPhone wallpapers
noooo space is beautiful nooooooooo


particle and nuclear physicists
we literally have no idea what it looks like, here's an ugly diagram in comic sans
sweet

References

- Elmar Biernat's notes on QCD
- HADRON PHYSICS,FROM NUCLEI TO QUARKS AND GLUONS,Elmar P. Biernat (presentation
- QCD and Hadron Physics, Gernot Eichmann, Lecture notes, 2021
- Mass function from Dyson-Schwinger equation with confinement and OGE interaction,Elmar P. Biernat(on-going)