Hyperfine interactions in heavy quarkonia

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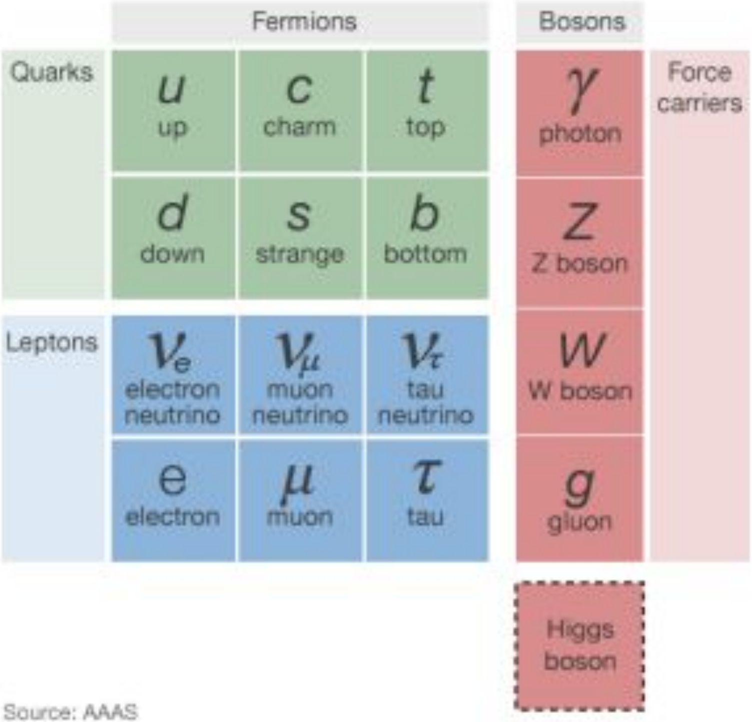
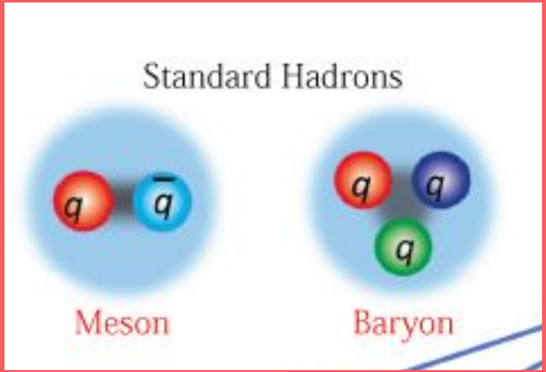
Standard Model



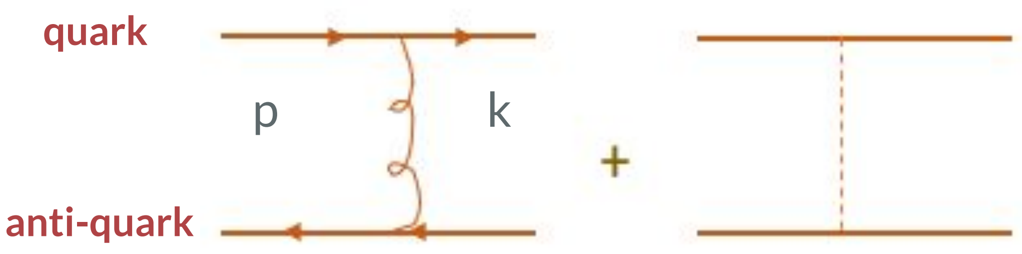
Source: AAAS

- → **hadrons:** bound states of strongly interacting quarks
- → **mesons:** quark-antiquark
- → **baryons**: 3 quarks



Quark and Anti-quark Potential

- ➔ One Gluon Exchange + Linear Confinement: $V(r) = -\frac{\alpha_s}{r} + \sigma r$
- ➔ Good description of the bottomonium spectrum
- ➔ Main purpose : study effects of hyperfine interaction



$$\frac{\alpha_s}{r} + \sigma r$$

First Step

- ➔ Write the base of the code to solve the Schrödinger equation in momentum space
- ➔ First test case : Calculate the binding energy of deuteron and the hydrogen atom
- ➔ Gauss Legendre Quadrature

$$E_b = -2.22452$$

$$\int_0^\infty dk\, f(k) \longrightarrow \sum_{i=1}^{N} w_i f(k_i)$$

nº points = 50	nº points = 100	nº points = 200	nº points = 500	nº points = 1000
-2.225825	-2.224652	-2.224646	-2.224646	-2.224646

Bound state Schrödinger equation in momentum space is a homogeneous integral equation

S-waves (l=0):

One-gluon-exchange — Linear confinement

$$\frac{p^2}{2\mu}\psi_0(p) - \frac{\alpha}{2\pi}\int_0^\infty dk\frac{k}{p}\ln\left(\frac{p+k}{p-k}\right)^2\psi_0(k) - \frac{4\sigma}{\pi}\int_0^\infty dk k^2\frac{\psi_0(k)-\psi_0(p)}{(p^2-k^2)^2} = E\psi_0(p)$$

Singular at $k = p$

With a subtraction technique, we can make the kernel regular

$$\frac{p^2}{2\mu}\psi_0(p) - \frac{\alpha\pi p}{2}\psi_0(p) - \frac{\alpha}{2\pi}\int_0^\infty dk\ln\left(\frac{p+k}{p-k}\right)^2\left[\frac{k}{p}\psi_0(k) - \frac{p}{k}\psi_0(p)\right] - \frac{2\sigma}{\pi}\int_0^\infty\frac{dk}{k^2-p^2}\left[\frac{2k^2}{k^2-p^2}\Big(\psi_0(k)-\psi_0(p)\Big) - p\psi_0'(p)\right] = E\psi_0(p)\,.$$

Regular (one-gluon-exchange term)

Regular (linear confinement term)

Coulomb Potential

n	l = 0		l = 1		l = 2	
	Model Values	Exact Values	Model Values	Exact Values	Model Values	Exact Values
1	-13.598320	-13.598321	-3.3995801	-3.3995802	-1.510924	-1.510924
2	-3.3995802	-3.3995802	-1.5109246	-1.510924	-0.849895	-0.849895
3	-1.5109247	-1.5109245	-0.8498951	-0.849895	-0.543933	-0.849895
4	-0.8498953	-0.849895	-0.5439334	-0.543933	-0.377732	-0.377731
5	-0.5439333	-0.849895	-0.377731	-0.377731	-0.277517	-0.277516

➔ Apply this to meson bound state problem

Linear Potential

Test case of the Schrodinger equation with linear potential : exact solution is known Airy Functions

n	l = 0		l = 1		l = 2	
	Model Values	Exact Values	Model Values	Exact Values	Model Values	Exact Values
1	2.334960	2.338107	3.359266	3.361258	4.443187	4.248187
2	4.080205	4.087949	4.877074	4.884455	5.652748	5.629114
3	5.506225	5.520560	6.192766	6.207626	6.964232	6.868888
4	6.763920	6.786708	7.381525	7.405667	8.017647	8.009707
5	7.911185	7.944134	8.480134	8.515235	9.126500	9.077007

Linear + Coulomb Potential

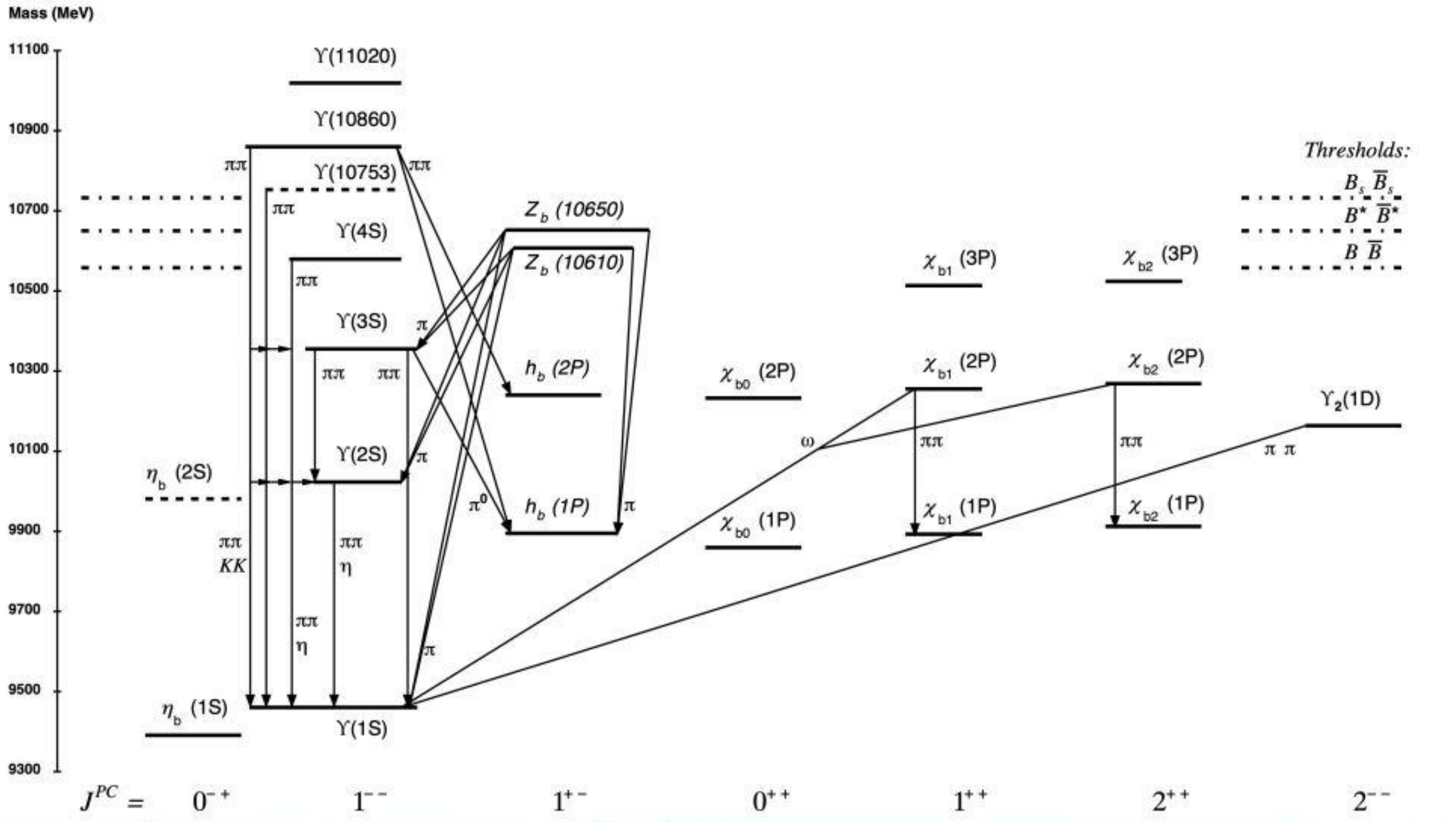
$$\left[\frac{p^2}{2m_R}+\frac{\sigma\pi}{2p}P'_\ell(1)-\frac{\alpha\pi p}{2}\right]\psi_\ell(p)-\frac{2\sigma}{\pi}\int_0^\infty dk\left\{\left[\frac{2k^2}{(k^2-p^2)^2}(P_\ell(y)\psi_\ell(k)-\psi_\ell(p))-\frac{p\psi'_\ell(p)}{k^2-p^2}\right]\right.$$
$$-\frac{1}{4p^2}\ln\left(\frac{p+k}{p-k}\right)^2\left[P'_\ell(y)\psi_\ell(k)-P'_\ell(1)\frac{p}{k}\psi_\ell(p)\right]+\left.\frac{w'_{\ell-1}(y)}{2p^2}\psi_\ell(k)\right\}$$
$$-\frac{\alpha}{\pi}\int_0^\infty dk\left\{\frac{1}{2}\ln\left(\frac{p+k}{p-k}\right)^2\left[\frac{k}{p}P_\ell(y)\psi_\ell(k)-\frac{p}{k}\psi_\ell(p)\right]-\frac{k}{p}w_{\ell-1}(y)\psi_\ell(k)\right\}=E\psi_\ell(p).$$

- ➔ Apply to bottomonium and charmonium

$m_b = 4.7477$ $\quad\sigma = 0.2140$

$m_c = 1.3703$ $\quad\alpha_s = 0.4174$

Bottomonium System



HyperFine Interactions in One Gluon Exchange

Spin-orbit

$$V(\vec{q}^{\,2}) = -\frac{4\pi\alpha_s}{q^2}\left\{1 - \frac{\vec{k}\cdot\vec{p}}{m^2} + \frac{3i}{4m^2}(\vec{p}\times\vec{k})\cdot(\vec{\sigma}_1 + \vec{\sigma}_2)\right.$$

$$\left. -\frac{1}{6m^2}\vec{q}^{\,2}(\vec{\sigma}_1\cdot\vec{\sigma}_2) + \frac{1}{4m^2}\left[\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q} - \frac{1}{3}\vec{q}^{\,2}(\vec{\sigma}_1\cdot\vec{\sigma}_2)\right]\right\}$$

spin-spin

Tensor Force

Pauli-Villars subtraction (form factors)

$$V(\vec{q}^{\,2}) = -4\alpha_s\pi\left(\frac{1}{q^2} - \frac{\Lambda^2}{q^2 + \Lambda^2}\right)$$

$$= -\frac{4\alpha_s\pi}{q^2}\left(\frac{\Lambda^2}{q^2 + \Lambda^2}\right)$$

$$\Lambda = 2\mathrm{m}_b$$

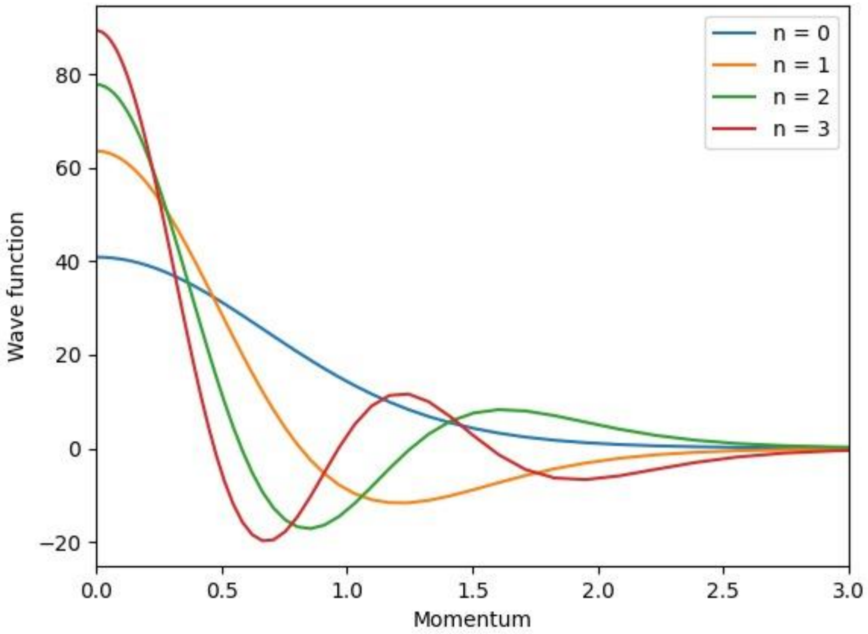
Bottomonium

n	Non-relativistic (without form factor)	Non-relativistic (with form factor)	Spin-spin dependence	Fully Relativistic
1	9.544758	9.543773	9.494522	9.15776
2	10.094968	10.083422	10.059072	9.85597
3	10.457809	10.443936	10.424913	10.20683
4	10.757798	10.742810	10.726324	10.47873
5	11.023094	11.007414	10.992496	10.71291

Charmonium

n	Non-relativistic (without form factor)	Non-relativistic (with form factor)	Spin-spin dependence	Fully Relativistic
1	3.236602	3.183976	3.133244	2.7707
2	3.886227	3.825244	3.791603	3.3697
3	4.379722	4.315851	4.287100	3.8003
4	4.804384	4.738877	4.714281	4.1651
5	5.187036	5.120418	5.098023	4.4921

Bottomonium Pseudoscalar Wave Functions



Summary and Conclusions

-Solved the schrodinger equation using a linear +Coulomb potential for bottomonium and charmonium

- For pseudoscalar mesons : Included the hyperfine interaction to the potential and we observed substantial effects which makes the masses smaller

- We compared our results with hyperfine interaction with fully relativistic calculations: they lie between a non-relativistic and fully relativistic as expected

What's next?

- ➔ Look to vector, scalar, axial, vector, ect mesons
- ➔ Include higher partial waves therefore Spin-orbit and tensor force dependence.
- ➔ Heavy-light mesons
- ➔ Improve the code for linear potential : use better method to calculate the derivative

Questions?