Quantum Simulation of Scattering

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Introduction

- There is a growing interest in quantum technologies within the HEP community. For example, at CERN there is the **Quantum Technology Initiative** (quantum.cern).
- Studying real-time dynamics is notoriously hard in quantum systems.
- If properly developed, quantum computers would offer the possibility of making **efficient simulations of real-time dynamics**.
- While modern quantum computers are not enough to go beyond the limitations of classical computers, progress is being made.
- The aim of my work is to develop methods to simulate **scattering processes** relying on the gate model of quantum computation.

Gate model of quantum computation

- A qubit is a quantum system with Hilbert space $\mathbb{C}^2$:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

- A gate is a simple unitary operator acting on a small number of qubits (1,2 or 3 typically).
- Gates are combined to approximate complicated unitary operators acting on many qubits.
- Measurements are performed to read qubits, collapsing them on either $|0\rangle$ or $|1\rangle$.
- A quantum computer is a system of N qubits on which it is possible to perform gates and measurements in a programmable way

$$|00\dots0\rangle \underset{\text{gates}}{\longrightarrow} \sum_{j_1\dots j_N} \alpha_{j_1\dots j_N} |j_1\dots j_N\rangle \underset{\text{measurements}}{\longrightarrow} |01101\dots01\rangle$$

Quantum simulation

First of all, we need to map our quantum field theory to a qubit system:

- Spacetime $\to$ space lattice of $\mathcal{V}$ sites in the Hamiltonian formulation.
- Each lattice site has a (typically infinite) local Hilbert space. After truncation, k qubits are used to represent it.
- The total Hilbert space is a tensor product of the local Hilbert spaces. $k\mathcal{V}$ qubits are used in total.

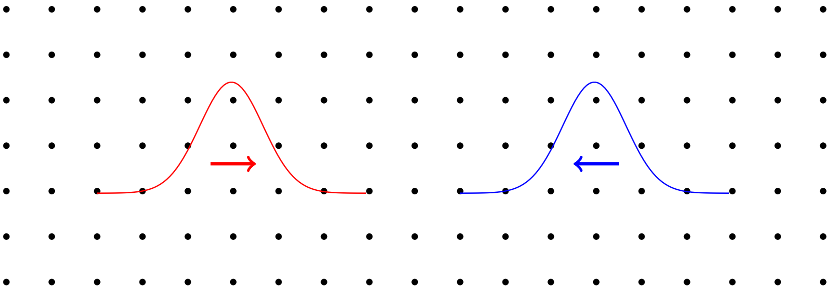
After that, a quantum simulation consists of

1. initial state preparation;
2. simulation of the time evolution $U = e^{-iHt}$;
3. measurements.

Initial state preparation

On the quantum computer, we want to prepare a state corresponding to two incoming wavepackets moving on top of the vacuum state

$$|00\ldots0\rangle \to |\alpha^{\text{in}}\rangle = \sum_{j_1\ldots j_N} \alpha^{\text{in}}_{j_1\ldots j_N} |j_1\ldots j_N\rangle$$



Haag-Ruelle scattering theory for creation of wavepackets

The initial state preparation is the most difficult part.

My idea, PRX Quantum 5, 020311 (2024), is to approach it using the Haag-Ruelle scattering theory.
Basically, we can create a wavepacket acting on the vacuum with the operator

$$\hat{a}^\dagger_\psi = \int d^4x \, \psi(x) \hat{O}(x),$$

where

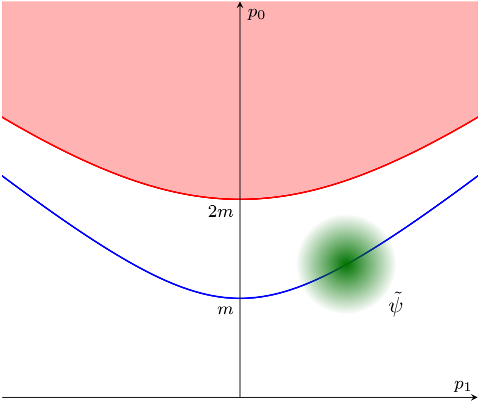
- ψ is a suitably chosen wavefunction (its Fourier transform has support only on the one-particle mass hyperboloid);
- $\hat{O}(x)$ is a local operator carrying the quantum numbers of the particle

$$\hat{O}(x) = e^{iHt} \hat{O}(\vec{x}) e^{-iHt}$$

Joint energy-momentum spectrum

Energy-momentum spectrum:

- one-particle mass hyperboloid $p^2 = m^2$;
- multi-particle continuum $p^2 \geq 4m^2$.



Thanks for listening!