TAAD Project

Deciphering the Dark: A Machine Learning Approach to Detecting Dark Matter Influence in Neutron Star Observables

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What is a Compact Star?

Compact stars (CSs) are the final stage in the evolution of ordinary stars, they are formed when a star ceases its nuclear fuel. This star must have between **8-20 $M_\odot$** to form a **neutron star (NS)**. Below this, we get a white dwarf.

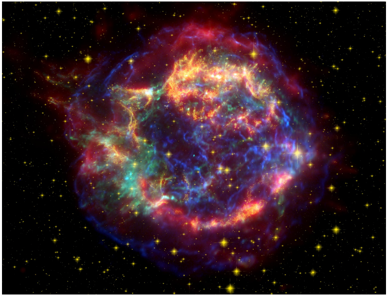


Figure 1: Cassiopeia A (SNR) [Chandra, 2017]

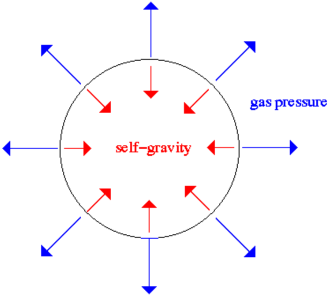


Figure 2: Ilustration of Hydrostatic Equilibrium

Neutron Star Composition

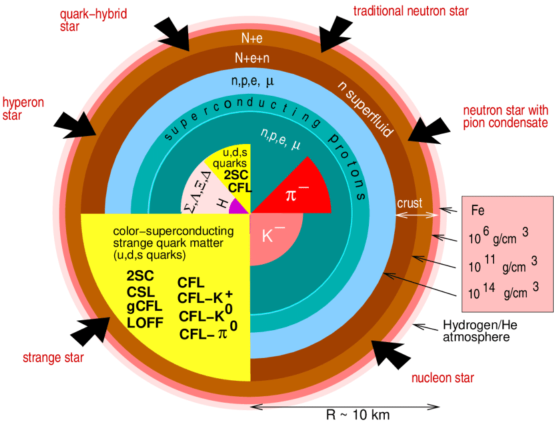
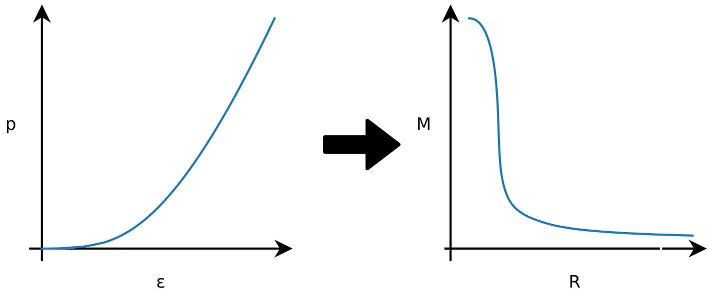


Figure 3: Internal composition of neutron stars (schematic) [Weber (2005)].

How to model such objects?

Equation of State (EoS) **Observable Mass-Radius**

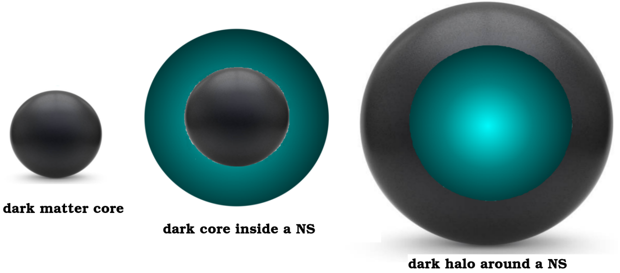


The **Tolman–Oppenheimer–Volkoff (TOV) equation** constrains the structure of a spherically symmetric body of isotropic material which is in static gravitational equilibrium $[h = G = c = 1]$.

$$\frac{dp}{dr} = -\frac{(\varepsilon + p)(M + 4\pi r^3 p)}{r^2\,(1 - 2M/r)} \qquad M(r) = 4\pi \int_0^r \varepsilon(r') r'^2 dr' \tag{1}$$

Why DM in NSs?

- The proto-cloud may already present traces of DM
- DM could be accreted by the main sequence star
- In the supernova explosion, DM might be created and accrued inside the remnant
- **DM is trapped in the gravitational field of a NS** [Brito et al. (2015); Kouvaris and Tinyakov (2011)]



Initial Dataset

Our inital dataset have **17810** nucleonic (without DM) and **66002** with DM mass-radius-Λ (MRΛ) observables.

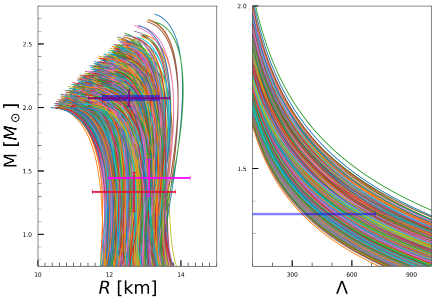


Figure 4: Nucleonic MRΛ

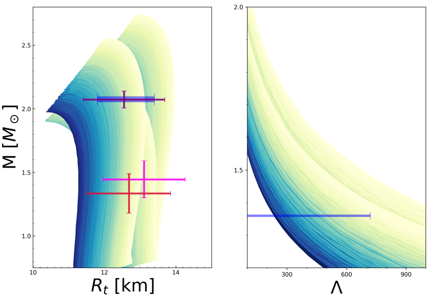


Figure 5: With DM MRΛ

Generation of Datasets

From the intial data, it was randomly selected **17000** nucleonic and **17000** with DM MRΛ observables.

Training and Testing | **Class Prediction**

$$\mathbf{30k}\ (15k + 15k) \Longleftarrow \mathbf{34000\ EoSs} \Longrightarrow 4k\ (2k + 2k)$$

The **train and test** and the **class prediction** datasets were constructed by extrating 5 random MRΛ points ($[1, \mathrm{M}_{\mathrm{max}}]\ \mathrm{M}_{\odot}$), in which

- It it **doesn't have DM** $\Longrightarrow y = 0$
- If it **has DM** $\Longrightarrow y = 1$

$$\mathbf{X_i} = [M_1, ..., M_5, R_1, ..., R_5, \Lambda_1, ..., \Lambda_5, y]$$

[Carvalho et al. (2023)]

Generation of Datasets

$$\mathbf{X_i} = [M_1, ..., M_5, R_1, ..., R_5, \Lambda_1, ..., \Lambda_5, y]$$

$$[1, \mathrm{M_{max}}]\ \mathrm{M_\odot}$$

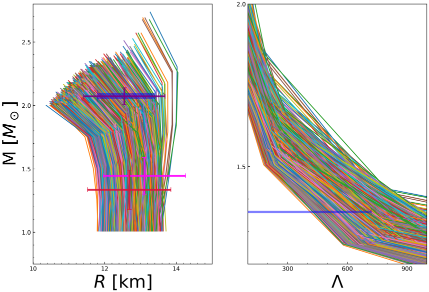


Figure 6: Nucleonic 5p MRΛ

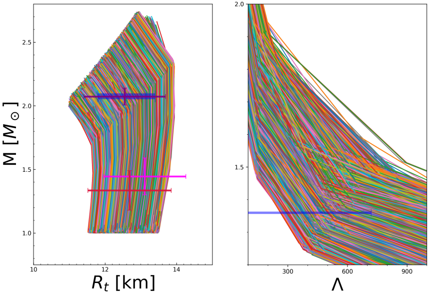


Figure 7: With DM 5p MRΛ

Train and Test Dataset

This dataset was mixed up randomly in order for the training to not be bias.

$$\mathbf{X_i} = [M_1, ..., M_5, R_1, ..., R_5, \Lambda_1, ..., \Lambda_5, y]$$

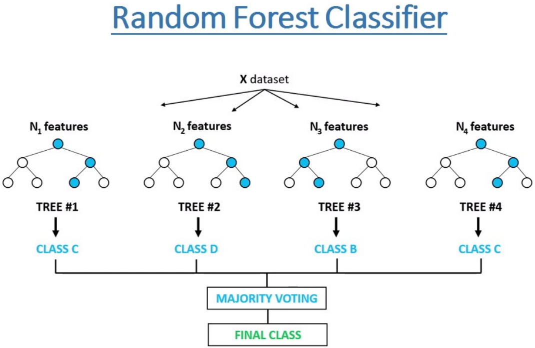
	M1	M2	M3	M4	M5	R(M1)	R(M2)	...	R(M5)	LAM(M1)	LAM(M2)	LAM(M3)	LAM(M4)	LAM(M5)	y
0	1.010	1.26125	1.5125	1.76375	2.015	12.708444	12.637214	...	11.297831	2953.802516	837.588252	279.737802	85.285676	15.304562	0
1	1.061	1.34375	1.6265	1.90925	2.192	12.160000	12.280000	...	11.540000	1510.689413	417.139150	140.828908	43.520143	7.424024	1
2	1.033	1.45850	1.8840	2.30950	2.735	13.510000	13.770000	...	12.950000	3500.271067	509.882566	132.665325	38.058792	4.380459	1
3	1.010	1.26875	1.5275	1.78625	2.045	12.597349	12.552588	...	11.068980	2698.392396	789.074739	253.127438	73.610589	11.171262	0
4	1.010	1.28000	1.5500	1.82000	2.090	12.269663	12.287386	...	11.096665	2508.137098	691.695553	210.606651	62.408135	10.344236	0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..
29995	1.003	1.33850	1.6740	2.00950	2.345	12.470000	12.630000	...	11.870000	2705.160896	509.000785	163.328537	41.585915	6.030242	1
29996	1.008	1.40300	1.7980	2.19300	2.588	13.150000	13.320000	...	12.570000	3293.379720	762.882509	146.930261	37.987680	4.892461	1
29997	1.010	1.28375	1.5575	1.83125	2.105	12.623463	12.553645	...	11.235397	2612.669839	715.349636	203.380552	60.038648	10.388661	0
29998	1.050	1.39525	1.7405	2.08575	2.431	12.810000	12.980000	...	12.360000	2159.068434	501.875428	136.804005	41.145024	6.741356	1
29999	1.010	1.27250	1.5350	1.79750	2.060	12.380717	12.301033	...	10.653555	2499.312324	702.881166	203.541722	56.764788	7.478324	0

[30000 rows x 16 columns]

Random Forest: 80% (24k) Training and 20% (6k) Testing

Random Forest Classifier

- A **random forest** is a meta estimator: fits a number of decision tree classifiers on various sub-samples of the dataset.
- A **decision tree** is a flowchart-like structure in which each internal node represents a "test" on an attribute of the dataset $\Longrightarrow$ **It uses classification and regression tasks**



Train and Test Dataset - 30k

The accuracy obtained with the **default parameters**[1] was 99.967 %.

```
-----------------------------------------------------
TRAINING AND TESTING
-----------------------------------------------------

Accuracy: 0.9996666666666667

Confusion Matrix:
[[3024    1]
 [   1 2974]]

Classification Report:
              precision    recall  f1-score   support

           0       1.00      1.00      1.00      3025
           1       1.00      1.00      1.00      2975

    accuracy                           1.00      6000
   macro avg       1.00      1.00      1.00      6000
weighted avg       1.00      1.00      1.00      6000
```

[1] max_depth: None, n_estimators: 100

Train and Test Dataset with Best Hyperparameters

The accuracy obtained **now** with the **the best hyperparameters**[2] was 99.983 %.

```
-----------------------------------------------------
WITH BEST HYPER-PARAMETERS
-----------------------------------------------------

Accuracy: 0.9998333333333334

Confusion Matrix:
[[3024    1]
 [   0 2975]]

Classification Report:
              precision    recall  f1-score   support

           0       1.00      1.00      1.00      3025
           1       1.00      1.00      1.00      2975

    accuracy                           1.00      6000
   macro avg       1.00      1.00      1.00      6000
weighted avg       1.00      1.00      1.00      6000
```

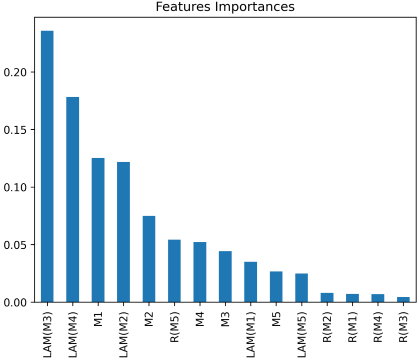
[2] max_depth: 19, n_estimators: 191

Features Importances

- Provides a way to rank the features based on **their contribution** to the final prediction

$\Downarrow$

How effective each feature is at reducing uncertainty!



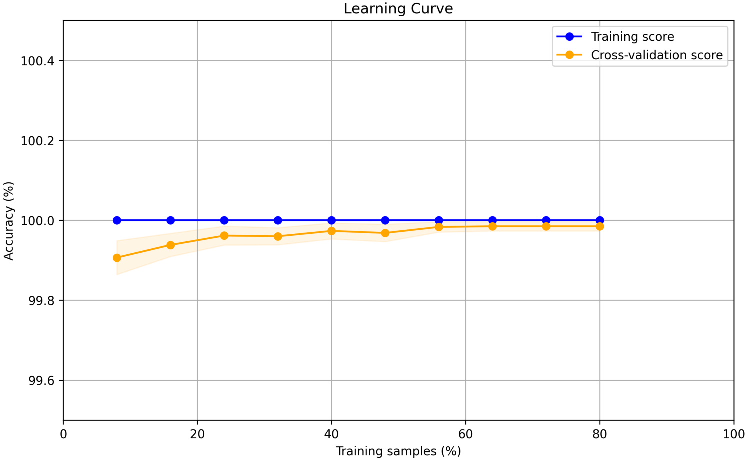
- **LAM(M3): 0.2358**
- **LAM(M4): 0.1781**
- **M1: 0.1253**
- LAM(M2): 0.1219
- M2: 0.0751
- **R(M5): 0.054**

  ...

- R(M3): 0.004

Learning Curve

- train_scores_mean: [1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
- test_scores_mean: [0.99907 0.99938 0.99962 0.9996 0.99973 0.99968 0.99983 0.99985 0.99985 0.99985]



Validation Curve - max_depth

- **Recall: max_depth = 19**

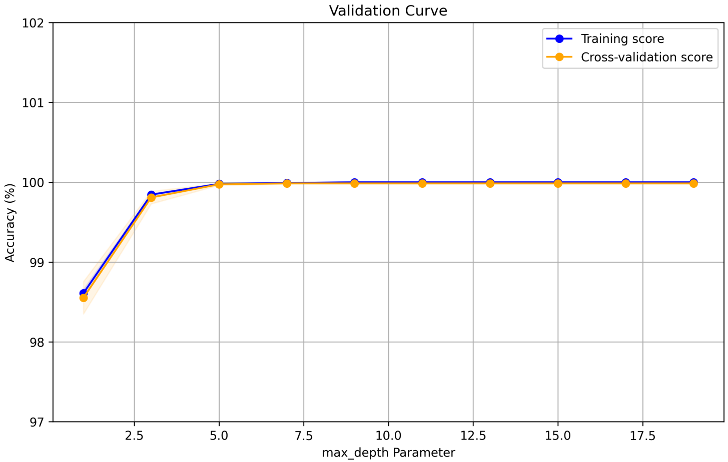


Figure 8: max_depth validation curve

Validation Curve - n_estimators

- **Recall: n_estimators = 191**

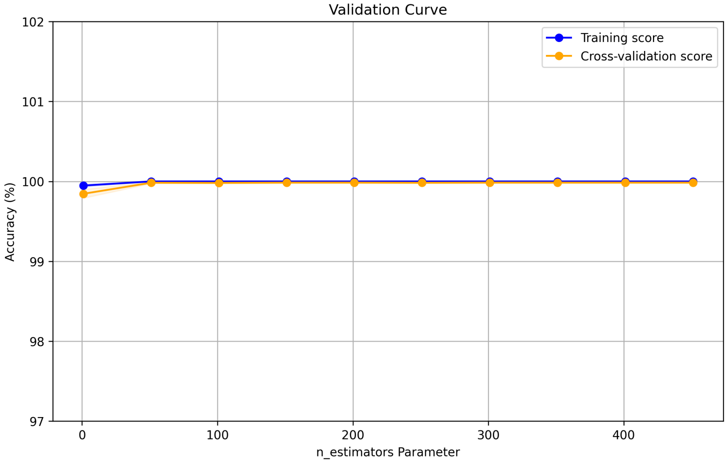


Figure 9: n_estimators validation curve

Class Prediction - 4k

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CLASS PREDICTION
--------------------------------------------------

         M1       M2      M3       M4     M5      R(M1)      R(M2)  ...      R(M5)      LAM(M1)     LAM(M2)     LAM(M3)    LAM(M4)    LAM(M5)  y
0     1.019  1.36800  1.7170  2.06600  2.415  12.750000  12.910000  ...  12.200000  2441.437362  605.953993  139.691126  41.927249   6.186687  1
1     1.010  1.28750  1.5650  1.84250  2.120  12.660051  12.627100  ...  11.477929  2697.890201  754.234736  221.995297  69.347760  12.058428  0
2     1.010  1.32125  1.6325  1.94375  2.255  12.436340  12.586854  ...  11.723861  3089.608676  730.087386  202.672308  60.525458   9.598781  0
3     1.075  1.35775  1.6405  1.92325  2.206  12.210000  12.330000  ...  11.600000  1417.234883  421.121385  125.596857  42.481917   7.409802  1
4     1.010  1.28750  1.5650  1.84250  2.120  12.732521  12.655193  ...  11.504638  2651.386146  733.846808  214.526027  66.840817  12.203105  0
...     ...      ...     ...      ...    ...        ...        ...  ...        ...          ...         ...         ...        ...        ... ..
3995  1.076  1.38275  1.6895  1.99625  2.303  12.380000  12.520000  ...  11.740000  1809.086838  410.739129  147.570263  41.600137   6.243576  1
3996  1.010  1.28000  1.5500  1.82000  2.090  12.469007  12.471304  ...  11.469575  2634.870681  733.923572  228.800539  70.934422  13.921236  0
3997  1.010  1.26125  1.5125  1.76375  2.015  12.522064  12.413655  ...  11.209168  2779.868050  757.056094  247.807923  75.563499  14.812697  0
3998  1.010  1.26500  1.5200  1.77500  2.030  12.743348  12.632446  ...  11.369355  2561.867507  749.933476  241.182025  76.163586  15.001370  0
3999  1.009  1.30600  1.6030  1.90000  2.197  12.130000  12.270000  ...  11.510000  1963.014456  474.881562  146.574704  46.905043   7.125357  1

[4000 rows x 16 columns]
```

Figure 10: Class Prediction dataset

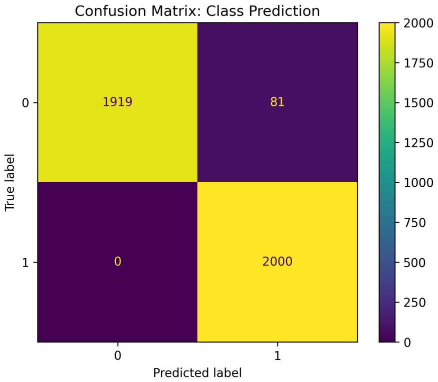
Final Results

- The trained model was able to correctly predict **97.975 %** of the MRΛ observables selected.

- False Positives: 2.025 %

Accuracy: 0.97975
Classification Report:

	precision	recall	f1-score	support
0	1.00	0.96	0.98	2000
1	0.96	1.00	0.98	2000
accuracy			0.98	4000
macro avg	0.98	0.98	0.98	4000
weighted avg	0.98	0.98	0.98	4000



Conclusion

- Using the Random Forest Classifier, the model got an accuracy of **97.975 %**
- The "freedom" of the model allows for a fine-tune of the hyperparameters, resulting in a **high accuracy** and **preventing over-fitting**

Regarding the physics:

- The false positives and the matrix confusion allows to state that the points collected from the nucleonic observables **mimic** the behaviour of the DM observables
- From the features importances, it's clear that the main distinction between NSs **with** and **without DM** will be on the **tidal deformability (Λ)** behaviour, especilly at high mass configurations[3]
- The model had predicted **all the DM observables** correctly!

[3] LAM(M3), LAM(M4)

References I

R. Brito, V. Cardoso, and H. Okawa. Accretion of dark matter by stars. *Physical Review Letters*, 115(11), sep 2015. doi: 10.1103/physrevlett.115.111301. URL `https://doi.org/10.11032Fphysrevlett.115.111301`.

V. Carvalho, M. Ferreira, T. Malik, and C. Providência. Decoding neutron star observations: Revealing composition through Bayesian neural networks. *Phys. Rev. D*, 108(4):043031, 2023. doi: 10.1103/PhysRevD.108.043031.

C. Kouvaris and P. Tinyakov. Constraining Asymmetric Dark Matter through observations of compact stars. *Phys. Rev. D*, 83:083512, 2011. doi: 10.1103/PhysRevD.83.083512.

F. Weber. Strange quark matter and compact stars. *Prog. Part. Nucl. Phys.*, 54:193–288, 2005. doi: 10.1016/j.ppnp.2004.07.001.