Effective Field Theory

High Energy Physics

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Table of Contents

Starting point

"Every theory is an effective field theory"

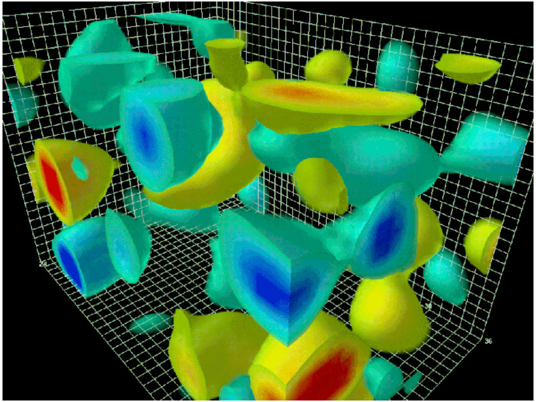

Figure: QFT visualization

"Every theory is an effective field theory"

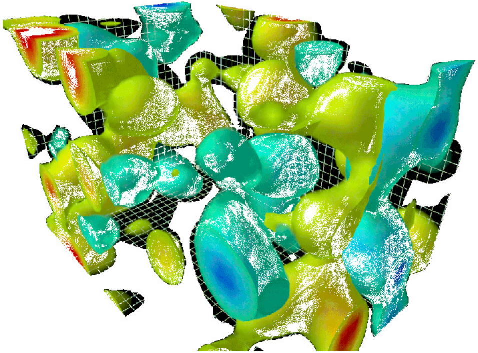

Figure: QFT visualization of low-energy effects

In the low energy limit of any theory, the emphasis is on the dominant low-energy effects!

The sigma model

'Effective' Lagrangian

The purpose of an effective field theory is to represent (in a simple way) the dynamical content of a theory in the low-energy limit. The linear sigma model provides a 'user-friendly' introduction to EFT.

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \pi \cdot \partial^\mu \pi - g\bar{\psi}(\sigma - i\tau \cdot \pi\gamma_5)\psi + \frac{1}{2}\mu^2(\sigma^2 + \pi^2) - \frac{\lambda}{4}(\sigma^2 + \pi^2)^2 \tag{1}$$

Alternately, when working at low-energy (E $\ll \mu^2$), all matrix elements of pions are contained in a rather different 'effective lagrangian'

$$\mathcal{L}_{eff} = \frac{F^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger), \qquad U = \exp i\tau \cdot \pi / F, \tag{2}$$

Representations of the sigma model

Linear Sigma model

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - g\nu) + \frac{1}{2}[\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma} - 2\mu^2\tilde{\sigma}^2] + \frac{1}{2}\partial_\mu\pi\cdot\partial^\mu\pi$$
$$- g\bar{\psi}(\tilde{\sigma} - i\tau\cdot\pi\gamma_5)\psi - \lambda\nu\tilde{\sigma}(\tilde{\sigma}^2 + \pi^2) - \frac{\lambda}{4}(\tilde{\sigma}^2 + \pi^2)^2 \quad (3)$$

After symmetry breaking and the redefinition of the σ field,

$$\sigma = \nu + \tilde{\sigma}, \qquad \nu = \sqrt{\frac{\mu^2}{\lambda}}, \quad (4)$$

Representations of the sigma model

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EFT

Square-root Sigma model

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}[(\partial_\mu S)^2 - 2\mu^2 S^2] + \frac{1}{2}(\frac{\nu + S}{\nu})^2[(\partial_\mu \phi)^2 + \frac{(\phi \cdot \partial_\mu \phi)^2}{\nu^2 - \phi^2}] \\ &- \lambda \nu S^3 - \frac{\lambda}{4} S^4 - \bar{\psi} i \partial\!\!\!/ \psi - g(\frac{\nu + S}{\nu})\bar{\psi}[(\nu^2 - \phi^2)^{1/2} - i\phi \cdot \tau \gamma_5]\psi\end{aligned} \tag{5}$$

After symmetry breaking and the definition of the ϕ and S field,

$$S \equiv \sqrt{(\tilde{\sigma} + \nu)^2 + \pi^2} - \nu = \tilde{\sigma} + \ldots, \qquad \phi \equiv \frac{\nu \pi}{\sqrt{(\tilde{\sigma} + \nu)^2 + \pi^2}} = \pi + \ldots, \tag{6}$$

Representations of the sigma model

Exponential Sigma model

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu S)^2 - 2\mu^2 S^2] + \frac{(\nu + S)^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger)^2 \\ - \lambda \nu S^3 - \frac{\lambda}{4} S^4 - \bar{\psi} i \partial\!\!\!/ \psi - g(\nu + S)(\bar{\psi}_L U \psi_R + \bar{\psi}_R U^\dagger \psi_L) \tag{7}$$

We redefine the fields as,

$$\sum = \sigma + i\tau \cdot \pi = (\nu + S)U, \qquad U = exp(i\tau \cdot \pi'/\nu) \tag{8}$$

General framework techniques

Framework

1. Representation independence
2. Integrating out heavy fields
3. Matching the model

(...) and all this will be carried out at tree level.

$\pi^+\pi^0 \to \pi^+\pi^0$ scattering.

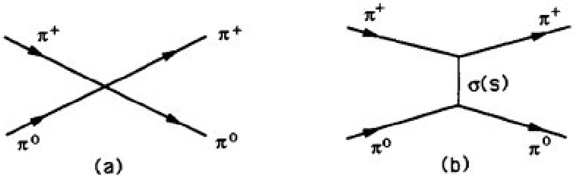
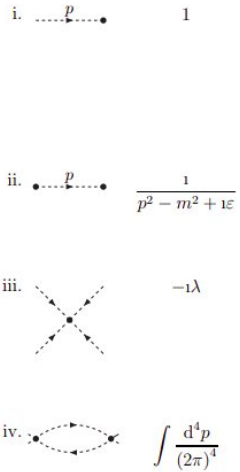


Figure: Tree level diagrams of contributions to the scattering of the Goldstone bosons, specifically $\pi^+\pi^0$.

$\pi^+\pi^0 \to \pi^+\pi^0$ scattering.

From the interaction Lagrangian and the Feynman rules we get **tree-level scattering amplitudes in every representation**

$$i\mathcal{M} = i\frac{q^2}{\nu^2} + \ldots \qquad (9)$$



Representation Independence

All three representations give the same answer despite very different forms and diagrams. A similar conclusion would follow for any other observable that one might want to calculate.

Integrating out heavy fields

When using effective low-energy theory, one does not include the heavy fields in the lagrangian, but their virtual effects are represented by various couplings between light fields.

Decoupling theorem

If the remaining low-energy theory is renormalizable, then all effects of the heavy-particle appear either as renormalization of the coupling constant or else are suppressed by powers of heavy-particle mass [1].

$$W_{eff}[J] = \int d^4x \frac{1}{2m_H^2} J(x)J(x) + \dots \tag{10}$$

where the heavy-particle propagator peaked at small distances, of order $1/m_H^2$

[1] e.g, $W^{\pm}$ exchange amplitudes where $G_F \propto W^{\pm}$

Matching the sigma model

Matching at tree level

All we have left is a 'non-renormalizable' nonlinear lagrangian that's also incomplete because loop diagrams have not been included yet. For sure loops for a 'non-renormalizable' theory would cause trouble. Or would they?

This situation helps demonstrate the effectiveness of EFT - showing how loop processes are reproduced in a much simpler manner.

Matching at tree level

What we learned:

- Integrating out heavy fields

$$\mathcal{L}_{eff} = \frac{\nu^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger)^2 + \frac{\nu^2}{8m_S}[Tr(\partial_\mu U \partial^\mu U^\dagger)]^2 + \dots \quad (11)$$

- Accomplishing renormalization and matching (for loops)

$$\begin{aligned}\mathcal{L}_{eff} =& \frac{\nu^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger)^2 \\ &+ \alpha_1 [Tr(\partial_\mu U \partial^\mu U^\dagger)]^2 + \alpha_2 Tr(\partial_\mu U \partial_\nu U^\dagger) Tr(\partial^\mu U \partial^\nu U^\dagger)\end{aligned} \quad (12)$$

$\pi^+\pi^0 \to \pi^+\pi^0$ to one loop.

The full linear sigma model is renormalizable and yields finite predictions in terms of the (renormalizable) parameters. The effective theory has been constructed with the same vertices at lowest energies, but will have quite different high-energy properties (due to missing to the missing degrees of freedom). New divergences will rise but the low-energy effects will be similar in both calculations.

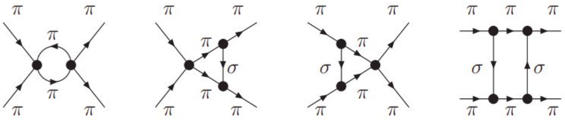


Figure: Subset of one-loop diagrams contributing to $\pi^+\pi^0$ elastic scattering.

$\pi^+\pi^0 \to \pi^+\pi^0$ to one loop.

Considering the set of diagrams depicted in 5, we can write the combined amplitudes as

$$i\mathcal{M}_{full} = \int \frac{d^4k}{(2\pi)^4}[-2i\lambda + (-2i\lambda)^2 \frac{i}{(k^2+p_+)^2 - m_\sigma^2}] \frac{i}{(k^2+p_+ + p_0)^2} \frac{i}{k^2} \times [-2i\lambda + (-2i\lambda)^2 \frac{i}{(k^2+p'_+)^2 - m_\sigma^2}] \tag{13}$$

For EFT, we apply the low-energy limit of the vertex

$$i\mathcal{M}_{eff} = \int \frac{d^4k}{(2\pi)^4} \frac{i(k^2+p_+)^2}{\nu^2} \frac{i}{(k^2+p_+ + p_0)^2} \frac{i}{(k^2)} \frac{i(k^2+p'_+)^2}{\nu^2} \tag{14}$$

$\pi^+\pi^0 \to \pi^+\pi^0$ to one loop.

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$$i\mathcal{M}_{eff} = \frac{i}{96\pi^2\nu^4} s(s-u)[\frac{2}{4-d} - \gamma + ln4\pi - ln\frac{-s-i\epsilon}{\mu^2}] + \frac{i}{288\pi^2\nu^4}[-s^2 - 5su], \quad (15)$$

This diagram has a different divergence than the full one and kinematically simpler[2].

[2] $s = (p_+ + p_0)^2$, $t = (p_+ - p'_+)^2$, $u = (p_0 - p'_+)^2$

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Chiral Perturbation Theory to One Loop*

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$\pi^+\pi^0 \to \pi^+\pi^0$ to one loop.

The comparison of the full theory and the effective theory can be carried out directly for this reaction

$$\alpha_1^r = \underbrace{\frac{\nu^2}{8m_\sigma^2}}_{\alpha_1} + \frac{1}{192\pi^2}[\ln\frac{m_\sigma^2}{\nu^2} - \frac{35}{6}]$$
$$\alpha_2^r = \frac{1}{384\pi^2}[\ln\frac{m_\sigma^2}{\nu^2} - \frac{11}{6}] \qquad (16)$$

$\pi^+\pi^0 \to \pi^+\pi^0$ to one loop.

From the paper, it is seen that

- more precise matching
- kinematic dependence in the scattering amplitudes
- full theory is produced with only light DOF
- this holds for all observables

General features of EFT

Is any of this reliable?

- Intermediate states contain all energies
- More particles and interactions?
- Most often coefficients have to be measured
- Effects from high physics contained in the parameters
- All theories are EFT

Effective lagrangians and symmetries

Focus: Generation of candidate interactions that are invariant under *SU(2)* chiral transformations. The effect of an n-derivative vertex is of order E^n/M^{n-4}.
The general lagrangian can be organized by dimensionality of the operators,

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \mathcal{L}_8 + \ldots \\ =& \frac{F^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger) + \alpha_1 [Tr(\partial_\mu U \partial^\mu U^\dagger)]^2 \\ &+ \alpha_2 Tr(\partial_\mu U \partial_\nu U^\dagger) \cdot Tr(\partial^\mu U \partial^\nu U^\dagger) + \ldots \end{aligned} \tag{17}$$

Power counting

In the sigma model, all the lower-energy couplings of the pions are proportional $1/\nu^2 \propto 1/m_\sigma^2$; and the low-energy expansion gives us, up to one loop

$$\mathcal{M}^{(loop)}_{\pi^+\pi^0\rightarrow\pi^+\pi^0} \equiv \frac{1}{\nu^4} I(p_+, p_0, p_+^{'}) \tag{18}$$

Due to the dimensional analysis of this framework, we can define *power counting*[3]

$$\mathcal{A} \sim (\frac{p}{\Lambda})^{n-4} \tag{19}$$

where n is the dimension number - or n-derivative number.

[3] It's just a matter of notation, but it has the same meaning.

The limits of EFT

The EFT of linear sigma model is valid for energies well below the mass of the scalar particle, σ or S. Once there is enough energy to directly excite the S particle, EF is not adequate. Scattering matrix elements are an expansion in the energy

$$\boxed{\mathcal{M} \sim \frac{q^2}{\nu^2}[1 + \frac{q^2}{m_\sigma^2} + ...]} \tag{20}$$

and the scale of the energy dependence is determined largely by the scalar mass. Increasing energy results in the corrections to the lowest-order grow *making all terms equally important* and EF breaks down.

New Physics?

EFT as probes of New Physics

Trying to represent the low-energy effects from the 'heavy' sector of the theory.

$$\mathcal{L}_{eff} = \sum_n C_n O_n, \tag{21}$$

The lagrangian itself has mass dimension 4, so for an operator $\{O\}$ has dimension d_n the coefficient must have mass dimension

$$C_n \sim M^{4-d_n}, \tag{22}$$

Operators of higher dimensions will be suppressed by powers of the heavy mass

EFT as probes of New Physics

Dimensional analysis supplies an estimate for the magnitude of the energy scales of possible New Physics.
Why do we call it 'New Physics'?

$$\mathcal{L}_{eff} = \underbrace{\mathcal{L}_{SM}}_{d_n \leq 4} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 \tag{23}$$

On some scales there are possible violations of some of the symmetries of the Standard Model.

Outlook

As we delve further into the mysteries of the subatomic world, the Outlook of Effective Field Theory presents an exciting path forward for particle physics research. By leveraging the power of EFT, we can extend our knowledge, explore uncharted territories, and ultimately uncover new fundamental principles that govern the cosmos.