Little Higgs (in a nutshell)

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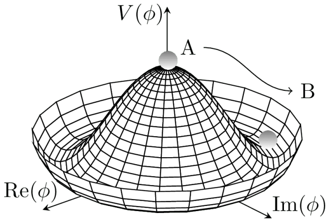
Contents

The Higgs mechanism

- Spontaneous breaking of the electroweak symmetry $SU(2)_L \times U(1)_Y$ leads to mass terms for the $W^\pm$, Z^0 and Higgs (H) bosons

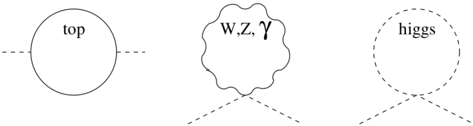
Masses of the $W^\pm$, Z^0 and H bosons

$$\begin{cases} m_W = \frac{g}{2}\, v \\ m_Z = \frac{\sqrt{g^2+g'^2}}{2}\, v \\ m_H = \sqrt{2\lambda}\, v \end{cases} \qquad (v \text{ is the VEV of the Higgs field}) \qquad (1)$$



The hierarchy problem

- We know from experiment that $m_W, m_Z, m_H \sim 100$ GeV (before the Higgs was discovered, this was already expected)
- The Standard Model (SM) is an effective theory, valid up to some cut-off scale $\Lambda \sim$ TeV
- Hence, m_H^2 receives one-loop radiative corrections proportional to Λ^2, i.e. $m_H^2 = m_{H,\text{bare}}^2 + \delta m_H^2$, with $\delta m_H^2 \sim \text{TeV}^2$



- Either $m_{H,\text{bare}}^2$ is fine-tuned ($\sim -\Lambda^2$) or a BSM theory removes the one-loop corrections (via the embedding of the SM in some symmetry group)

Supersymmetry as a solution

- Each SM particle has a *superpartner* of the opposite statistics

Some examples of superpartners

$$\text{top quark } (s = \tfrac{1}{2}) \longrightarrow \text{stop squark } (s = 0)$$
$$\text{W boson } (s = 1) \longrightarrow \text{Wino } (s = \tfrac{1}{2})$$
$$\text{Higgs boson } (s = 0) \longrightarrow \text{Higgsino } (s = \tfrac{1}{2})$$

- The SM one-loop corrections to m_H^2 are cancelled out exactly by their supersymmetric analogues $\Longrightarrow m_H$ remains small
- However, no superpartners have been detected so far – in particular, low-energy supersymmetric models ($m_{\text{SUSY}} \sim$ TeV) can be tested using the LHC

Little Higgs models

- The Higgs boson is a pseudo-Nambu-Goldstone boson (PNGB) corresponding to some spontaneously broken approximate global symmetry
- Loop corrections involving interactions that explicitly break the symmetry generate a potential, including a mass term, for the Higgs
- If the symmetry is *collectively* broken, a Λ^2-proportional one-loop contribution to the squared Higgs mass is *not* generated

$$\Downarrow$$

m_H remains small, even at loop level

Little Higgs models

- **Why approximate? Why pseudo?**
  - The breakdown of an *exact* global symmetry leads to *massless*, proper NGBs (Goldstone's theorem), which only have derivative interactions
  - Introduce terms that *explicitly* break the symmetry (e.g. gauge and Yukawa interactions)
  - No mass term for the Higgs at tree level: it arises from one-loop corrections, which make it a PNGB with a small mass
- **Why collective?**
  - Later...

Little Higgs models: the simplest little Higgs

- Let us consider an *exact* $SU(3)$ globally invariant theory with a complex scalar triplet ϕ. This symmetry is spontaneously broken down to a global $SU(2)$, which produces $8 - 3 = 5$ NGBs

Ground state parametrization of ϕ and of the NGBs

$$\phi = e^{\frac{i}{f}\pi}\phi_0 \tag{2}$$

$$\pi = \left(\begin{array}{cc|c} -\eta/2 & 0 & \multirow{2}{*}{h} \\ 0 & -\eta/2 & \\ \hline \multicolumn{2}{c|}{h^\dagger} & \eta \end{array}\right) \qquad \phi_0 = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \tag{3}$$

- $f \sim$ TeV is the VEV of ϕ and h is an $SU(2)$ complex doublet

Little Higgs models: the simplest little Higgs

- The most general $SU(3)$-invariant Lagrangian that we can write for this field theory is then

Lagrangians for ϕ and for h

$$\mathcal{L} = \text{const.} + f^2|\partial_\mu \phi|^2 + \mathcal{O}(\partial^4) \approx \\ \approx \text{const.} + |\partial_\mu h|^2 + \frac{1}{f^2}|\partial_\mu h|^2 h^\dagger h + \ldots \tag{4}$$

- Since the $SU(3)$ symmetry is exact, we only have derivative interactions for h, which is then a strictly massless NGB $\longrightarrow$ doesn't work: Higgs is massive and has other types of interactions

Little Higgs models: the simplest little Higgs

- Let us then introduce $SU(2)$ gauge interactions for h, which explicitly break the global $SU(3)$ symmetry of $\mathcal{L}$

First attempt

Simply replace $\partial_\mu \longrightarrow D_\mu = \partial_\mu - igW_\mu^a(x)Q^a$ in $\mathcal{L}$, with $Q^a \in \mathfrak{su}(2)$, leading to

$$\mathcal{L} \quad \supset \quad |\partial_\mu h|^2 \, , \; |gW_\mu h|^2 \, , \; \ldots \tag{5}$$

- The additional interaction terms lead to one-loop diagrams that generate a mass term for h

Mass term from gauge one-loop diagrams

$$\mathcal{L} \quad \supset \quad \frac{g^2}{16\pi^2}\Lambda^2 \, h^\dagger h \quad \Longrightarrow \quad m_h^2 \propto \Lambda^2 \qquad \textbf{Failed!} \tag{6}$$

Little Higgs models: the simplest little Higgs

- Let us try to gauge the full $SU(3)$ symmetry instead

Second attempt

Replace $\partial_\mu \longrightarrow D_\mu = \partial_\mu - igW^a_\mu(x)Q^a$, but now with $Q^a \in \mathfrak{su}(3)$

- This leads to the following one-loop contribution

One-loop contribution to $\mathcal{L}$

$$\mathcal{L} \supset \frac{g^2}{16\pi^2}\Lambda^2\phi^\dagger\phi = \frac{g^2}{16\pi^2}\Lambda^2 f^2 = \text{const.} \tag{7}$$

- Hence, <u>no mass term</u> for h is generated by one-loop diagrams and all the NGBs are "eaten" by the five $SU(3)$ gauge bosons that have now become massive

Failed!

Little Higgs models: the simplest little Higgs

- The way to do it is via the introduction of *two* independent ϕ fields and hence *two* sets of NGBs (total of 10 NGBs)

Parametrization of two independent ϕ fields and NGBs

$$\begin{cases} \phi_1 = e^{\frac{i}{f}\pi_1} \begin{pmatrix} \\ f \end{pmatrix} \\ \phi_2 = e^{\frac{i}{f}\pi_2} \begin{pmatrix} \\ f \end{pmatrix} \end{cases} \Longleftrightarrow \begin{cases} \phi_1 = e^{\frac{i}{f}\kappa} e^{\frac{i}{f}\eta} \begin{pmatrix} \\ f \end{pmatrix} \\ \phi_2 = e^{\frac{i}{f}\kappa} e^{-\frac{i}{f}\eta} \begin{pmatrix} \\ f \end{pmatrix} \end{cases} \tag{8}$$

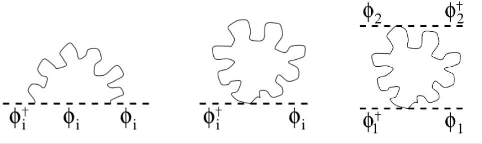
- We now gauge a single $SU(3)$ symmetry to get

Lagrangian for ϕ_1 and ϕ_2

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 \qquad (g_1 = g_2 \equiv g) \tag{9}$$

Little Higgs models: the simplest little Higgs

- We find the following set of one-loop diagrams



- The first two lead to the constant contributions found in the second attempt, whereas the third leads to

One-loop contribution involving both ϕ_1 and ϕ_2

$$\mathcal{L} \supset \frac{g^4}{16\pi^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) |\phi_1^\dagger \phi_2|^2 \qquad (10)$$

Little Higgs models: the simplest little Higgs

- The global phase factor $e^{\frac{i}{f}\kappa}$ corresponds to the "eaten" NGBs and can be removed by an $SU(3)$ gauge transformation. The relative phases $e^{\pm\frac{i}{f}\eta}$ are always kept, leading in particular to a mass term for h

Mass term for h

$$\mathcal{L} \supset -\frac{g^4}{16\pi^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) f^2\, h^\dagger h \qquad (11)$$

- For $f \sim$ TeV and $\Lambda \sim \mu \sim$ TeV, we find $m_h \sim M_{\text{weak}} \sim 100$ GeV

Success!

Little Higgs models: the simplest little Higgs

- **Note!** This was only possible because we took $g_1 = g_2 \equiv g$ in the $SU(3)$ gauge covariant derivatives:
  - This ensures that there is a single $SU(3)$ symmetry in $\mathcal{L}$, rather than two independent ones (one for each ϕ) – the latter case would reduce to our second attempt
  - That is, the breakdown of the $SU(3)$ symmetry must be done *collectively*, rather than independently

Little Higgs models: other models

- A more realistic model should feature Yukawa couplings, hypercharge and colour, as well as a Higgs potential with a quartic coupling
- Moreover, little Higgs models can be based on other symmetry breaking patterns, among which are $SU(3)_L \times SU(3)_R \longrightarrow SU(3)_V$ (Minimal Moose) and $SU(5) \longrightarrow SO(5)$ (Littlest Higgs)

Little Higgs models: experimental tests

- The spectrum of new particles varies between models
- However, all of them predict at least one vector-like quark at the TeV scale, as well as extra gauge bosons (both vector and scalar)
- In particular, heavy $SU(2)$ gauge bosons ($m_{SU(2)} \gtrsim 2$ TeV) are predicted to be produced in pp collisions at the LHC, mostly via $q\bar{q}$ annihilation
- So far, no experimental evidence for a little Higgs has been discovered

Summary

- It is possible to find a naturally small mass term for the Higgs boson by modelling it as a pseudo-Nambu-Goldstone boson arising from the collective breakdown of an approximate global symmetry, thus solving the hierarchy problem
- However, no experimental evidence has yet been found for little Higgs models

References

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