TAAD Project

Deciphering the Dark: A Machine Learning Approach to Detecting Dark Matter Influence in Neutron Star Observables

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What is a Compact Star?

Compact stars (CSs) are the final stage in the evolution of ordinary stars, they are formed when a star ceases its nuclear fuel. This star must have between 8-20 M_{\odot} to form a **neutron star (NS)**. Below this, we get a white dwarf.



Figure 1: Cassiopeia A (SNR) [Chandra, 2017]

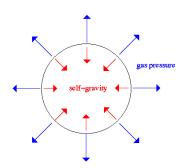


Figure 2: Ilustration of Hydrostatic Equilibrium

Neutron Star Composition

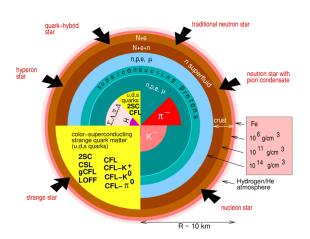
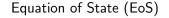


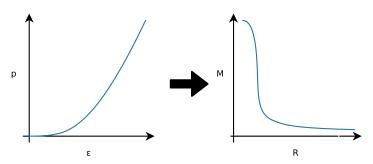
Figure 3: Internal composition of neutron stars (schematic) [Weber (2005)].

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How to model such objects?



Observable Mass-Radius



The **Tolman–Oppenheimer–Volkoff (TOV) equation** constrains the structure of a spherically symmetric body of isotropic material which is in static gravitational equilibrium [h = G = c = 1].

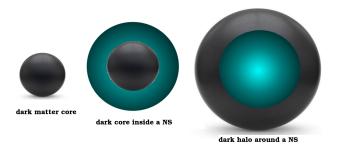
$$\frac{dp}{dr} = -\frac{(\varepsilon + p)(M + 4\pi r^3 p)}{r^2 (1 - 2M/r)}$$

$$M(r) = 4\pi \int_0^r \varepsilon(r') r'^2 dr' \qquad (1)$$

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Why DM in NSs?

- The proto-cloud may already present traces of DM
- DM could be accreted by the main sequence star
- In the supernova explosion, DM might be created and accrued inside the remnant
- DM is trapped in the gravitational field of a NS [Brito et al. (2015); Kouvaris and Tinyakov (2011)]



Initial Dataset

Our inital dataset have 17810 nucleonic (without DM) and 66002 with DM mass-radius- Λ (MR Λ) observables.

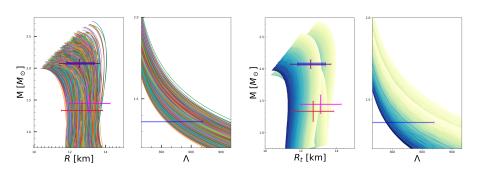


Figure 4: Nucleonic MRΛ

Figure 5: With DM MRΛ

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Generation of Datasets

From the intial data, it was randomly selected 17000 nucleonic and 17000 with DM MR Λ observables.

Training and Testing

Class Prediction

$$30k (15k+15k) \iff 34000 \text{ EoSs} \implies 4k (2k+2k)$$

The train and test and the class prediction datasets were constructed by extrating 5 random MRA points ([1, $M_{\rm max}$] M_{\odot}), in which

- It it doesn't have DM $\Longrightarrow y = 0$
- If it has $DM \Longrightarrow y = 1$

$$\mathbf{X_i} = [M_1, ..., M_5, R_1, ..., R_5, \Lambda_1, ..., \Lambda_5, y]$$
[Carvalho et al. (2023)]

Generation of Datasets

$$\begin{split} \boldsymbol{X_i} = [\textit{M}_1,...,\textit{M}_5,\textit{R}_1,...,\textit{R}_5,\Lambda_1,...,\Lambda_5,\textit{y}] \\ [1,\mathrm{M}_{\mathrm{max}}] \ \mathrm{M}_{\odot} \end{split}$$

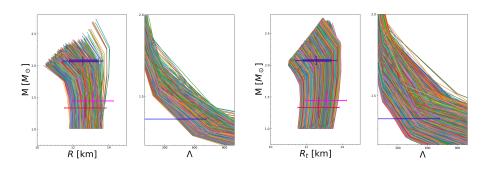


Figure 6: Nucleonic 5p MRA

Figure 7: With DM 5p MRA

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Train and Test Dataset

This dataset was mixed up randomly in order for the training to not be bias.

$$X_i = [M_1, ..., M_5, R_1, ..., R_5, \Lambda_1, ..., \Lambda_5, y]$$

M1	M2	M3	M4	M5	R(M1)	R(M2)		R(M5)	LAM(M1)	LAM(M2)	LAM(M3)	LAM(M4)	LAM(M5)	у
1.010	1.26125	1.5125	1.76375	2.015	12.708444	12.637214		11.297831	2953.802516	837.588252	279.737802	85.285676	15.304562	0
1.061	1.34375	1.6265	1.90925	2.192	12.160000	12.280000		11.540000	1510.689413	417.139150	140.828908	43.520143	7.424024	1
1.033	1.45850	1.8840	2.30950	2.735	13.510000	13.770000		12.950000	3500.271067	509.882566	132.665325	38.058792	4.380459	1
1.010	1.26875	1.5275	1.78625	2.045	12.597349	12.552588		11.068980	2698.392396	789.074739	253.127438	73.610589	11.171262	6
1.010	1.28000	1.5500	1.82000	2.090	12.269663	12.287386		11.096665	2508.137098	691.695553	210.606651	62.408135	10.344236	6
														٠.
1.003	1.33850	1.6740	2.00950	2.345	12.470000	12.630000		11.870000	2705.160896	509.000785	163.328537	41.585915	6.030242	1
1.008	1.40300	1.7980	2.19300	2.588	13.150000	13.320000		12.570000	3293.379720	762.882509	146.930261	37.987680	4.892461	3
1.010	1.28375	1.5575	1.83125	2.105	12.623463	12.553645		11.235397	2612.669839	715.349636	203.380552	60.038648	10.388661	6
1.050	1.39525	1.7405	2.08575	2.431	12.810000	12.980000		12.360000	2159.068434	501.875428	136.804005	41.145024	6.741356	1
								10.653555		702 004466	202 544722		7.478324	
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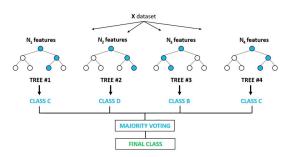
Random Forest: 80% (24k) Training and 20% (6k) Testing

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Random Forest Classifier

- A random forest is a <u>meta estimator</u>: fits a number of decision tree classifiers on various sub-samples of the dataset.
- A decision tree is a flowchart-like structure in which each internal node represents a "test" on an attribute of the dataset ⇒ It uses classification and regression tasks

Random Forest Classifier



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Train and Test Dataset - 30k

The accuracy obtained with the **default parameters**¹ was 99.967 %.

TRAINING AND	TESTING				
Accuracy: 0.9	996666666666	667			
Confusion Mat [[3024 1] [1 2974]]					
Classificatio	n Report: precision	recall	f1-score	support	
0	1.00	1.00	1.00	3025	
1	1.00	1.00	1.00	2975	
accuracy macro avg weighted avg	1.00 1.00	1.00 1.00	1.00 1.00 1.00	6000 6000 6000	

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¹max_depth: None, n_estimators: 100

Train and Test Dataset with Best Hyperparameters

The accuracy obtained **now** with the **the best hyperparameters**² was 99.983 %.

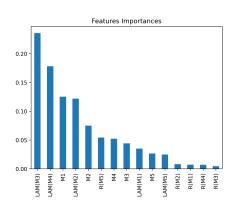
WITH BEST HYP	ER-PARAMETER	 S 			
Accuracy: 0.9	998333333333	334			
Confusion Mat [[3024 1] [0 2975]]	rix:				
Classificatio	n Report: precision	recall	f1-score	support	
0	1.00	1.00	1.00	3025	
1	1.00	1.00	1.00	2975	
accuracy			1.00	6000	
macro avg	1.00	1.00	1.00	6000	
weighted avg	1.00	1.00	1.00	6000	

Features Importances

 Provides a way to rank the features based on their contribution to the final prediction



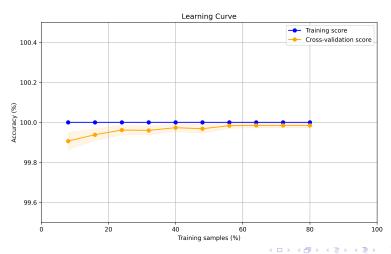
How effective each feature is at reducing uncertainty!



- LAM(M3): 0.2358
- LAM(M4): 0.1781
- M1: 0.1253
- LAM(M2): 0.1219
- M2: 0.0751
- R(M5): 0.054
 - D/M2
- R(M3): 0.004

Learning Curve

- train_scores_mean: [1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
- test_scores_mean: [0.99907 0.99938 0.99962 0.9996 0.99973 0.99968 0.99983 0.99985 0.99985 0.99985]



Validation Curve - max_depth

• Recall: max_depth = 19

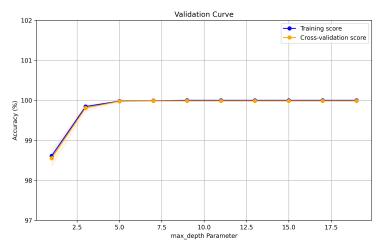


Figure 8: max_depth validation curve

Validation Curve - n_estimators

• Recall: n_estimators = 191

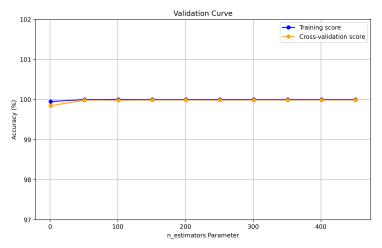


Figure 9: n_estimators validation curve

Class Prediction - 4k

	PREDIC	TION												
	M1	M2	МЗ	M4	M5	R(M1)	R(M2)	 R(M5)	LAM(M1)	LAM(M2)	LAM(M3)	LAM(M4)	LAM(M5)	
	1.019	1.36800	1.7170	2.06600	2.415	12.750000	12.910000	 12.200000	2441.437362	605.953993	139.691126	41.927249	6.186687	
	1.010	1.28750	1.5650	1.84250	2.120	12.660051	12.627100	 11.477929	2697.890201	754.234736	221.995297	69.347760	12.058428	
	1.010	1.32125	1.6325	1.94375	2.255	12.436340	12.586854	 11.723861	3089.608676	730.087386	202.672308	60.525458	9.598781	
	1.075	1.35775	1.6405	1.92325	2.206	12.210000	12.330000	 11.600000	1417.234883	421.121385	125.596857	42.481917	7.409802	
1	1.010	1.28750	1.5650	1.84250	2.120	12.732521	12.655193	 11.504638	2651.386146	733.846808	214.526027	66.840817	12.203105	,
3995	1.076	1.38275	1.6895	1.99625	2.303	12.380000	12.520000	 11.740000	1809.086838	410.739129	147.570263	41.600137	6.243576	,
3996	1.010	1.28000	1.5500	1.82000	2.090	12.469007	12.471304	 11.469575	2634.870681	733.923572	228.800539	70.934422	13.921236	,
3997	1.010	1.26125	1.5125	1.76375	2.015	12.522064	12.413655	 11.209168	2779.868050	757.056094	247.807923	75.563499	14.812697	
998	1.010	1.26500	1.5200	1.77500	2.030	12.743348	12.632446	 11.369355	2561.867507	749.933476	241.182025	76.163586	15.001370	į
3999	1.009	1.30600	1.6030	1.90000	2.197	12.130000	12.270000	 11.510000	1963.014456	474.881562	146.574704	46.905043	7.125357	٠.

Figure 10: Class Prediction dataset

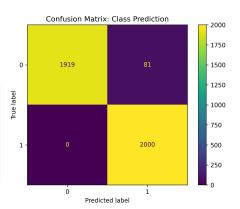
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Final Results

 The trained model was able to correctly predict 97.975 % of the MRΛ observables selected.

• False Positives: 2.025 %

Accuracy: 0.9 Classification		recall	f1-score	support
0 1	1.00 0.96	0.96 1.00	0.98 0.98	2000 2000
accuracy macro avg weighted avg	0.98 0.98	0.98 0.98	0.98 0.98 0.98	4000 4000 4000



Conclusion

- Using the Random Forest Classifier, the model got an accuracy of 97.975 %
- The "freedom" of the model allows for a fine-tune of the hyperparameters, resulting in a high accuracy and preventing over-fitting

Regarding the physics:

- The false positives and the matrix confusion allows to state that the points collected from the nucleonic observables mimic the behaviour of the DM observables
- From the features importances, it's clear that the main distinction between NSs with and without DM will be on the tidal deformability (Λ) behaviour, especilly at high mass configurations³
- The model had predicted all the DM observables correctly!

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