Exploring the conformal bootstrap at higher points

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Quantum Field Theory

QFT has had great success. However, it is still reliant on perturbative calculations. These work well for weakly coupled theories like QED. This breaks down in strongly coupled theories like QCD.

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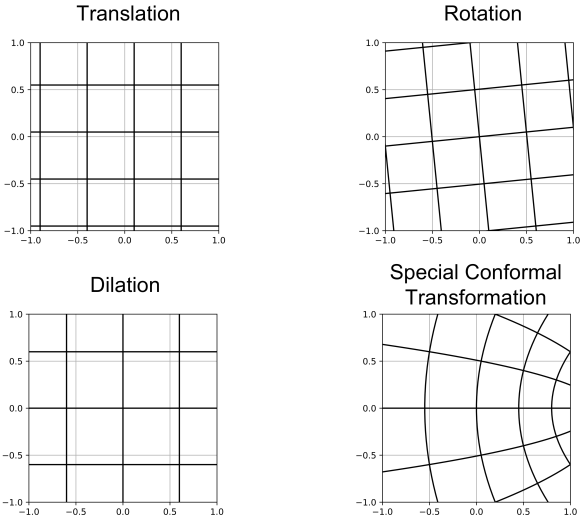
This signals the need for a nonperturbative approach. While general QFTs are still out of the reach of current methods, a subset known as Conformal Field Theories are simple enough that this is possible.

Conformal symmetry



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Conformal symmetry

Theories invariant under these transformations are called **conformal field theories (CFTs)**. In a CFT, the two-point function of scalar operators is fixed to be

$$\langle O_1 O_2 \rangle = \frac{\delta_{\Delta_1 \Delta_2}}{(x_{12}^2)^{\frac{\Delta_1 + \Delta_2}{2}}} \tag{1}$$

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while the three-point function is

$$\langle O_1 O_2 O_3 \rangle = \frac{c_{123}}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2-\Delta_3}{2}} (x_{13}^2)^{\frac{\Delta_1+\Delta_3-\Delta_2}{2}} (x_{23}^2)^{\frac{\Delta_2+\Delta_3-\Delta_1}{2}}} \tag{2}$$

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Remark

The CFT data $\{\Delta_i, c_{ijk}\}$ encodes all the information about a CFT.

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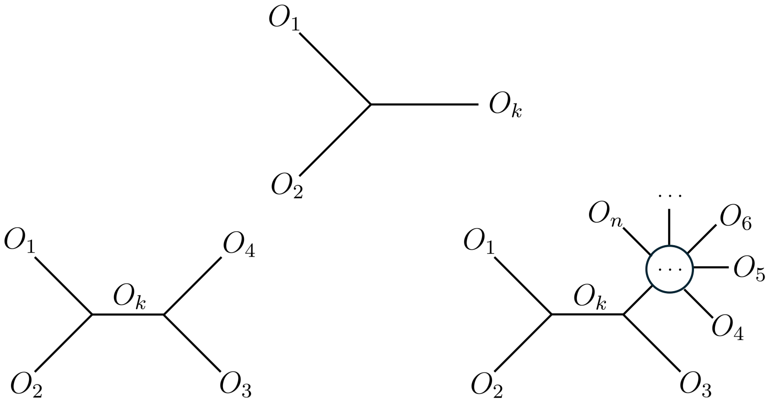
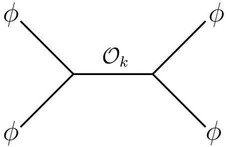


Bootstrap

In general it is easier to consider correlators of identical scalars



$$\sim \quad c^2_{\phi\phi k}$$

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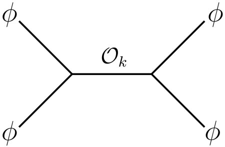

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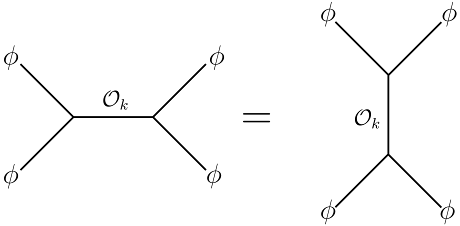
These functions satisfy the **bootstrap equations**



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Five-point functions contain information that isn't present in four-point functions. For example OPE coefficients of two spinning operators.

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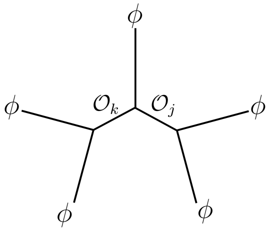

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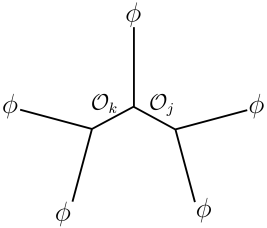

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Five-point functions also contain information about an infinite number of four-point functions through the exchanged operators.

Work plan

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Thank you for your time!