LOW-*x* PROJECTILE DENSITY CONTRIBUTIONS IN THE DILUTE-DENSE CGC FRAMEWORK FOR TWO-PARTICLE CORRELATIONS

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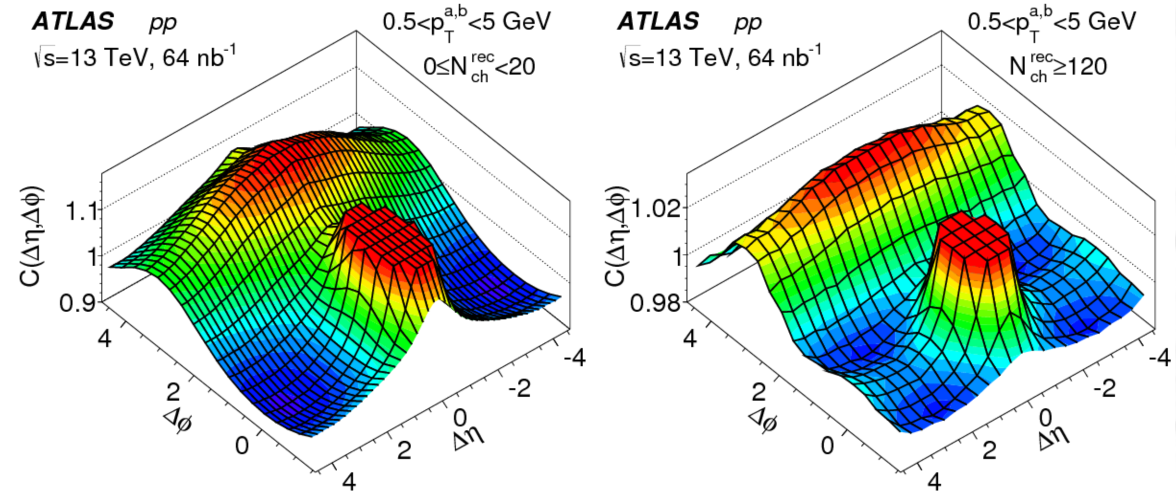
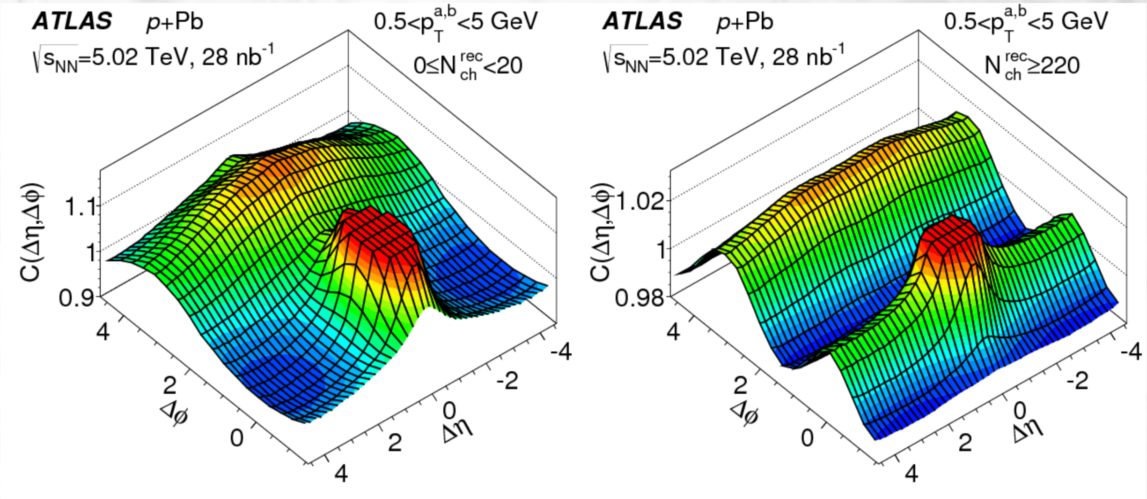
[based on arXiv:2303.08711, in collaboration with Cyrille Marquet and Anderson Kendi]

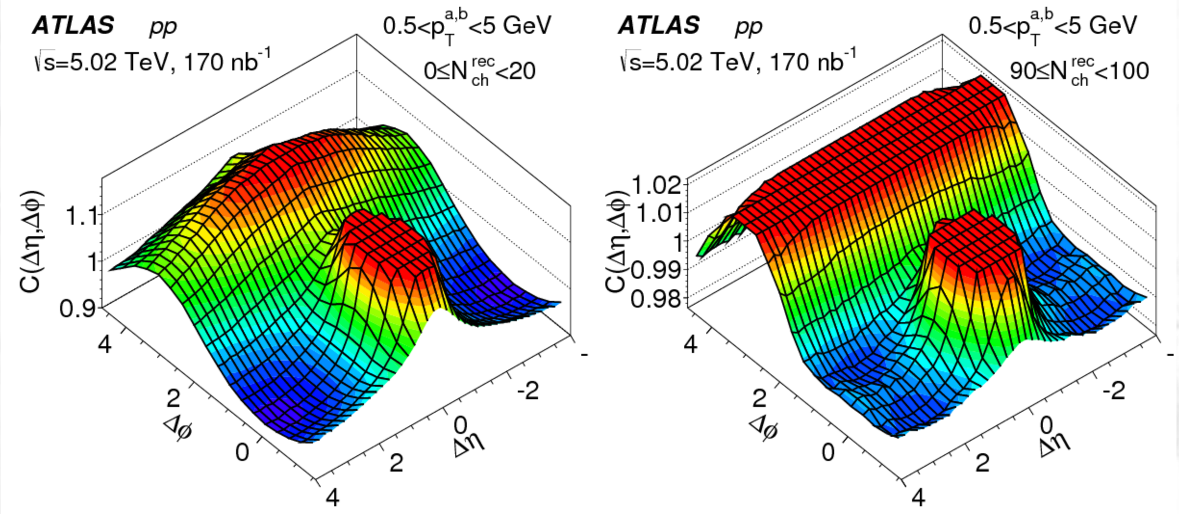
LIP — Lisbon Seminar, November 2

Motivation | The ridge





ATLAS Collaboration, "Measurements of long-range azimuthal anisotropies and associated Fourier coefficients for *pp* collisions at $s^{1/2}$=5.02 and *13 TeV* and *p+Pb* collisions at $s_{NN}^{1/2}$=5.02 *TeV* with the ATLAS detector", ATLAS-CONF-2016-026 [arXiv:1609.06213 [nucl-ex]].



- **One of the main unveilings of the heavy-ion programs at RHIC and the LHC.**
- **First noticed in relativistic heavy-ion data from RHIC.**
- **An incentive to seek the origin of particle correlations.**

MECHANISMS behind the RIDGE in *pp* and *pA* collisions:

- **PARTICLE CORRELATIONS: ROOTS in the VERY EARLY STAGES of the COLLISION.**
- **GLASMA GRAPH APPROXIMATION — DILUTE-DILUTE LIMIT of the CGC framework:**
  - **EXTREMELY EFFECTIVE.**
  - **ANISOTROPIC DISTRIBUTION of particles MOTIVATES decomposition in FOURIER HARMONICS with the corresponding COEFFICIENTS v_n.**
- **ALTERNATIVE MECHANISM: HYDRODYNAMICAL EVOLUTION logic consistent with data.**

[argument taken from AA collisions]

CHALLENGE: $\boxed{\sigma(k,p) = \sigma(-k,p)}$

ACCIDENTAL SYMMETRY OF THE CGC → $v_{\mathrm{odd}}=0$

- SURFACING ODD AZIMUTHAL HARMONICS:
  - BEYOND THE PROJECTILE AND TARGET MV AVERAGES.
  - LOW PROJECTILE DENSITY CONTRIBUTIONS.

→ GOAL OF THIS WORK

ρ_P → PROJECTILE CHARGE DENSITY

ρ_T → TARGET CHARGE DENSITY

DENSE HADRON OR NUCLEUS: $\rho \sim 1/g_s^2$

DILUTE HADRON OR NUCLEUS: $\rho \sim 1$

THE DILUTE PROJECTILE LIMIT OF THE DENSE-DENSE RESULT DOES NOT YIELD THE CORRECT DILUTE-DENSE ANSWER, BUT IT HAS BEEN WIDELY USED IN THE CASE OF TWO-GLUON PRODUCTION

DILUTE-DENSE LIMIT OF THE DENSE-DENSE RESULT: $g_s^4 \rho_P^4$ **¡INCOMPLETE!**

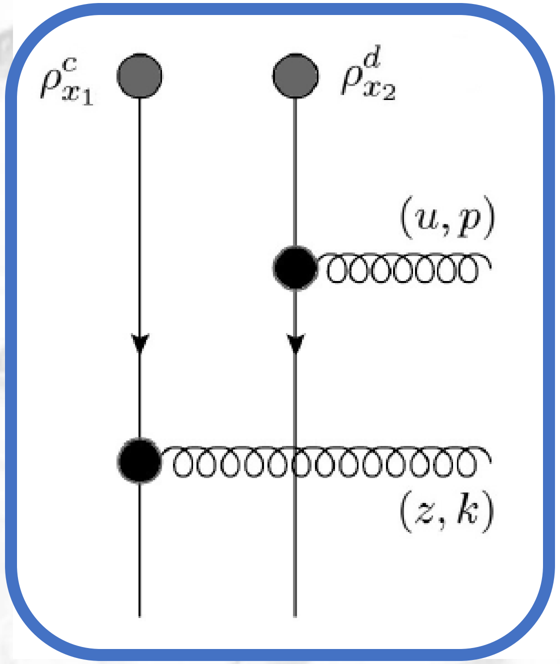
FULL DILUTE-DENSE RESULT: $g_s^4 \rho_P^4 \quad \boxed{g_s^4 \rho_P^3 \quad g_s^4 \rho_P^2}$ **ODD HARMONICS?**

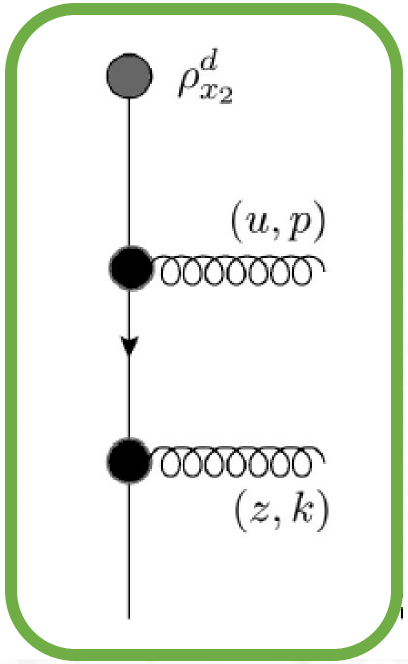
$$A^{ab}_{ij}(k,p) = \int_{u,z} e^{ik\cdot z + ip\cdot u}\Bigg\{\int_{x_1,x_2}\Big\{f_i(z-x_1)[S_z - S_{x_1}]^{ac}\rho^c_{x_1}\Big\}\Big\{f_j(u-x_2)[S_u - S_{x_2}]^{bd}\rho^d_{x_2}\Big\}$$

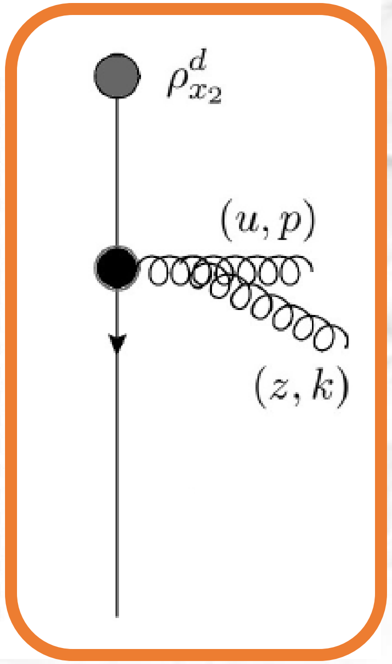
$$-\frac{1}{2}\int_{x_1} f_i(z-x_1)f_j(u-x_1)\Big\{[S_z - S_{x_1}]\bar{\rho}_{x_1}[S^\dagger_u + S^\dagger_{x_1}]\Big\}^{ab}$$

$$+\int_{x_1} f_i(z-u)f_j(u-x_1)\Big\{[S_z - S_u]\bar{\rho}_{x_1}S^\dagger_u\Big\}^{ab}\Bigg\}$$

A. Kovner and M. Lublinsky, "Angular Correlations in Gluon Production at High Energy", Phys.Rev. D**83** (2011) 034017 [arXiv:1012.3398 [hep-ph]].







Dilute-dense two-gluon production cross-section

$$\frac{dN}{d^2p\,d^2k\,d\eta\,d\xi} = \left\langle \boxed{\sigma^4} + \boxed{\sigma^3} + \boxed{\sigma^2} \right\rangle_{P,T}$$

A. Kovner and M. Lublinsky, "Angular Correlations in Gluon Production at High Energy", Phys.Rev. D**83** (2011) 034017 [arXiv:1012.3398 [hep-ph]].

$$\sigma^4 = \int_{u,z,\bar u,\bar z} e^{ik\cdot(z-\bar z)+ip\cdot(u-\bar u)} \int_{x_1,x_2,\bar x_1,\bar x_2} f(\bar z-\bar x_1)\cdot f(z-x_1)\; f(\bar u-\bar x_2)\cdot f(u-x_2)$$
$$\times \left\{\rho_{x_1}[S^\dagger_z - S^\dagger_{x_1}][S_{\bar z} - S_{\bar x_1}]\rho_{\bar x_1}\right\}\left\{\rho_{x_2}[S^\dagger_u - S^\dagger_{x_2}][S_{\bar u} - S_{\bar x_2}]\rho_{\bar x_2}\right\}$$

$$\sigma^3 = \int_{u,z,\bar u,\bar z} e^{ik\cdot(z-\bar z)+ip\cdot(u-\bar u)} \int_{x_1,\bar x_1,x_2(\bar x_2)}$$
$$-\frac{1}{2} f(\bar z-\bar x_1)\cdot f(z-x_1) f(\bar u-\bar x_1)\cdot f(u-x_2)$$
$$\times \mathrm{Tr}\left\{\bar\rho_{x_1}[S^\dagger_z - S^\dagger_{x_1}][S_{\bar z} - S_{\bar x_1}]\bar\rho_{\bar x_1}[S^\dagger_{\bar u} + S^\dagger_{\bar x_1}][S_u - S_{x_2}]\bar\rho_{x_2}\right\}$$
$$+\frac{1}{2} f(\bar z-\bar x_1)\cdot f(z-x_1) f(\bar u-\bar x_2)\cdot f(u-x_1)$$
$$\times \mathrm{Tr}\left\{\bar\rho_{\bar x_2}[S^\dagger_{\bar u} - S^\dagger_{\bar x_2}][S_u + S_{x_1}]\bar\rho_{x_1}[S^\dagger_z - S^\dagger_{x_1}][S_{\bar z} - S_{\bar x_1}]\bar\rho_{\bar x_1}\right\}$$
$$+ f(\bar z-\bar u)\cdot f(z-x_1) f(\bar u-\bar x_1)\cdot f(u-x_2)$$
$$\times \mathrm{Tr}\left\{\bar\rho_{x_1}[S^\dagger_z - S^\dagger_{x_1}][S_{\bar z} - S_{\bar u}]\bar\rho_{\bar x_1} S^\dagger_{\bar u}[S_u - S_{x_2}]\bar\rho_{x_2}\right\}$$
$$- f(\bar z-\bar x_1)\cdot f(z-u) f(\bar u-\bar x_2)\cdot f(u-x_1)$$
$$\times \mathrm{Tr}\left\{\bar\rho_{\bar x_2}[S^\dagger_{\bar u} - S^\dagger_{\bar x_2}]S_u\bar\rho_{x_1}[S^\dagger_z - S^\dagger_u][S_{\bar z} - S_{\bar x_1}]\bar\rho_{\bar x_1}\right\}$$

$$\sigma^2 = \int_{u,z,\bar u,\bar z} e^{ik\cdot(z-\bar z)+ip\cdot(u-\bar u)} \int_{x_1,\bar x_1}$$
$$\frac{1}{4} f(\bar z-\bar x_1)\cdot f(z-x_1) f(\bar u-\bar x_1)\cdot f(u-x_1)$$
$$\times \mathrm{Tr}\left\{[S_u + S_{x_1}]\bar\rho_{x_1}[S^\dagger_z - S^\dagger_{x_1}][S_{\bar z} - S_{\bar x_1}]\bar\rho_{\bar x_1}[S^\dagger_{\bar u} + S^\dagger_{\bar x_1}]\right\}$$
$$+ f(\bar z-\bar u)\cdot f(z-u) f(\bar u-\bar x_1)\cdot f(u-x_1)$$
$$\times \mathrm{Tr}\left\{S_u\bar\rho_{x_1}[S^\dagger_z - S^\dagger_u][S_{\bar z} - S_{\bar u}]\bar\rho_{\bar x_1} S^\dagger_{\bar u}\right\}$$
$$-\frac{1}{2} f(\bar z-\bar x_1)\cdot f(z-u) f(\bar u-\bar x_1)\cdot f(u-x_1)$$
$$\times \mathrm{Tr}\left\{S_u\bar\rho_{x_1}[S^\dagger_z - S^\dagger_u][S_{\bar z} - S_{\bar x_1}]\bar\rho_{\bar x_1}[S^\dagger_{\bar u} + S^\dagger_{\bar x_1}]\right\}$$
$$-\frac{1}{2} f(\bar z-\bar u)\cdot f(z-x_1) f(\bar u-\bar x_1)\cdot f(u-x_1)$$
$$\times \mathrm{Tr}\left\{[S_u + S_{x_1}]\bar\rho_{x_1}[S^\dagger_z - S^\dagger_{x_1}][S_{\bar z} - S_{\bar u}]\bar\rho_{\bar x_1} S^\dagger_{\bar u}\right\}$$

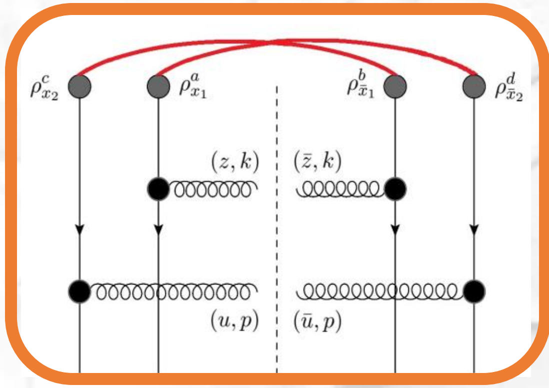
Kendi Kohara, A., Marquet, C., Vila, V., arXiv:2303.08711 [hep-ph]

$$\left\langle \hat\rho^c_{x_2}\hat\rho^a_{x_1}\hat\rho^d_{\bar x_2}\hat\rho^b_{\bar x_1}\right\rangle_P = \left\langle \hat\rho^c_{x_2}\hat\rho^a_{x_1}\right\rangle_P\left\langle\hat\rho^d_{\bar x_2}\hat\rho^b_{\bar x_1}\right\rangle_P + \left\langle\hat\rho^c_{x_2}\hat\rho^d_{\bar x_2}\right\rangle_P\left\langle\hat\rho^a_{x_1}\hat\rho^b_{\bar x_1}\right\rangle_P + \left\langle\hat\rho^c_{x_2}\hat\rho^b_{\bar x_1}\right\rangle_P\left\langle\hat\rho^a_{x_1}\hat\rho^d_{\bar x_2}\right\rangle_P$$

$$\left\langle\hat\rho^a_{x_1}\hat\rho^b_{x_2}\right\rangle_P = \delta^{ab}\mu^2\delta^{(2)}(x_1-x_2)$$

MV MODEL PROJECTILE AVERAGING

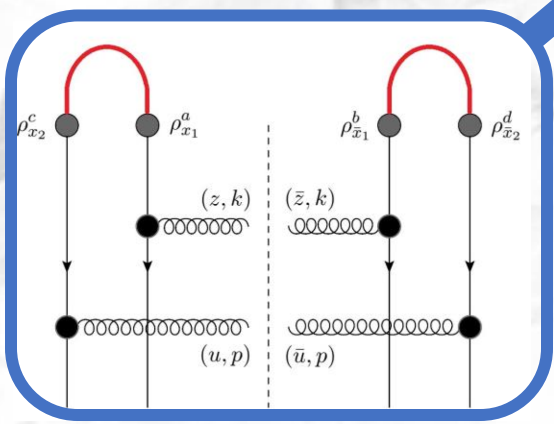
$$\begin{aligned}
\langle\sigma_4\rangle_{P,T} &= \int_{u,z,\bar u,\bar z} e^{ik\cdot(z-\bar z)+ip\cdot(u-\bar u)}\int_{x_1,x_2,\bar x_1,\bar x_2} f(\bar z-\bar x_1)\cdot f(z-x_1)\, f(\bar u-\bar x_2)\cdot f(u-x_2)\\
&\times\left\langle\rho^c_{x_2}\rho^a_{x_1}\rho^d_{\bar x_2}\rho^b_{\bar x_1}\right\rangle_P\left\langle\left[S^\dagger_z-S^\dagger_{x_1}\right]\left[S_{\bar z}-S_{\bar x_1}\right]^{ab}\left[S^\dagger_u-S^\dagger_{x_2}\right]\left[S_{\bar u}-S_{\bar x_2}\right]^{cd}\right\rangle_T\\
&= \mu^4\int_{u,z,\bar u,\bar z} e^{ik\cdot(z-\bar z)+ip\cdot(u-\bar u)}\\
&\left\{\int_{x_1,\bar x_1} f(\bar z-\bar x_1)\cdot f(z-x_1)\, f(\bar u-\bar x_1)\cdot f(u-x_1)\right.\\
&\quad\times\left\langle\left[S^\dagger_z-S^\dagger_{x_1}\right]\left[S_{\bar z}-S_{\bar x_1}\right]^{ab}\left[S^\dagger_u-S^\dagger_{x_1}\right]\left[S_{\bar u}-S_{\bar x_1}\right]^{ab}\right\rangle_{T\ [(i)]} \qquad \sigma^{4(i)}\\
&+\int_{x_1,x_2} f(\bar z-x_1)\cdot f(z-x_1)\, f(\bar u-x_2)\cdot f(u-x_2)\\
&\quad\times\left\langle\left[S^\dagger_z-S^\dagger_{x_1}\right]\left[S_{\bar z}-S_{x_1}\right]^{aa}\left[S^\dagger_u-S^\dagger_{x_2}\right]\left[S_{\bar u}-S_{x_2}\right]^{cc}\right\rangle_{T\ [(ii)]} \qquad \sigma^{4(ii)}\\
&+\int_{x_1,x_2} f(\bar z-x_2)\cdot f(z-x_1)\, f(\bar u-x_1)\cdot f(u-x_2)\\
&\quad\left.\times\left\langle\left[S^\dagger_z-S^\dagger_{x_1}\right]\left[S_{\bar z}-S_{x_2}\right]^{ab}\left[S^\dagger_u-S^\dagger_{x_2}\right]\left[S_{\bar u}-S_{x_1}\right]^{ba}\right\rangle_{T\ [(iii)]}\right\} \qquad \sigma^{4(iii)}
\end{aligned}$$

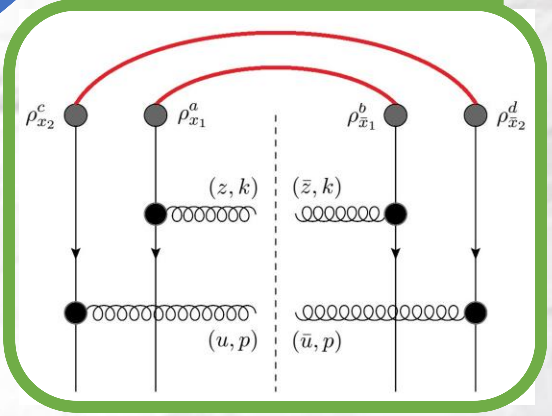


$\sigma^{4(iii)}$

$\sigma^{4(i)}$

$\sigma^{4(ii)}$



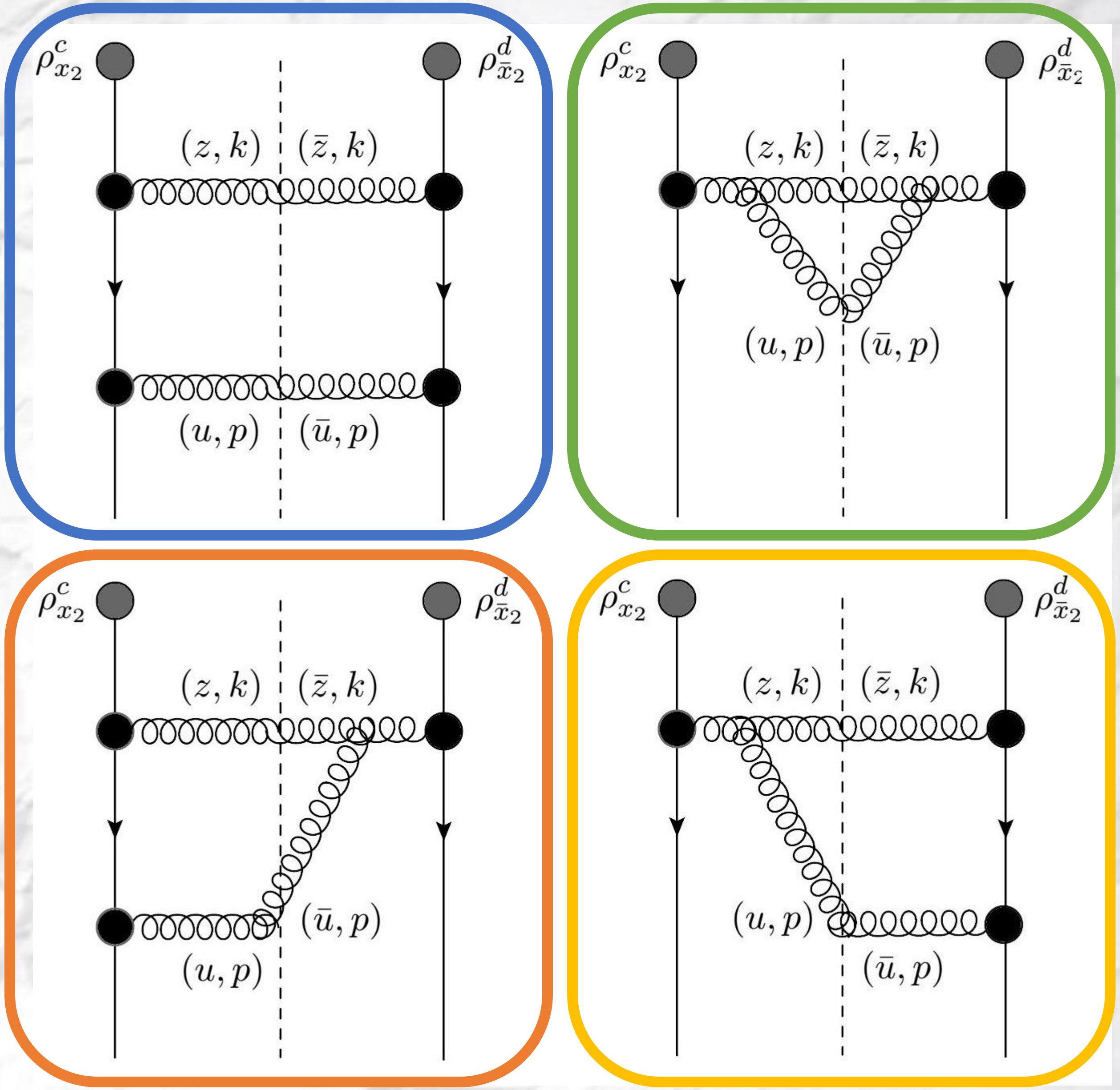


$$
\begin{aligned}
\langle\sigma_2\rangle_{P,T} = \int_{u,z,\bar u,\bar z} e^{ik\cdot(z-\bar z)+ip\cdot(u-\bar u)} \int_{x_1,\bar x_1}
& \frac{1}{4} f(\bar z-\bar x_1)\cdot f(z-x_1) f(\bar u-\bar x_1)\cdot f(u-x_1) \\
& \times \mathrm{Tr}\Big\{[S_u+S_{x_1}]\bar\rho_{x_1}[S_z^\dagger - S_{x_1}^\dagger][S_{\bar z}-S_{\bar x_1}]\bar\rho_{\bar x_1}[S_{\bar u}^\dagger + S_{\bar x_1}^\dagger]\Big\}_{[(i)]} \\
& + f(\bar z-\bar u)\cdot f(z-u) f(\bar u-\bar x_1)\cdot f(u-x_1) \\
& \times \mathrm{Tr}\Big\{S_u\bar\rho_{x_1}[S_z^\dagger - S_u^\dagger][S_{\bar z}-S_{\bar u}]\bar\rho_{\bar x_1}S_{\bar u}^\dagger\Big\}_{[(ii)]} \\
& - \frac{1}{2} f(\bar z-\bar x_1)\cdot f(z-u) f(\bar u-\bar x_1)\cdot f(u-x_1) \\
& \times \mathrm{Tr}\Big\{S_u\bar\rho_{x_1}[S_z^\dagger - S_u^\dagger][S_{\bar z}-S_{\bar x_1}]\bar\rho_{\bar x_1}[S_{\bar u}^\dagger + S_{\bar x_1}^\dagger]\Big\}_{[(iii)]} \\
& - \frac{1}{2} f(\bar z-\bar u)\cdot f(z-x_1) f(\bar u-\bar x_1)\cdot f(u-x_1) \\
& \times \mathrm{Tr}\Big\{[S_u+S_{x_1}]\bar\rho_{x_1}[S_z^\dagger - S_{x_1}^\dagger][S_{\bar z}-S_{\bar u}]\bar\rho_{\bar x_1}S_{\bar u}^\dagger\Big\}_{[(iv)]}
\end{aligned}
$$

Kendi Kohara, A., Marquet, C., Vila, V., arXiv:2303.08711 [hep-ph]

$$\left\langle \hat\rho^a_{x_1}\hat\rho^b_{\bar x_1}\hat\rho^c_{\bar x_2}\right\rangle_P = 0$$

MV MODEL PROJECTILE AVERAGING



$$D(\bar{u},u) = \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\ S_{\bar{u}}^{\dagger} S_u \right\rangle_T$$

$$Q(\bar{u},u,z,\bar{z}) = \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\ S_{\bar{u}}^{\dagger} S_u S_z^{\dagger} S_{\bar{z}} \right\rangle_T$$

$$DD(\bar{u},u,z,\bar{z}) = \frac{1}{(N_c^2-1)^2}\left\langle \mathrm{Tr}\ S_{\bar{u}}^{\dagger} S_u\ \mathrm{Tr}\ S_z^{\dagger} S_{\bar{z}} \right\rangle_T$$

TARGET AVERAGES OF ADJOINT WILSON LINES

$$\sigma_4^{(ii)}(k,p) = \langle \tilde{\sigma}(k)\tilde{\sigma}(p)\rangle_T$$

$$\tilde{\sigma}(k) = \mu^2 \int_{z,\bar{z}} e^{ik\cdot(z-\bar{z})} \int_x f(\bar{z}-x)\cdot f(z-x)\ \mathrm{Tr}[S_z^{\dagger} - S_x^{\dagger}][S_{\bar{z}} - S_x]$$

UNCORRELATED PIECE

$$\begin{aligned}
\sigma_4^{(i)}(k,p) = &\int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \\
&\times \int_{x_1,\bar{x}_1} f(\bar{z}-\bar{x}_1)\cdot f(z-x_1)\ f(\bar{u}-\bar{x}_1)\cdot f(u-x_1) \\
&\times (N_c^2-1)\mu^4 \Big\{ Q(\bar{z},z,u,\bar{u}) - Q(\bar{z},z,u,\bar{x}_1) - Q(\bar{z},z,x_1,\bar{u}) \\
&+ Q(\bar{z},z,x_1,\bar{x}_1) - Q(\bar{x}_1,z,u,\bar{u}) + D(z,u) + Q(\bar{x}_1,z,x_1,\bar{u}) - D(z,x_1) \\
&- Q(\bar{z},x_1,u,\bar{u}) + Q(\bar{z},x_1,u,\bar{x}_1) + D(\bar{z},\bar{u}) - D(\bar{z},\bar{x}_1) + Q(\bar{x}_1,x_1,u,\bar{u}) \\
&- D(x_1,u) - D(\bar{x}_1,\bar{u}) + 1 \Big\}
\end{aligned}$$

$$\sigma_4^{(iii)}(k,p) = \sigma_4^{(i)}(k,-p)$$

CORRELATED PIECES

Kendi Kohara, A., Marquet, C., Vila, V., arXiv:2303.08711 [hep-ph]

$$(N_c^2-1)^2\left\langle S_{\bar u}^{ab}S_u^{cd}S_z^{ef}S_{\bar z}^{gh}\right\rangle_T \overset{AEM}{\Longrightarrow} \delta_{ac}\delta_{bd}\delta_{eg}\delta_{fh}\; D(\bar u,u)D(z,\bar z) + \delta_{ag}\delta_{bh}\delta_{ce}\delta_{df}\; D(\bar u,\bar z)D(z,u) + \delta_{ae}\delta_{bf}\delta_{cg}\delta_{dh}\; D(\bar u,z)D(u,\bar z)$$

$$Q(\bar u,u,z,\bar z)\longrightarrow D(\bar u,u)D(z,\bar z)+D(\bar u,\bar z)D(z,u)+\frac{1}{N_c^2-1}D(\bar u,z)D(u,\bar z)$$

$$DD(\bar u,u,z,\bar z)\longrightarrow D(\bar u,u)D(z,\bar z)+\frac{1}{(N_c^2-1)^2}\left[D(\bar u,\bar z)D(z,u)+D(\bar u,z)D(u,\bar z)\right]$$

$$\left\langle S_u^{b'm}T^c_{ma}S_z^{\dagger ac'}S_{\bar z}^{c'b}T^c_{bn}S_{\bar u}^{\dagger nb'}\right\rangle_T \longrightarrow N_c(N_c^2-1)D(\bar u,u)D(z,\bar z)-N_cD(\bar u,z)D(u,\bar z)$$

AEM

Kendi Kohara, A., Marquet, C., Vila, V., arXiv:2303.08711 [hep-ph]

Projectile and target averaging

$$\sigma_4^{(i)}(k,p) = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \int_{x,\bar{x}} f(\bar{z}-\bar{x})\cdot f(z-x)\; f(\bar{u}-\bar{x})\cdot f(u-x)$$
$$\times (N_c^2-1)\mu^4\Big\{D(\bar{z},z)D(u,\bar{u}) + D(\bar{z},\bar{u})D(z,u) - D(\bar{z},z)D(u,\bar{x})$$
$$- D(\bar{z},\bar{x})D(z,u) - D(\bar{z},z)D(x,\bar{u}) - D(\bar{z},\bar{u})D(z,x)$$
$$+ D(\bar{z},z)D(x,\bar{x}) + D(\bar{z},\bar{x})D(z,x) - D(\bar{x},z)D(u,\bar{u})$$
$$- D(\bar{x},\bar{u})D(z,u) + D(z,u) + D(\bar{x},z)D(x,\bar{u}) + D(\bar{x},\bar{u})D(z,x)$$
$$- D(z,x) - D(\bar{z},x)D(u,\bar{u}) - D(\bar{z},\bar{u})D(x,u) + D(\bar{z},x)D(u,\bar{x})$$
$$+ D(\bar{z},\bar{x})D(x,u) + D(\bar{z},\bar{u}) - D(\bar{z},\bar{x}) + D(\bar{x},x)D(u,\bar{u})$$
$$+ D(\bar{x},\bar{u})D(x,u) - D(x,u) - D(\bar{x},\bar{u}) + 1\Big\}$$
$$= \sigma_4^{(iii)}(k,-p)$$

$$\sigma_4^{(ii)}(k,p) = \bar{\sigma}(k)\bar{\sigma}(p),$$
$$\bar{\sigma}(k) = \mu^2(N_c^2-1)\int_{z,\bar{z}} e^{ik\cdot(z-\bar{z})}\int_x f(\bar{z}-x)\cdot f(z-x)$$
$$\times\Big[D(z,\bar{z}) - D(z,x) - D(x,\bar{z}) + 1\Big]$$

AVERAGED SIGMA^4

$$\sigma_2^{(i)}(k,p) = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})}\int_x f(\bar{z}-x)\cdot f(z-x)\; f(\bar{u}-x)\cdot f(u-x)$$
$$\times \frac{1}{4}N_c^3\mu^2\Big\{D(u,\bar{u})D(z,\bar{z}) + D(u,x)D(z,\bar{z}) - D(u,\bar{u})D(z,x)$$
$$- D(u,x)D(z,x) + D(x,\bar{u})D(z,\bar{z}) + D(z,\bar{z}) - D(x,\bar{u})D(z,x) - D(z,x)$$
$$- D(u,\bar{u})D(x,\bar{z}) - D(u,x)D(x,\bar{z}) + D(u,\bar{u}) + D(u,x) - D(x,\bar{u})D(x,\bar{z})$$
$$- D(x,\bar{z}) + D(x,\bar{u}) + 1\Big\}$$

$$\sigma_2^{(ii)}(k,p) = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \times \int_x f(\bar{z}-\bar{u})\cdot f(z-u)\; f(\bar{u}-x)\cdot f(u-x)$$
$$\times N_c^3\mu^2 D(u,\bar{u})\Big\{D(z,\bar{z}) - D(z,\bar{u}) - D(u,\bar{z}) + D(u,\bar{u})\Big\}$$

$$\sigma_2^{(iii)}(k,p) = -\int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \times \int_x f(\bar{z}-x)\cdot f(z-u)\; f(\bar{u}-x)\cdot f(u-x)$$
$$\times \frac{1}{2}N_c^3\mu^2\Big\{D(u,\bar{u})D(z,\bar{z}) - D(u,\bar{u})D(z,x) - D(u,\bar{u})D(u,\bar{z})$$
$$+ D(u,\bar{u})D(u,x) + D(u,x)D(z,\bar{z}) - D(u,x)D(z,x) - D(u,x)D(u,\bar{z})$$
$$+ D^2(u,x)\Big\}$$

$$\sigma_2^{(iv)}(k,p) = -\int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \times \int_x f(\bar{z}-\bar{u})\cdot f(z-x)\; f(\bar{u}-x)\cdot f(u-x)$$
$$\times \frac{1}{2}N_c^3\mu^2\Big\{D(u,\bar{u})D(z,\bar{z}) - D(u,\bar{u})D(z,\bar{u}) - D(u,\bar{u})D(x,\bar{z})$$
$$+ D(u,\bar{u})D(x,\bar{u}) + D(x,\bar{u})D(z,\bar{z}) - D(x,\bar{u})D(z,\bar{u}) - D(x,\bar{u})D(x,\bar{z})$$
$$+ D^2(x,\bar{u})\Big\}$$

AVERAGED SIGMA^2

Kendi Kohara, A., Marquet, C., Vila, V., arXiv:2303.08711 [hep-ph]

$$f_i(x-y) = \frac{g_s}{2\pi}\frac{(x-y)_i}{(x-y)^2} = g_s\int\frac{d^2q}{(2\pi)^2}(-i)e^{iq\cdot(x-y)}\frac{q^i}{q^2}$$

$$D(r) = \int\frac{d^2q}{2\pi}e^{-iq\cdot r}\frac{1}{Q_s^2}e^{-\frac{q^2}{2Q_s^2}}$$

GBW MODEL

BOSE-ENHANCEMENT CONTRIBUTION

$$\sigma_4^{([(i)+(iii)].\mathrm{I})}(k,p) = 2\pi\frac{\sigma_0^2}{N_4}\,e^{-\frac{(k+p)^2}{4Q_s^2}} \qquad N_4 = (N_c^2-1)S_\perp Q_s^2$$

$$\times\left[\frac{2Q_s^4\left(k^4+p^4+2\,(k\cdot p)^2\right)}{k^2p^2\,(k-p)^4}+\frac{8Q_s^6\,(k+p)^4}{k^2p^2\,(k-p)^6}\right.$$

$$\left.+\frac{64Q_s^8\left(k^4+4(k\cdot p)^2+p^4+8(k\cdot p)(k^2+p^2)+14k^2p^2\right)}{k^2p^2\,(k-p)^8}\right]$$

$$+\,(p\to -p)$$

UNCORRELATED PIECE

$$\sigma_4^{(ii)}(k,p) = \sigma_0^2\,e^{-\frac{k^2+p^2}{2Q_s^2}}$$

$$\times\left\{\frac{2Q_s^2}{k^2}-\frac{Q_s^2}{k^2}e^{\frac{k^2}{2Q_s^2}}+\frac{1}{2}\left[\mathrm{Ei}\left(\frac{k^2}{2Q_s^2}\right)-\mathrm{Ei}\left(\frac{k^2\lambda}{2Q_s^2}\right)\right]\right\}$$

$$\times\left\{\frac{2Q_s^2}{p^2}-\frac{Q_s^2}{p^2}e^{\frac{p^2}{2Q_s^2}}+\frac{1}{2}\left[\mathrm{Ei}\left(\frac{p^2}{2Q_s^2}\right)-\mathrm{Ei}\left(\frac{p^2\lambda}{2Q_s^2}\right)\right]\right\}$$

$$\sigma_0 = g_s^2(N_c^2-1)S_\perp\frac{\mu^2}{Q_s^2}\ ,\ \lambda\sim 1/(S_\perp Q_s^2)$$

$$\sigma_4^{([(i)+(iii)].\mathrm{II})}(k,p) = \frac{\sigma_0^2}{N_4}Q_s^2(2\pi)^2\left[\delta^{(2)}(k+p)+\delta^{(2)}(k-p)\right]$$

$$N_4 = (N_c^2-1)S_\perp Q_s^2 \qquad \times\,\frac{4Q_s^4}{k^{12}}e^{-\frac{k^2}{Q_s^2}}\left(k^4+k^2e^{\frac{k^2}{2Q_s^2}}Q_s^2+4e^{\frac{k^2}{2Q_s^2}}Q_s^4\right)^2$$

HANBURY-BROWN-TWISS CONTRIBUTION

Kendi Kohara, A., Marquet, C., Vila, V., arXiv:2303.08711 [hep-ph]

$$\sigma_2^{(i)}(k,p) = \frac{\sigma_0^2}{2N_2}\frac{Q_s^4}{k^6p^6}e^{-\frac{k^2+p^2}{2Q_s^2}} \qquad N_2 = \frac{(N_c^2-1)^2}{N_c^3}S_\perp\mu^2$$

$$\times\left[k^4 + e^{\frac{k^2}{2Q_s^2}}k^2Q_s^2 + 4e^{\frac{k^2}{2Q_s^2}}Q_s^4\right]\left[\left(2e^{\frac{p^2}{2Q_s^2}}-1\right)p^4 + e^{\frac{p^2}{2Q_s^2}}p^2Q_s^2 + 4e^{\frac{p^2}{2Q_s^2}}Q_s^4\right]$$

$$+\,(k\leftrightarrow p)$$

Kendi Kohara, A., Marquet, C., Vila, V., arXiv:2303.08711 [hep-ph]

$$N_2 = \frac{(N_c^2-1)^2}{N_c^3}S_\perp\mu^2$$

$$\sigma_2^{(ii)}(k,p) = \frac{\sigma_0^2}{2N_2}\Bigg\{\frac{1}{(2\pi)^2}e^{-\frac{k^2+p^2}{2Q_s^2}}\int_{s_1,s_2}\frac{1}{s_1^2}\frac{1}{(s_1-s_2)^2}e^{-\frac{s_1^2+2k\cdot s_1}{2Q_s^2}}e^{-\frac{s_2^2-2p\cdot s_2}{2Q_s^2}}$$

$$+\frac{1}{2\pi^2}e^{-\frac{k^2+p^2}{2Q_s^2}}\frac{k^i}{k^2}\int_{s_1,s_2}\frac{s_1^i}{s_1^2}\frac{1}{(s_1-s_2)^2}e^{-\frac{s_1^2+2k\cdot s_1}{2Q_s^2}}e^{-\frac{s_2^2-2p\cdot s_2}{2Q_s^2}}$$

$$+\frac{Q_s^2}{4k^2}e^{-\frac{(k+p)^2}{4Q_s^2}}\left[\mathrm{Ei}\left(\frac{(k+p)^2}{4Q_s^2}\right)-\mathrm{Ei}\left(\frac{(k+p)^2\lambda}{4Q_s^2}\right)\right]\Bigg\}$$

$$+\,(k\leftrightarrow p)$$

$$N_2 = \frac{(N_c^2-1)^2}{N_c^3}S_\perp\mu^2$$

$$\sigma_2^{[(iii)+(iv)]}(k,p) = \frac{\sigma_0^2}{2N_2}$$

$$\times\Bigg\{\frac{1}{(2\pi)^2}e^{-\frac{k^2+p^2}{2Q_s^2}}\int_{s_1,s_2}\left[\left(\frac{s_1^i}{s_1^2}+\frac{k^i}{k^2}\right)^2\left(\frac{s_2^j}{s_2^2}+\frac{p^j}{p^2}\right)-\frac{1}{k^2}\frac{p^j}{p^2}\right]$$

$$\times\frac{(s_1-s_2)^j}{(s_1-s_2)^2}e^{-\frac{s_1^2+2k\cdot s_1}{2Q_s^2}}e^{-\frac{s_2^2-2p\cdot s_2}{2Q_s^2}}$$

$$+\frac{Q_s^4}{k^2}\frac{p\cdot(k+p)}{p^2(k+p)^2}\left(e^{-\frac{(k+p)^2}{4Q_s^2}}-1\right)\Bigg\}$$

$$+\,(k\leftrightarrow p)$$

Some details on the integration procedure

$$\sigma_4^{(i)} = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \int_{x,\bar{x}} f(\bar{z}-\bar{x})\cdot f(z-x)\ f(\bar{u}-\bar{x})\cdot f(u-x)$$
$$\times (N_c^2-1)\mu^4\Big\{D(\bar{z},z)D(u,\bar{u})_{[1]} + D(\bar{z},\bar{u})D(z,u)_{[2]} - D(\bar{z},z)D(u,\bar{x})_{[3]}$$
$$- D(\bar{z},\bar{x})D(z,u)_{[4]} - D(\bar{z},z)D(x,\bar{u})_{[5]} - D(\bar{z},\bar{u})D(z,x)_{[6]} + D(\bar{z},z)D(x,\bar{x})_{[7]}$$
$$+ D(\bar{z},\bar{x})D(z,x)_{[8]} - D(\bar{x},z)D(u,\bar{u})_{[9]} - D(\bar{x},\bar{u})D(z,u)_{[10]} + D(z,u)_{[11]}$$
$$+ D(\bar{x},z)D(x,\bar{u})_{[12]} + D(\bar{x},\bar{u})D(z,x)_{[13]} - D(z,x)_{[14]} - D(\bar{z},x)D(u,\bar{u})_{[15]}$$
$$- D(\bar{z},\bar{u})D(x,u)_{[16]} + D(\bar{z},x)D(u,\bar{x})_{[17]} + D(\bar{z},\bar{x})D(x,u)_{[18]} + D(\bar{z},\bar{u})_{[19]}$$
$$- D(\bar{z},\bar{x})_{[20]} + D(\bar{x},x)D(u,\bar{u})_{[21]} + D(\bar{x},\bar{u})D(x,u)_{[22]} - D(x,u)_{[23]}$$
$$- D(\bar{x},\bar{u})_{[24]} + 1_{[25]}\Big\}$$

$$\sigma_4^{(i.[1])} = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \int_{x,\bar{x}} f(\bar{z}-\bar{x})\cdot f(z-x)\ f(\bar{u}-\bar{x})\cdot f(u-x)$$
$$\times (N_c^2-1)\mu^4 D(\bar{z},z)D(u,\bar{u})$$

$$\sigma_4^{(i.[1])} = g_s^4(N_c^2-1)\mu^4\frac{1}{(2\pi)^4}\int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})}\int_{x,\bar{x}}$$
$$\times \frac{d^2q_1}{2\pi}(-i)e^{iq_1\cdot(\bar{z}-\bar{x})}\frac{q_1^i}{q_1^2}\frac{d^2q_2}{2\pi}(-i)e^{iq_2\cdot(z-x)}\frac{q_2^i}{q_2^2}\frac{d^2q_3}{2\pi}(-i)e^{iq_3\cdot(\bar{u}-\bar{x})}\frac{q_3^j}{q_3^2}$$
$$\times \frac{d^2q_4}{2\pi}(-i)e^{iq_4\cdot(u-x)}\frac{q_4^j}{q_4^2}\frac{d^2t_1}{2\pi}e^{-it_1\cdot(\bar{z}-z)}\frac{1}{Q_s^2}e^{-\frac{t_1^2}{2Q_s^2}}\frac{d^2t_2}{2\pi}e^{-it_2\cdot(u-\bar{u})}\frac{1}{Q_s^2}e^{-\frac{t_2^2}{2Q_s^2}}$$

$$\int d^2x e^{i(k-p)\cdot x} = (2\pi)^2\delta^{(2)}(k-p) \qquad S_\perp \equiv \int_{\text{proton}} d^2x$$

$$\sigma_4^{(i.[1])} = g_s^4(N_c^2-1)\mu^4\frac{1}{Q_s^4}\int\frac{d^2t_1}{2\pi}\frac{(t_1+k)^i}{(t_1+k)^2}\frac{(t_1+k)^i}{(t_1+k)^2}e^{-\frac{t_1^2}{2Q_s^2}}$$
$$\times\frac{d^2t_2}{2\pi}\frac{(t_2-p)^j}{(t_2-p)^2}\frac{(t_2-p)^j}{(t_2-p)^2}e^{-\frac{t_2^2}{2Q_s^2}}\int_{x,\bar{x}} e^{-i[(t_1+k)-(t_2-p)]\cdot(x-\bar{x})}$$

$$\sigma_4^{(i.[1])} = g_s^4(N_c^2-1)\mu^4\frac{1}{Q_s^4}e^{-\frac{k^2+p^2}{2Q_s^2}}\int\frac{d^2s_1}{2\pi}e^{-\frac{s_1^2-2k\cdot s_1}{2Q_s^2}}\frac{1}{s_1^2}\frac{d^2s_2}{2\pi}e^{-\frac{s_2^2+2p\cdot s_2}{2Q_s^2}}\frac{1}{s_2^2}\int_{x,\bar{x}}e^{i(s_1-s_2)\cdot(x-\bar{x})}$$
$$= g_s^4(N_c^2-1)\mu^4\frac{1}{Q_s^4}e^{-\frac{k^2+p^2}{2Q_s^2}}\int d^2s e^{-\frac{2s^2+2(p-k)\cdot s}{2Q_s^2}}\frac{1}{s^4}\int d^2x$$

$$T_{0,4} = \int d^2q e^{-\frac{2q^2+2(k+p)\cdot q}{Q_s^2}}\frac{1}{q^4}$$
$$\approx \pi Q_s^2 e^{\frac{(k+p)^2}{2Q_s^2}}\frac{2^4}{(k+p)^4}\left[\frac{1}{2}+\frac{2^2Q_s^2}{(k+p)^2}+\frac{9}{4}\frac{2^4Q_s^4}{(k+p)^4}\right]+\frac{1}{Q_s^2}\mathcal{O}\left(\frac{Q_s^{10}}{(k+p)^{10}}\right)$$

$$\sigma_4^{(i.[1])} \approx g_s^4(N_c^2-1)\mu^4 S_\perp\frac{2\pi}{Q_s^2}e^{-\frac{(k+p)^2}{4Q_s^2}}\frac{2^4}{(k-p)^4}\left[\frac{1}{2}+\frac{2^3Q_s^2}{(k-p)^2}+\frac{9}{4}\frac{2^6Q_s^4}{(k-p)^4}\right]$$

Kendi Kohara, A., Marquet, C., Vila, V., arXiv:2303.08711 [hep-ph]

Some details on the integration procedure

$$I^i_{1,2} = \int d^2q\, e^{-\frac{q^2+2k\cdot q}{Q_s^2}} \frac{q^i}{q^2} = \pi Q_s^2 \frac{k^i}{k^2}\left(1-e^{\frac{k^2}{Q_s^2}}\right)$$

$$T_{0,2} = \int d^2q\, e^{-\frac{2q^2+2(k+p)\cdot q}{Q_s^2}} \frac{1}{q^2} \approx \pi Q_s^2 e^{\frac{(k+p)^2}{2Q_s^2}} \frac{2^2}{(k+p)^2}\left[\frac{1}{2}+\frac{1}{4}\frac{2^2Q_s^2}{(k+p)^2}+\frac{1}{4}\frac{2^4Q_s^4}{(k+p)^4}\right]$$

$$T^i_{1,4} = \int d^2q\, e^{-\frac{2q^2+2(k+p)\cdot q}{Q_s^2}} \frac{q^i}{q^4} \approx -\pi Q_s^2 e^{\frac{(k+p)^2}{2Q_s^2}} \frac{(k+p)^i}{(k+p)^2}\frac{2^2}{(k+p)^2}\left[1+\frac{2^2Q_s^2}{(k+p)^2}+\frac{3}{2}\frac{2^4Q_s^4}{(k+p)^4}\right]$$

$$T^{ij}_{2,4} = \int d^2q\, e^{-\frac{2q^2+2(k+p)\cdot q}{Q_s^2}} \frac{q^iq^j}{q^4}$$
$$\approx \pi Q_s^2 e^{\frac{(k+p)^2}{2Q_s^2}} \frac{2^2}{(k+p)^2}\left[\frac{1}{2}\frac{(k+p)^i(k+p)^j}{(k+p)^2}+\frac{\delta^{ij}}{8}\frac{2^2Q_s^2}{(k+p)^2}\left(1+\frac{2^2Q_s^2}{(k+p)^2}\right)\right]$$

$$\sigma_4^{(i.[2])} \approx g_s^4(N_c^2-1)\mu^4 S_\perp \pi^2\delta^{(2)}(k+p)\left\{\frac{2^2}{k^2}\left[\frac{1}{2}\frac{k^ik^j}{k^2}+\frac{\delta^{ij}}{8}\frac{2^2Q_s^2}{k^2}\left(1+\frac{2^2Q_s^2}{k^2}\right)\right]\right\}^2$$

$$\sigma_4^{(i.[3])} = \sigma_4^{(i.[5])} \approx g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2}e^{-\frac{(k+p)^2}{4Q_s^2}}\frac{p\cdot(k-p)}{p^2(k-p)^2}\frac{2^2}{(k-p)^2}\left[1+\frac{2^3Q_s^2}{(k-p)^2}+\frac{3}{2}\frac{2^6Q_s^4}{(k-p)^4}\right]$$

$$\sigma_4^{(i.[4])} = \sigma_4^{(i.[6])} = \sigma_4^{(i.[10])} = \sigma_4^{(i.[16])} \approx g_s^4(N_c^2-1)\mu^4 S_\perp\left(2\pi^2\right)\delta^{(2)}(k+p)$$
$$\times\left(e^{-\frac{k^2}{2Q_s^2}}-1\right)\frac{k^ik^j}{k^4}\left\{\frac{2^2}{k^2}\left[\frac{1}{2}\frac{k^ik^j}{k^2}+\frac{\delta^{ij}}{8}\frac{2^2Q_s^2}{k^2}\left(1+\frac{2^2Q_s^2}{k^2}\right)\right]\right\}$$

$$\sigma_4^{(i.[7])} \approx g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2}e^{-\frac{(k+p)^2}{4Q_s^2}}\frac{1}{p^2}\frac{2^2}{(k-p)^2}\left[\frac{1}{2}+\frac{1}{4}\frac{2^3Q_s^2}{(k-p)^2}+\frac{1}{4}\frac{2^6Q_s^4}{(k-p)^4}\right]$$

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$$\sigma_4^{(i.[8])} = \sigma_4^{(i.[13])} = \sigma_4^{(i.[18])} = \sigma_4^{(i.[22])} = g_s^4(N_c^2-1)\mu^4 S_\perp(2\pi)^2\delta^{(2)}(k+p)\frac{1}{k^4}\left(e^{-\frac{k^2}{2Q_s^2}}-1\right)^2$$

$$\sigma_4^{(i.[9])} = \sigma_4^{(i.[15])} \approx g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2}e^{-\frac{(k+p)^2}{4Q_s^2}}$$
$$\times\frac{k\cdot(p-k)}{k^2(k-p)^2}\frac{2^2}{(k-p)^2}\left[1+\frac{2^3Q_s^2}{(k-p)^2}+\frac{3}{2}\frac{2^6Q_s^4}{(k-p)^4}\right]$$

$$\sigma_4^{(i.[11])} = \sigma_4^{(i.[19])} \approx g_s^4(N_c^2-1)\mu^4 S_\perp\left(2\pi^2\right)\delta^{(2)}(k+p)\frac{k^ik^j}{k^4}\left\{\frac{2^2}{k^2}\left[\frac{1}{2}\frac{k^ik^j}{k^2}+\frac{\delta^{ij}}{8}\frac{2^2Q_s^2}{k^2}\left(1+\frac{2^2Q_s^2}{k^2}\right)\right]\right\}$$

$$\sigma_4^{(i.[12])} = \sigma_4^{(i.[17])} \approx -g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2}e^{-\frac{(k+p)^2}{4Q_s^2}}\frac{k^i}{k^2}\frac{p^j}{p^2}$$
$$\times\left\{\frac{2^2}{(k-p)^2}\left[\frac{1}{2}\frac{(k-p)^i(k-p)^j}{(k-p)^2}+\frac{\delta^{ij}}{8}\frac{2^3Q_s^2}{(k-p)^2}\left(1+\frac{2^3Q_s^2}{(k-p)^2}\right)\right]\right\}$$

$$\sigma_4^{(i.[14])} = \sigma_4^{(i.[20])} = \sigma_4^{(i.[23])} = \sigma_4^{(i.[24])} = g_s^4(N_c^2-1)\mu^4 S_\perp(2\pi)^2\delta^{(2)}(k+p)\frac{1}{k^4}\left(e^{-\frac{k^2}{2Q_s^2}}-1\right)$$

$$\sigma_4^{(i.[21])} \approx g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2}e^{-\frac{(k+p)^2}{4Q_s^2}}\frac{1}{k^2}\frac{2^2}{(k-p)^2}\left[\frac{1}{2}+\frac{1}{4}\frac{2^3Q_s^2}{(k+p)^2}+\frac{1}{4}\frac{2^6Q_s^4}{(k+p)^4}\right]$$

$$\sigma_4^{(i.[25])} = g_s^4(N_c^2-1)\mu^4 S_\perp(2\pi)^2\delta^{(2)}(k+p)\frac{1}{k^4}$$

Some details on the integration procedure

$$\sigma_4^{(i.\mathrm{I})} \approx g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2} e^{-\frac{(k+p)^2}{4Q_s^2}} \frac{2^4}{(k-p)^4}\left[\frac{1}{2}+\frac{2^3Q_s^2}{(k-p)^2}+\frac{9}{4}\frac{2^6Q_s^4}{(k-p)^4}\right]$$
$$+ g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2} e^{-\frac{(k+p)^2}{4Q_s^2}}\, 2\left(\frac{p\cdot(k-p)}{p^2(k-p)^2}+\frac{k\cdot(p-k)}{k^2(k-p)^2}\right)$$
$$\times \frac{2^2}{(k-p)^2}\left[1+\frac{2^3Q_s^2}{(k-p)^2}+\frac{3}{2}\frac{2^6Q_s^4}{(k-p)^4}\right]$$
$$+ g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2} e^{-\frac{(k+p)^2}{4Q_s^2}}\left(\frac{1}{k^2}+\frac{1}{p^2}\right)\frac{2^2}{(k-p)^2}\left[\frac{1}{2}+\frac{1}{4}\frac{2^3Q_s^2}{(k-p)^2}+\frac{1}{4}\frac{2^6Q_s^4}{(k-p)^4}\right]$$
$$- g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2} e^{-\frac{(k+p)^2}{4Q_s^2}}$$
$$\times \frac{2^2}{(k-p)^2}\left[\frac{k\cdot(k-p)\; p\cdot(k-p)}{k^2p^2(k-p)^2}+\frac{2k\cdot p}{k^2p^2}\frac{1}{4}\frac{2^3Q_s^2}{(k-p)^2}+\frac{2k\cdot p}{k^2p^2}\frac{1}{4}\frac{2^6Q_s^4}{(k-p)^4}\right]$$

$$\sigma_4^{(i.\mathrm{II})} \approx g_s^4(N_c^2-1)\mu^4 S_\perp (2\pi)^2\delta^{(2)}(k+p)\frac{2^2}{k^4}\left\{\left[\frac{1}{2}\frac{k^ik^j}{k^2}+\frac{\delta^{ij}}{8}\frac{2^2Q_s^2}{k^2}\left(1+\frac{2^2Q_s^2}{k^2}\right)\right]\right\}^2$$
$$+ g_s^4(N_c^2-1)\mu^4 S_\perp (2\pi)^2\delta^{(2)}(k+p)\frac{2^2}{k^2}\left(e^{-\frac{k^2}{2Q_s^2}}-1\right)$$
$$\times \frac{2k^ik^j}{k^4}\left\{\left[\frac{1}{2}\frac{k^ik^j}{k^2}+\frac{\delta^{ij}}{8}\frac{2^2Q_s^2}{k^2}\left(1+\frac{2^2Q_s^2}{k^2}\right)\right]\right\}$$
$$+ g_s^4(N_c^2-1)\mu^4 S_\perp (2\pi)^2\delta^{(2)}(k+p)\frac{2^2}{k^4}\left(e^{-\frac{k^2}{2Q_s^2}}-1\right)^2$$
$$+ g_s^4(N_c^2-1)\mu^4 S_\perp (2\pi)^2\delta^{(2)}(k+p)\frac{2^2}{k^2}\frac{k^ik^j}{k^4}\left\{\left[\frac{1}{2}\frac{k^ik^j}{k^2}+\frac{\delta^{ij}}{8}\frac{2^2Q_s^2}{k^2}\left(1+\frac{2^2Q_s^2}{k^2}\right)\right]\right\}$$
$$+ g_s^4(N_c^2-1)\mu^4 S_\perp (2\pi)^2\delta^{(2)}(k+p)\frac{2^2}{k^4}\left(e^{-\frac{k^2}{2Q_s^2}}-1\right)$$
$$+ g_s^4(N_c^2-1)\mu^4 S_\perp (2\pi)^2\delta^{(2)}(k+p)\frac{1}{k^4}$$

Kendi Kohara, A., Marquet, C., Vila, V., arXiv:2303.08711 [hep-ph]

$$\sigma_4^{(i.\mathrm{I})} \approx g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2} e^{-\frac{(k+p)^2}{4Q_s^2}}$$
$$\times\left[\frac{2\left(k^4+p^4+2k^2p^2\cos^2\phi\right)}{k^2p^2\left(k^2+p^2-2kp\cos\phi\right)^2}+\frac{8Q_s^2\left(k^2+p^2+2kp\cos\phi\right)^2}{k^2p^2\left(k^2+p^2-2kp\cos\phi\right)^3}\right.$$
$$\left.+\frac{64Q_s^4\left(k^4+16k^2p^2+p^4+8kp\cos\phi(k^2+p^2)+2k^2p^2\cos(2\phi)\right)}{k^2p^2\left(k^2+p^2-2kp\cos\phi\right)^4}\right]$$

$$\sigma_4^{(i.\mathrm{I})} = g_s^4(N_c^2-1)\mu^4 S_\perp \frac{2\pi}{Q_s^2} e^{-\frac{(k+p)^2}{4Q_s^2}}$$
$$\times\left[\frac{2\left(k^4+p^4+2\left(k\cdot p\right)^2\right)}{k^2p^2\left(k-p\right)^4}+\frac{8Q_s^2\left(k+p\right)^4}{k^2p^2\left(k-p\right)^6}\right.$$
$$\left.+\frac{64Q_s^4\left(k^4+4(k\cdot p)^2+p^4+8(k\cdot p)(k^2+p^2)+14k^2p^2\right)}{k^2p^2\left(k-p\right)^8}\right]$$

$$\sigma_4^{(i.\mathrm{II})} \approx g_s^4(N_c^2-1)\mu^4 S_\perp (2\pi)^2\delta^{(2)}(k+p)\frac{2^2}{k^{12}}e^{-\frac{k^2}{Q_s^2}}\left(k^4+k^2e^{\frac{k^2}{2Q_s^2}}Q_s^2+2^2e^{\frac{k^2}{2Q_s^2}}Q_s^4\right)^2$$

$$\bar{\sigma} = \int_{z,\bar{z}} e^{ik\cdot(z-\bar{z})} \int_x f(\bar{z}-x)\cdot f(z-x)$$
$$\times (N_c^2-1)\mu^2 \Big[D(z,\bar{z})_{[1]} - D(z,x)_{[2]} - D(x,\bar{z})_{[3]} + 1_{[4]} \Big]$$

$$\sigma_4^{(ii)} = g_s^4 (N_c^2-1)^2 \mu^4 S_\perp^2$$
$$\times e^{-\frac{k^2}{2Q_s^2}} \left\{ \frac{2}{k^2} - \frac{1}{k^2} e^{\frac{k^2}{2Q_s^2}} + \frac{1}{2Q_s^2} \left[\mathrm{Ei}\left(\frac{k^2}{2Q_s^2}\right) - \mathrm{Ei}\left(\frac{k^2\lambda}{2Q_s^2}\right) \right] \right\}$$
$$\times \{(k \to p)\}$$

$$\sigma_4^{(iii)} = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \int_{x,\bar{x}} f(\bar{z}-\bar{x})\cdot f(z-x)\ f(\bar{u}-x)\cdot f(u-\bar{x})$$
$$\times (N_c^2-1)\mu^4 \Big\{ D(\bar{z},z)D(\bar{u},u) + D(\bar{z},u)D(z,\bar{u}) - D(\bar{z},z)D(x,u)$$
$$- D(\bar{z},u)D(z,x) - D(\bar{z},z)D(\bar{u},\bar{x}) - D(\bar{z},\bar{x})D(z,\bar{u})$$
$$+ D(\bar{z},z)D(x,\bar{x}) + D(\bar{z},\bar{x})D(z,x) - D(\bar{x},z)D(\bar{u},u)$$
$$- D(\bar{x},u)D(z,\bar{u}) + D(\bar{x},z)D(x,u) + D(\bar{x},u)D(z,x)$$
$$+ D(z,\bar{u}) - D(z,x) - D(\bar{z},x)D(\bar{u},u) - D(\bar{z},u)D(x,\bar{u})$$
$$+ D(\bar{z},u) + D(\bar{z},x)D(\bar{u},\bar{x}) + D(\bar{z},\bar{x})D(x,\bar{u}) - D(\bar{z},\bar{x})$$
$$+ D(\bar{x},x)D(\bar{u},u) + D(\bar{x},u)D(x,\bar{u}) - D(\bar{x},u) - D(x,\bar{u}) + 1 \Big\}$$

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$$\sigma_2^{(i)} = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \int_x f(\bar{z}-x)\cdot f(z-x)\ f(\bar{u}-x)\cdot f(u-x)$$
$$\times \frac{1}{4} N_c^3 \mu^2 \Big\{ D(u,\bar{u})D(z,\bar{z})_{[1]} + D(u,x)D(z,\bar{z})_{[2]} - D(u,\bar{u})D(z,x)_{[3]}$$
$$- D(u,x)D(z,x)_{[4]} + D(x,\bar{u})D(z,\bar{z})_{[5]} + D(z,\bar{z})_{[6]} - D(x,\bar{u})D(z,x)_{[7]}$$
$$- D(z,x)_{[8]} - D(u,\bar{u})D(x,\bar{z})_{[9]} - D(u,x)D(x,\bar{z})_{[10]} + D(u,\bar{u})_{[11]}$$
$$+ D(u,x)_{[12]} - D(x,\bar{u})D(x,\bar{z})_{[13]} - D(x,\bar{z})_{[14]} + D(x,\bar{u})_{[15]} + 1_{[16]} \Big\}$$

$$\sigma_2^{(i)} = g_s^4 N_c^3 \mu^2 S_\perp \frac{1}{k^6 p^6} e^{-\frac{k^2+p^2}{2Q_s^2}}$$
$$\times \left[k^4 + e^{\frac{k^2}{2Q_s^2}} k^2 Q_s^2 + 4e^{\frac{k^2}{2Q_s^2}} Q_s^4 \right] \left[\left(2e^{\frac{p^2}{2Q_s^2}} - 1 \right) p^4 + e^{\frac{p^2}{2Q_s^2}} p^2 Q_s^2 + 4e^{\frac{p^2}{2Q_s^2}} Q_s^4 \right]$$

$$\sigma_2^{(ii)} = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \times \int_x f(\bar{z}-\bar{u})\cdot f(z-u)\ f(\bar{u}-x)\cdot f(u-x)$$
$$\times N_c^3\mu^2 D(u,\bar{u})\left\{D(z,\bar{z})_{[1]} - D(z,\bar{u})_{[2]} - D(u,\bar{z})_{[3]} + D(u,\bar{u})_{[4]}\right\}$$

$$\sigma_2^{(ii)} = g_s^4 \frac{1}{(2\pi)^2 Q_s^4} N_c^3\mu^2 S_\perp e^{-\frac{k^2+p^2}{2Q_s^2}} \int_{s_1,s_2} \frac{1}{s_1^2}\frac{1}{(s_1-s_2)^2} e^{-\frac{s_1^2+2k\cdot s_1}{2Q_s^2}} e^{-\frac{s_2^2-2p\cdot s_2}{2Q_s^2}}$$
$$+ g_s^4 \frac{1}{(2\pi^2)\, Q_s^4} N_c^3\mu^2 S_\perp e^{-\frac{k^2+p^2}{2Q_s^2}} \frac{k^i}{k^2} \int_{s_1,s_2} \frac{s_1^i}{s_1^2}\frac{1}{(s_1-s_2)^2} e^{-\frac{s_1^2+2k\cdot s_1}{2Q_s^2}} e^{-\frac{s_2^2-2p\cdot s_2}{2Q_s^2}}$$
$$+ g_s^4 \frac{1}{4Q_s^2} N_c^3\mu^2 S_\perp \frac{1}{k^2} e^{-\frac{(k+p)^2}{4Q_s^2}} \left[\mathrm{Ei}\left(\frac{(k+p)^2}{4Q_s^2}\right) - \mathrm{Ei}\left(\frac{(k+p)^2\lambda}{4Q_s^2}\right)\right]$$

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$$\sigma_2^{(iii)} = -\int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \times \int_x f(\bar{z}-x)\cdot f(z-u)\ f(\bar{u}-x)\cdot f(u-x)$$
$$\times \frac{1}{2}N_c^3\mu^2\Big\{D(u,\bar{u})D(z,\bar{z})_{[1]} - D(u,\bar{u})D(z,x)_{[2]} - D(u,\bar{u})D(u,\bar{z})_{[3]}$$
$$+ D(u,\bar{u})D(u,x)_{[4]} + D(u,x)D(z,\bar{z})_{[5]} - D(u,x)D(z,x)_{[6]}$$
$$- D(u,x)D(u,\bar{z})_{[7]} + D^2(u,x)_{[8]}\Big\}$$

$$\sigma_2^{(iv)} = -\int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \times \int_x f(\bar{z}-\bar{u})\cdot f(z-x)\ f(\bar{u}-x)\cdot f(u-x)$$
$$\times \frac{1}{2}N_c^3\mu^2\Big\{D(u,\bar{u})D(z,\bar{z})_{[1]} - D(u,\bar{u})D(z,\bar{u})_{[2]} - D(u,\bar{u})D(x,\bar{z})_{[3]}$$
$$+ D(u,\bar{u})D(x,\bar{u})_{[4]} + D(x,\bar{u})D(z,\bar{z})_{[5]} - D(x,\bar{u})D(z,\bar{u})_{[6]}$$
$$- D(x,\bar{u})D(x,\bar{z})_{[7]} + D^2(x,\bar{u})_{[8]}\Big\}$$

$$\sigma_2^{[(iii)+(iv)]} = g_s^4 N_c^3\mu^2 S_\perp$$
$$\times \Bigg\{\frac{1}{(2\pi)^2 Q_s^4} e^{-\frac{k^2+p^2}{2Q_s^2}} \int_{s_1,s_2} \left[\left(\frac{s_1^i}{s_1^2}+\frac{k^i}{k^2}\right)^2\left(\frac{s_2^j}{s_2^2}+\frac{p^j}{p^2}\right) - \frac{1}{k^2}\frac{p^j}{p^2}\right]\frac{(s_1-s_2)^j}{(s_1-s_2)^2}$$
$$\times e^{-\frac{s_1^2+2k\cdot s_1}{2Q_s^2}} e^{-\frac{s_2^2-2p\cdot s_2}{2Q_s^2}}$$
$$+ \frac{1}{k^2}\frac{p\cdot(k+p)}{p^2(k+p)^2}\left(e^{-\frac{(k+p)^2}{4Q_s^2}} - 1\right)\Bigg\}$$

$$\sigma_4^{([(i)+(iii)].\mathrm{I})}(k,p) = 2\pi\frac{\sigma_0^2}{N_4}\, e^{-\frac{(k+p)^2}{4Q_s^2}} \qquad N_4 = (N_c^2-1)S_\perp Q_s^2$$
$$\times\left[\frac{2Q_s^4\left(k^4+p^4+2\,(k\cdot p)^2\right)}{k^2p^2\,(k-p)^4} + \frac{8Q_s^6\,(k+p)^4}{k^2p^2\,(k-p)^6}\right.$$
$$\left. + \frac{64Q_s^8\left(k^4+4(k\cdot p)^2+p^4+8(k\cdot p)(k^2+p^2)+14k^2p^2\right)}{k^2p^2\,(k-p)^8}\right]$$
$$+(p\to -p)$$

$$\sigma_4^{([(i)+(iii)].\mathrm{II})}(k,p) = \frac{\sigma_0^2}{N_4}Q_s^2(2\pi)^2\left[\delta^{(2)}(k+p)+\delta^{(2)}(k-p)\right]$$
$$N_4 = (N_c^2-1)S_\perp Q_s^2 \quad \times\frac{4Q_s^4}{k^{12}}e^{-\frac{k^2}{Q_s^2}}\left(k^4+k^2e^{\frac{k^2}{2Q_s^2}}Q_s^2+4e^{\frac{k^2}{2Q_s^2}}Q_s^4\right)^2$$

$$\sigma_4^{(ii)}(k,p) = \sigma_0^2\; e^{-\frac{k^2+p^2}{2Q_s^2}}$$
$$\times\left\{\frac{2Q_s^2}{k^2}-\frac{Q_s^2}{k^2}e^{\frac{k^2}{2Q_s^2}}+\frac{1}{2}\left[\mathrm{Ei}\left(\frac{k^2}{2Q_s^2}\right)-\mathrm{Ei}\left(\frac{k^2\lambda}{2Q_s^2}\right)\right]\right\}$$
$$\times\left\{\frac{2Q_s^2}{p^2}-\frac{Q_s^2}{p^2}e^{\frac{p^2}{2Q_s^2}}+\frac{1}{2}\left[\mathrm{Ei}\left(\frac{p^2}{2Q_s^2}\right)-\mathrm{Ei}\left(\frac{p^2\lambda}{2Q_s^2}\right)\right]\right\}$$
$$\sigma_0 = g_s^2(N_c^2-1)S_\perp\frac{\mu^2}{Q_s^2}\;,\;\; \lambda\sim 1/(S_\perp Q_s^2)$$

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$$\sigma_2^{(i)}(k,p) = \frac{\sigma_0^2}{2N_2}\frac{Q_s^4}{k^6p^6}e^{-\frac{k^2+p^2}{2Q_s^2}} \qquad N_2 = \frac{(N_c^2-1)^2}{N_c^3}S_\perp\mu^2$$
$$\times\left[k^4+e^{\frac{k^2}{2Q_s^2}}k^2Q_s^2+4e^{\frac{k^2}{2Q_s^2}}Q_s^4\right]\left[\left(2e^{\frac{p^2}{2Q_s^2}}-1\right)p^4+e^{\frac{p^2}{2Q_s^2}}p^2Q_s^2+4e^{\frac{p^2}{2Q_s^2}}Q_s^4\right]$$
$$+(k\leftrightarrow p)$$

$$N_2 = \frac{(N_c^2-1)^2}{N_c^3}S_\perp\mu^2$$
$$\sigma_2^{(ii)}(k,p) = \frac{\sigma_0^2}{2N_2}\left\{\frac{1}{(2\pi)^2}e^{-\frac{k^2+p^2}{2Q_s^2}}\int_{s_1,s_2}\frac{1}{s_1^2}\frac{1}{(s_1-s_2)^2}e^{-\frac{s_1^2+2k\cdot s_1}{2Q_s^2}}e^{-\frac{s_2^2-2p\cdot s_2}{2Q_s^2}}\right.$$
$$+\frac{1}{2\pi^2}e^{-\frac{k^2+p^2}{2Q_s^2}}\frac{k^i}{k^2}\int_{s_1,s_2}\frac{s_1^i}{s_1^2}\frac{1}{(s_1-s_2)^2}e^{-\frac{s_1^2+2k\cdot s_1}{2Q_s^2}}e^{-\frac{s_2^2-2p\cdot s_2}{2Q_s^2}}$$
$$\left.+\frac{Q_s^2}{4k^2}e^{-\frac{(k+p)^2}{4Q_s^2}}\left[\mathrm{Ei}\left(\frac{(k+p)^2}{4Q_s^2}\right)-\mathrm{Ei}\left(\frac{(k+p)^2\lambda}{4Q_s^2}\right)\right]\right\}$$
$$+(k\leftrightarrow p)$$

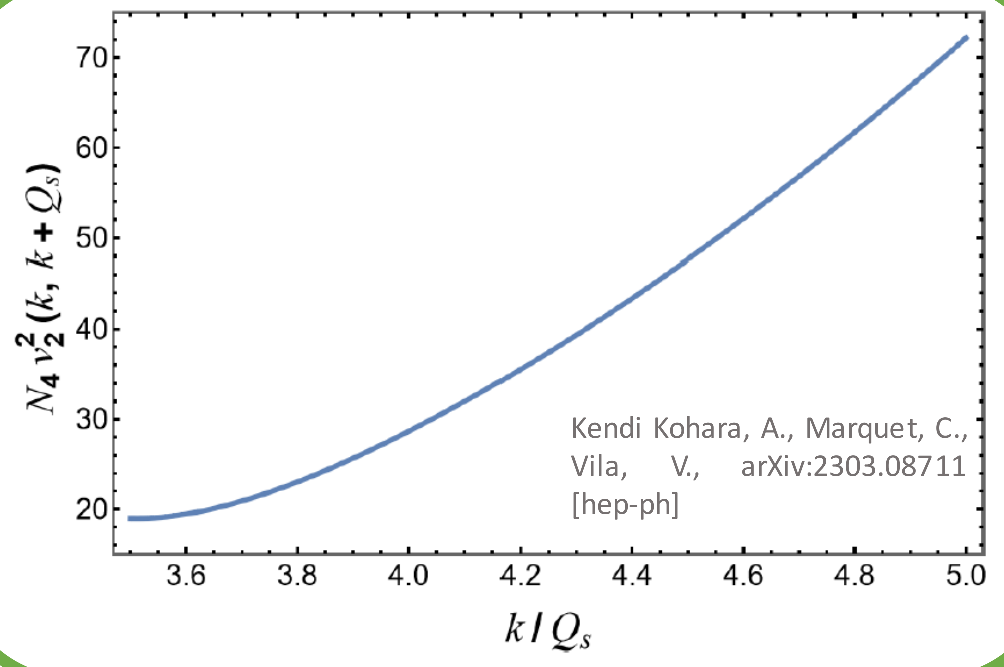
$$N_2 = \frac{(N_c^2-1)^2}{N_c^3}S_\perp\mu^2$$
$$\sigma_2^{[(iii)+(iv)]}(k,p) = \frac{\sigma_0^2}{2N_2}$$
$$\times\left\{\frac{1}{(2\pi)^2}e^{-\frac{k^2+p^2}{2Q_s^2}}\int_{s_1,s_2}\left[\left(\frac{s_1^i}{s_1^2}+\frac{k^i}{k^2}\right)^2\left(\frac{s_2^j}{s_2^2}+\frac{p^j}{p^2}\right)-\frac{1}{k^2}\frac{p^j}{p^2}\right]\right.$$
$$\times\frac{(s_1-s_2)^j}{(s_1-s_2)^2}\,e^{-\frac{s_1^2+2k\cdot s_1}{2Q_s^2}}e^{-\frac{s_2^2-2p\cdot s_2}{2Q_s^2}}$$
$$\left.+\frac{Q_s^4}{k^2}\frac{p\cdot(k+p)}{p^2(k+p)^2}\left(e^{-\frac{(k+p)^2}{4Q_s^2}}-1\right)\right\}$$
$$+(k\leftrightarrow p)$$

$\langle\sigma_2\rangle_{P,T}$ **CORRELATIONS are BIGGER than the** $\langle\sigma_4\rangle_{P,T}$ **ones.**

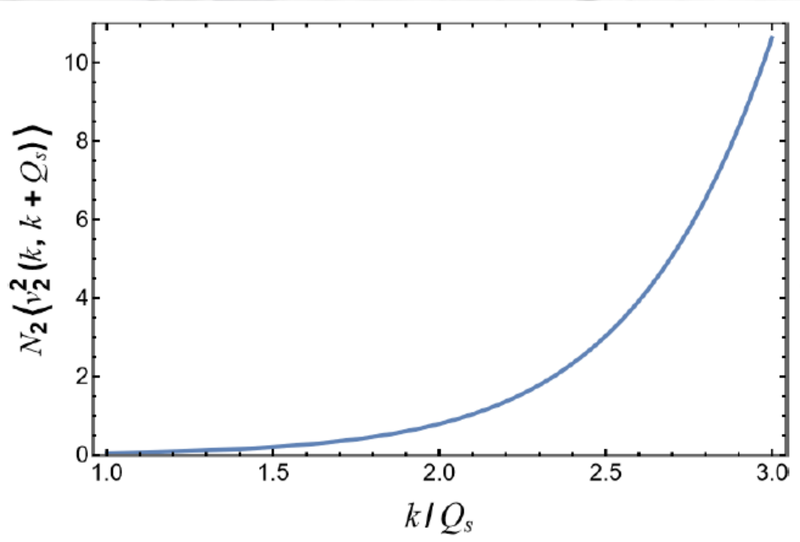
- $\sigma_4^{(ii)}$ is FULLY UNCORRELATED and reduces to the SQUARE of the S.I. C.S.
- $\sigma_4^{([(i)+(iii)].\mathrm{II})}$ embodies the HBT CORRELATIONS between the EMITTED GLUONS.
- $\langle\sigma_3\rangle_{P,T} = 0$ because of the MV MODEL ASSUMPTION.
- $\sigma_2^{(i)}$ does NOT CONTRIBUTE to the FH as it contains NO ANGULAR DEPENDENCE.
- $\sigma_2^{(ii),([(iii)+(iv)])}$ hold a STRONG CORRELATION: the DI-JET PEAK.

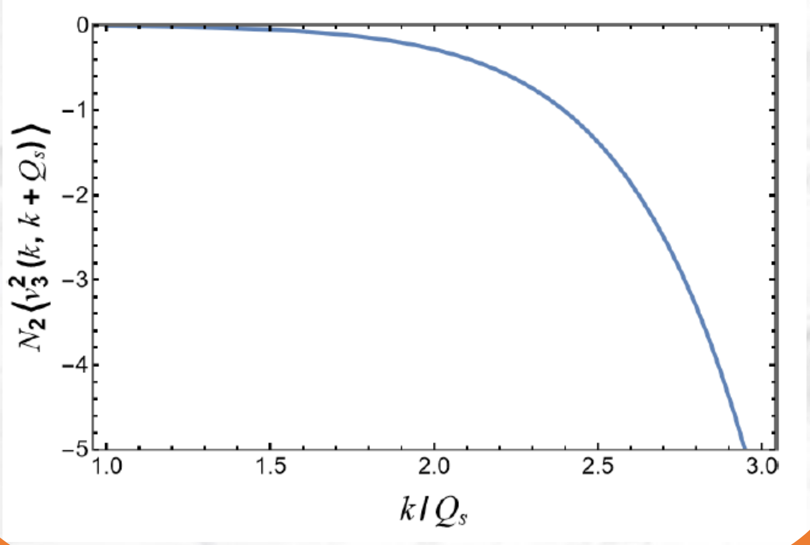
$$v_n^2(k,p) = \frac{\int d\phi_k d\phi_p e^{in(\phi_k - \phi_p)} \frac{d^2 N^{(2)}}{d^2 k d^2 p}}{\int d\phi_k d\phi_p \frac{d^2 N^{(2)}}{d^2 k d^2 p}}$$

$$\begin{aligned}\sigma_4^{([(i)+(iii)].\mathrm{I})}(k,p) = {} & 2\pi \frac{\sigma_0^2}{N_4}\, e^{-\frac{(k+p)^2}{4Q_s^2}} \\ & \times \left[\frac{2Q_s^4\left(k^4+p^4+2\,(k\cdot p)^2\right)}{k^2p^2\,(k-p)^4} + \frac{8Q_s^6\,(k+p)^4}{k^2p^2\,(k-p)^6} \right. \\ & \left. + \frac{64Q_s^8\left(k^4+4(k\cdot p)^2+p^4+8(k\cdot p)(k^2+p^2)+14k^2p^2\right)}{k^2p^2\,(k-p)^8} \right] \\ & + (p \to -p)\end{aligned}$$



- A numerical study **RESTRICTED TO** an **INTERMEDIATE WINDOW** of **TRANSVERSE MOMENTUM VALUES**.
- Obtained from the **TRADITIONAL PIECE**.
- **MOMENTUM VALUES** chosen for **BOSE-ENHANCEMENT EFECT CONTRIBUTION**, leaving the **HBT** effect **ASIDE**.





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$$\langle\sigma_2\rangle_{P,T}^{sub} = \frac{\sigma_0^2}{N_2}\left\{\frac{Q_s^2}{8}\left(\frac{1}{k^2}+\frac{1}{p^2}\right)e^{-\frac{(k+p)^2}{4Q_s^2}}\left[\mathrm{Ei}\left(\frac{(k+p)^2}{4Q_s^2}\right)-\mathrm{Ei}\left(\frac{(k+p)^2\lambda}{4Q_s^2}\right)\right] + \frac{Q_s^4}{2k^2p^2}\left(e^{-\frac{(k+p)^2}{4Q_s^2}}-1\right)\right\}$$

- $\sigma_2^{(i)}$ does **NOT SHOW** any **ANGULAR DEPENDENCE.**
- $\sigma_2^{(ii),([(iii)+(iv)])}$ contain **CORRELATIONS** associated with the **BACK-TO-BACK DI-JET PEAK.**
- v_2^2 values are **COMPARABLE** with the ones arising from $\sigma_4^{([(i)+(iii)].\mathrm{I})}$.
- v_3^2 shows **SIMILAR** magnitude as v_2^2 but with **OPPOSITE SIGN.**

- A work that extends existing studies on TWO-PARTICLE CORRELATIONS in PROTON-NUCLEUS COLLISIONS within the CGC APPROACH.

- This study incorporates RELEVANT CONTRIBUTIONS OVERLOOKED in the literature when implementing the DILUTE PROJECTILE LIMIT of the LEADING-ORDER DENSE-DENSE FORMULA.

- Averaging over the color charge configurations of the projectile via using the MV MODEL and invoking the AREA ENHANCEMENT MODEL to express all the TARGET AVERAGES in terms of the dipole scattering amplitude ALLOW TO WORK WITH TRACTABLE EXPRESSIONS.

- The GBW MODEL is employed to arrive at the FINAL EXPRESSIONS.

- $\langle\sigma_2\rangle_{P,T}$ CONTRIBUTES AS MUCH AS $\langle\sigma_4\rangle_{P,T}$ TO THE SECOND FOURIER COEFFICIENT, WHILE A NON-ZERO NEGATIVE v_3^2 IS OBTAINED FROM $\langle\sigma_2\rangle_{P,T}$.
- ORDERS OF MAGNITUDE OF OBTAINED NUMERICAL RESULTS MATCH THOSE RECENTLY MEASURED IN p+Au AND d+Au COLLISIONS BY THE PHENIX COLLABORATION.
- THE STUDY OF THE JET-LIKE CONTRIBUTIONS THAT SHOW UP $\langle\sigma_2\rangle_{P,T}$ DOES NOT FALL WITHIN THE SCOPE OF THIS WORK.
- IT WOULD BE WORTHWHILE TO GO A BIT FURTHER IN EXAMINING THE MAGNITUDE OF THE JET CORRELATION PRESENT ON TOP OF THE RIDGE STRUCTURE.

THANK YOU VERY MUCH FOR YOUR ATTENTION!