# Nucleon spin structure functions in a covariant quark model

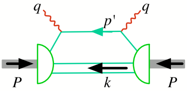
**Gilberto Ramalho**

**CFTP**, Instituto Superior Técnico, Universidade Técnica de Lisboa
gilberto.ramalho@cftp.ist.utl.pt

**In collaboration with** Franz Gross and M.T.Peña
**Phys. Rev. D85, 093006 (2012)**

LIP seminar, Lisbon
November 8, 2012

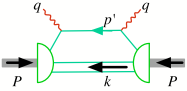
# Motivation and goals




- Describe **D**eep **I**nelastic **S**cattering (DIS)

$$W^{\mu\nu} = -2\pi \left\{ \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) W_1 - \left( P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left( P^\nu - \frac{P \cdot q}{q^2} q^\nu \right) \frac{W_2}{M^2} \right.$$
$$\left. + i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha S_\beta}{P \cdot q} (g_1 + g_2) - \frac{S \cdot q}{M(P \cdot q)} i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha P_\beta}{q \cdot P} g_2 \right\}$$

un**polarized PDF**: $W_1$, $W_2$; **polarized PDF**: $g_1$, $g_2$



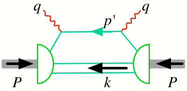

- Describe Deep Inelastic Scattering (DIS)

$$W^{\mu\nu} = -2\pi \left\{ \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) W_1 - \left( P^\mu - \frac{P\cdot q}{q^2} q^\mu \right) \left( P^\nu - \frac{P\cdot q}{q^2} q^\nu \right) \frac{W_2}{M^2} \right.$$
$$\left. + i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha S_\beta}{P\cdot q} (g_1 + g_2) - \frac{S\cdot q}{M(P\cdot q)} i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha P_\beta}{q\cdot P} g_2 \right\}$$

  un**polarized PDF**: $W_1$, $W_2$; **polarized PDF**: $g_1$, $g_2$

- ... using a constituent quark model (CQM)
  **Covariant spectator quark model:** PRC 77,015202 (2008)

# Motivation and goals



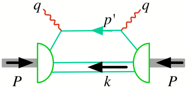

- Describe Deep Inelastic Scattering (DIS)

$$W^{\mu\nu} = -2\pi \left\{ \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) W_1 - \left( P^\mu - \frac{P\cdot q}{q^2} q^\mu \right) \left( P^\nu - \frac{P\cdot q}{q^2} q^\nu \right) \frac{W_2}{M^2} + i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha S_\beta}{P\cdot q}(g_1 + g_2) - \frac{S\cdot q}{M(P\cdot q)} i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha P_\beta}{q\cdot P} g_2 \right\}$$

un**polarized PDF**: $W_1$, $W_2$; **polarized PDF**: $g_1$, $g_2$

- ... using a constituent quark model (CQM)
  **Covariant spectator quark model:** PRC 77,015202 (2008)
- Valence quarks with orbital angular momentum

# Motivation and goals



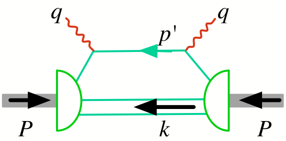

- Describe Deep Inelastic Scattering (DIS)

$$W^{\mu\nu} = -2\pi \left\{ \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) W_1 - \left( P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left( P^\nu - \frac{P \cdot q}{q^2} q^\nu \right) \frac{W_2}{M^2} + i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha S_\beta}{P \cdot q} (g_1 + g_2) - \frac{S \cdot q}{M(P \cdot q)} i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha P_\beta}{q \cdot P} g_2 \right\}$$

  un**polarized PDF**: $W_1$, $W_2$; **polarized PDF**: $g_1$, $g_2$

- ... using a constituent quark model (CQM)
  **Covariant spectator quark model:** PRC 77,015202 (2008)
- Valence quarks with orbital angular momentum
- Interpret the effects of orbital angular momentum states

# DIS in a quark model



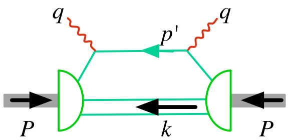

$$(J_{s_q,\lambda,\lambda_n})^\mu = -\bar{u}(p', s_q) j^\mu(q) (\Psi_N)_{\Lambda\lambda}(P,k)$$

**Hadronic tensor:**

$$W^{\mu\nu}(\lambda) = 3 \sum_{\Lambda, s_q} \iint_{p'k} (J^\dagger_{s_q,\Lambda,\lambda})^\mu (J_{s_q,\Lambda,\lambda})^\nu$$

$$\iint_{p'k} \equiv \iint \frac{d^3p'}{(2\pi)^3 2e_q} \frac{d^3k}{(2\pi)^3 2E_s} (2\pi)^4 \delta^4(p' + k - q - P)$$

# DIS in a quark model




$$(J_{s_q,\lambda,\lambda_n})^\mu = -\bar{u}(p', s_q) j^\mu(q) (\Psi_N)_{\Lambda\lambda}(P,k)$$

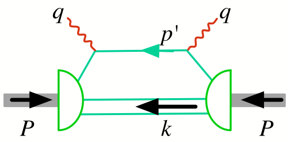
**Hadronic tensor:**

$$W^{\mu\nu}(\lambda) = 3 \sum_{\Lambda, s_q} \iint_{p'k} (J^\dagger_{s_q,\Lambda,\lambda})^\mu (J_{s_q,\Lambda,\lambda})^\nu$$

$$\iint_{p'k} \equiv \iint \frac{d^3p'}{(2\pi)^3 2e_q} \frac{d^3k}{(2\pi)^3 2E_s} (2\pi)^4 \delta^4(p' + k - q - P)$$

**Required:**

# DIS in a quark model




$$(J_{s_q,\lambda,\lambda_n})^\mu = -\bar{u}(p', s_q) j^\mu(q) (\Psi_N)_{\Lambda\lambda}(P,k)$$

**Hadronic tensor:**

$$W^{\mu\nu}(\lambda) = 3 \sum_{\Lambda, s_q} \iint_{p'k} (J^\dagger_{s_q,\Lambda,\lambda})^\mu (J_{s_q,\Lambda,\lambda})^\nu$$

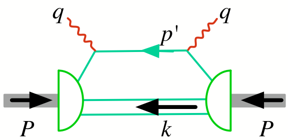
$$\iint_{p'k} \equiv \iint \frac{d^3p'}{(2\pi)^3 2e_q} \frac{d^3k}{(2\pi)^3 2E_s} (2\pi)^4 \delta^4(p' + k - q - P)$$

**Required:**

- Quark current: $\frac{\not{q} q^\mu}{q^2}$: include interaction currents behind impulse [Z. Batiz, F. Gross, PRC 58, 2963 (1998)]

$$j^\mu(q) = j_q(+\infty) \left( \gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + \mathcal{O}\left( \frac{i\sigma^{\mu\nu} q_\nu}{2M} \right)$$

# DIS in a quark model




$$(J_{s_q,\lambda,\lambda_n})^\mu = -\bar{u}(p', s_q) j^\mu(q) (\Psi_N)_{\Lambda\lambda}(P,k)$$

**Hadronic tensor:**

$$W^{\mu\nu}(\lambda) = 3 \sum_{\Lambda, s_q} \iint_{p'k} (J^\dagger_{s_q,\Lambda,\lambda})^\mu (J_{s_q,\Lambda,\lambda})^\nu$$

$$\iint_{p'k} \equiv \iint \frac{d^3p'}{(2\pi)^3 2e_q} \frac{d^3k}{(2\pi)^3 2E_s} (2\pi)^4 \delta^4(p' + k - q - P)$$

**Required:**

- Quark current: $\frac{\not{q} q^\mu}{q^2}$: include interaction currents behind impulse [Z. Batiz, F. Gross, PRC 58, 2963 (1998)]

$$j^\mu(q) = j_q(+\infty) \left( \gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + \mathcal{O}\left( \frac{i\sigma^{\mu\nu} q_\nu}{2M} \right)$$

- Nucleon wave function $(\Psi_N)_{\Lambda\lambda}$

# S-state (previous work): $W_1, W_2$

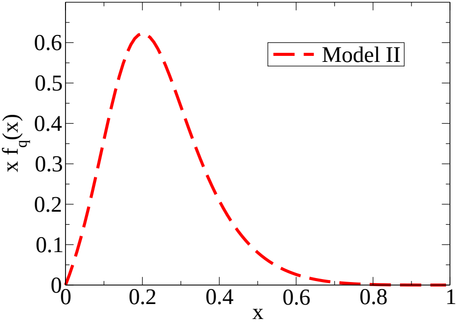
PRC 77,015202 (2008)

S-state approach

Qualitative description of DIS

**Callen-Gross scaling** $x = \frac{Q^2}{2M\nu}$

$$\nu W_2(x) = 2MxW_1(x)$$
$$= e_I^2 x f_q(x)$$




**Quark distribution function** (normalized to 1):

$$f_q(x) = \frac{\mathcal{N}}{4\pi} \int \frac{d^2k_\perp}{(2\pi)^2(1-x)} \psi_S^2(k_\perp; x).$$

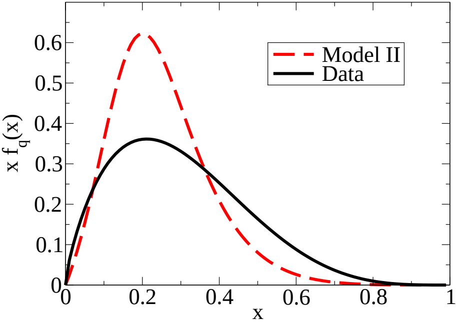
PRC 77,015202 (2008)

S-state approach

Qualitative description of DIS

**Callen-Gross scaling** $x = \frac{Q^2}{2M\nu}$

$$\nu W_2(x) = 2MxW_1(x)$$
$$= e_I^2 x f_q(x)$$




**Quark distribution function** (normalized to 1):

$$f_q(x) = \frac{\mathcal{N}}{4\pi} \int \frac{d^2k_\perp}{(2\pi)^2(1-x)} \psi_S^2(k_\perp; x).$$

# S-state (new result): $g_1$

Predicts $g_2 \equiv 0$, but

$$g_1(x) = \frac{5}{18} e_N f_q(x)$$

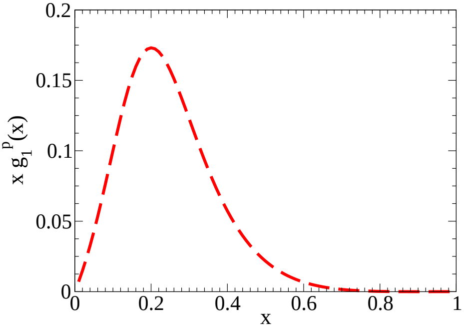
First moment (proton)

$$\Gamma_1 = \int dx g_1(x) = \frac{5}{18} = 0.28$$

$$>> \Gamma_1^{exp} = 0.17.$$

Proton spin problem $\Sigma \approx 0.3 \ll 1$

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Sigma}_{q \text{ spin}} + \underbrace{L_q}_{q \text{ OAM}} + \underbrace{L_q}_{\text{gluons}}$$



# S-state (new result): $g_1$

Predicts $g_2 \equiv 0$, but

$$g_1(x) = \frac{5}{18} e_N f_q(x)$$

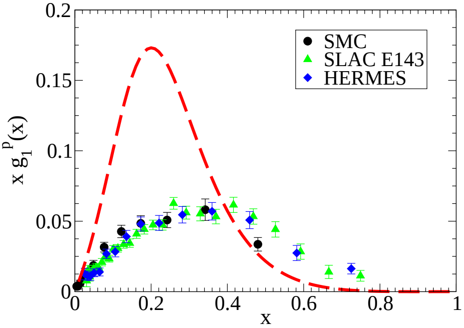
First moment (proton)

$$\Gamma_1 = \int dx g_1(x) = \frac{5}{18} = 0.28$$

$$>> \Gamma_1^{exp} = 0.17.$$

Proton spin problem $\Sigma \approx 0.3 \ll 1$

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Sigma}_{q \text{ spin}} + \underbrace{L_q}_{q \text{ OAM}} + \underbrace{L_q}_{\text{gluons}}$$



# S-state (new result): $g_1$

Predicts $g_2 \equiv 0$, but

$$g_1(x) = \frac{5}{18} e_N f_q(x)$$

First moment (proton)

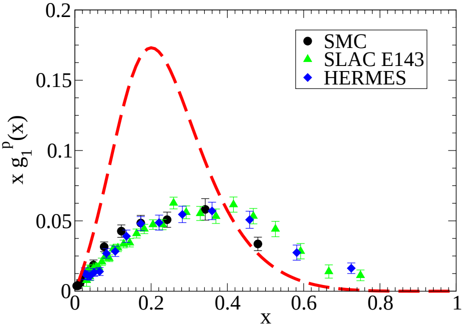
$$\Gamma_1 = \int dx g_1(x) = \frac{5}{18} = 0.28$$

$$>> \Gamma_1^{exp} = 0.17.$$

Proton spin problem $\Sigma \approx 0.3 \ll 1$

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Sigma}_{q \text{ spin}} + \underbrace{L_q}_{q \text{ OAM}} + \underbrace{L_q}_{\text{gluons}}$$

$S$-state quark are insuficient !!



# S-state (new result): $g_1$

Predicts $g_2 \equiv 0$, but

$$g_1(x) = \frac{5}{18} e_N f_q(x)$$

First moment (proton)

$$\Gamma_1 = \int dx g_1(x) = \frac{5}{18} = 0.28$$

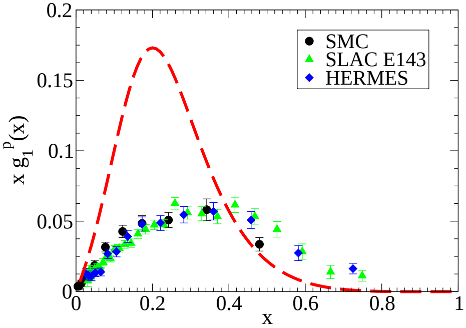
$$>> \Gamma_1^{exp} = 0.17.$$

Proton spin problem $\Sigma \approx 0.3 \ll 1$

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Sigma}_{q \text{ spin}} + \underbrace{L_q}_{q \text{ OAM}} + \underbrace{L_q}_{\text{gluons}}$$

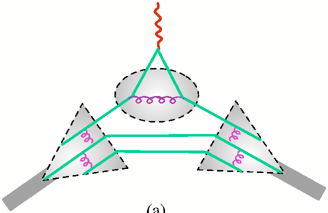
$S$-state quark are insuficient !!

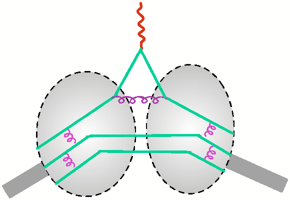
What about $P, D \oplus \Psi_u^L \neq \Psi_d^L$ ?




$$\Psi_N = \sum_q [n_S \Psi_q^S + n_P \Psi_q^P + n_D \Psi_q^D]$$

# Formalism (CSQM vs Light Front)


(a)


(b)

**Covariant Spectator QM view**

- Gluon interactions between $q\bar{q}$
  $\Rightarrow$ **quark form factors**
- Quarks dressed by gluons
  and $q\bar{q}$ interactions
- Massive quarks with anomalous magnetic moments $\kappa_u, \kappa_d$
- **Covariant formalism** with manifest rotational invariance

**Light Front view**

- Pointlike quarks
- Baryon states as a sum of Fock states:
  $qqq,\ qqqg,\ qqq(q\bar{q}), \ldots$
- **Light quarks**
  $\kappa_u, \kappa_d = 0$
- **Does not** handle rotational invariance

# Formalism (CSQM vs Light Front)

If **angular momentum** is the explanation for the **proton spin problem**, we need a **covariant formalism** with **manifest rotational invariance** and well defined **angular momentum states**

**Covariant Spectator QM view**

- Gluon interactions between $q\bar{q}$
  $\Rightarrow$ **quark form factors**
- Quarks dressed by gluons and $q\bar{q}$ interactions
- Massive quarks with anomalous magnetic moments $\kappa_u, \kappa_d$
- **Covariant formalism** with manifest rotational invariance

**Light Front view**

- Pointlike quarks
- Baryon states as a sum of Fock states:
  $qqq,\ qqqg,\ qqq(q\bar{q}), \ldots$
- **Light quarks**
  $\kappa_u, \kappa_d = 0$
- **Does not** handle rotational invariance

# Covariant Spectator Theory

**Covariant Spectator Theory** ©, Franz Gross *et al.*, applied to:
[See A. Stadler and F. Gross, FBS 49, 91 (2011)]

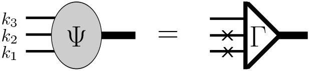
- $NN$ scattering, deuteron and three-nucleon bound states
- Deuteron and triton electromagnetic form factors
- $\pi N$ scattering and baryon resonances
- $q\bar{q}$ models of mesons

**Covariant Spectator Quark Model** (See arXiv:1008.0371 [hep-ph])
GR, F. Gross, M. T. Peña, K. Tsushima, ...

- Nucleon and $\Delta$ electromagnetic form factors
- Electromagnetic transition form factors $\gamma^* N \to N^*$
  $N^* = \Delta, N^*(1440), N^*(1535), \Delta(1600)$
- Octet baryon and decuplet baryon e. m. form factors:
  physical regime, nuclear matter and **extension to lattice QCD**
- $\Delta(1232)$ mass distribution on Dalitz decay: $\Delta \to N e^+ e^-$

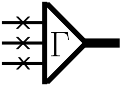
# Spectator QM: Baryon wave functions

- Baryon: 3 constituent quark system
- Covariant Spectator Theory: wave function $\Psi$ defined in terms of a 3-quark vertex $\Gamma$ with 2 on-mass-shell quarks




$$\Psi_\alpha(P,k_3) = \left(\frac{1}{m_q - \not{k}_3 - i\varepsilon}\right)_{\alpha\beta} \Gamma^\beta(P,k_1,k_2)$$

- Confinement insures that vertex $\Gamma$ vanishes when the 3 quarks are on-shell [$\Gamma$ cancels the quark propagator singularity]




$$= 0$$

Stadler, Gross and Frank PRC 56, 2396 (1998); Savkli and Gross PRC 63, 035208 (2001)

- $\Psi$ **free** of singularities $\Rightarrow$ modulate directly $\Psi$ (instead of $\Gamma$)

# Spectator QM: Baryon wave functions (2)

- Integrating over the on-mass-shell quark momenta:
  $k = k_1 + k_2$, $r = \frac{1}{2}(k_1 - k_2)$;
  reduce current integrals to the integration in $\mathbf{k}$ and $s = (k_1 + k_2)^2$
  F. Gross, GR and M. T. Peña: PRC 77, 015202 (2008); PRD 85, 093005 (2012)

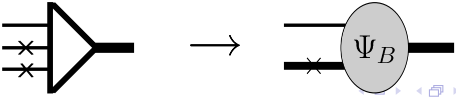
$$\int \frac{d^3 k_1}{2E_{k_1}} \int \frac{d^3 k_2}{2E_{k_2}} \quad = \quad \frac{\pi}{4} \int d\Omega_{\hat{\mathbf{r}}} \int_{4m_q^2}^{+\infty} ds \sqrt{\frac{s - 4m_q^2}{s}} \int \frac{d^3 \mathbf{k}}{2E_k}$$

  with $E_k = \sqrt{s + \mathbf{k}^2}$ as the energy of the diquark.

- Mean value theorem: average in diquark mass $\sqrt{s} \to m_D$

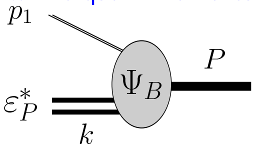
$$\int \frac{d^3 k_1}{2E_{k_1}} \int \frac{d^3 k_2}{2E_{k_2}} \to \int \frac{d^3 \mathbf{k}}{2\sqrt{m_D^2 + \mathbf{k}^2}}$$

  $m_D$=eff. mass; **covariant integration** in diquark **on-shell** momentum



S-state in quark-diquark [PRC 77,015202 (2008)]:

$k$ = diquark momentum




$$\Psi_S(P,k) = \frac{1}{\sqrt{2}}\left[\Phi_I^0 \Phi_S^0 + \Phi_I^1 \Phi_S^1\right] \psi_S(P,k)$$

$\Phi^0_{I,S}$ : anti-symmetric in the exchange of quark states $(12)$ - $M_A$

$\Phi^1_{I,S}$ : symmetric in the exchange of quark states $(12)$ - $M_S$

$$\Phi_I^0 \to \phi^0, \qquad \Phi_I^0 \to \phi^1$$

$$\Phi_S^0 \to u(P,\lambda), \qquad \Phi_S^1 \to -(\varepsilon^*_{\Lambda P})_\alpha U^\alpha(P,\lambda)$$

$\phi^{0,1}$ isospin operators acting in $\chi^I$ (nucleon isospin state)

$\varepsilon_P^\alpha$ diquark pol. vector: fixed-axis base [PRC 77, 015202 (2008)]

Vector spin 1/2 S-state $\left[1 \oplus \frac{1}{2} \to \frac{1}{2}\right]$: $U^\alpha(P,\lambda) = \frac{1}{\sqrt{3}}\gamma_5\left(\gamma^\alpha - \frac{P^\alpha}{M}\right) u(P,\lambda)$

# Nucleon wave function: P-state and D-states

S-state: quark-diquark [PRC 77, 015202 (2008)]: (review)

$$\Psi_S(P,k) = \frac{1}{\sqrt{2}}\left[\phi^0 U(P,\lambda) - \phi^1 (\varepsilon^*_\Lambda)_\alpha U^\alpha(P,\lambda)\right]\psi_S(P,k)$$

P-state: quark-diquark [PRD 77, 093005 (2012)]: $\tilde{k} = k - \frac{P\cdot k}{M^2}P$

$$\Psi_P(P,k) = \frac{1}{\sqrt{2}}\tilde{\not{k}}\left[\phi^0 U(P,\lambda) - \phi^1 (\varepsilon^*_\Lambda)_\alpha U^\alpha(P,\lambda)\right]\psi_P(P,k)$$

D-state: quark-diquark [PRD 77, 093005 (2012)]:

$$\Psi_D(P,k) \approx \frac{1}{\sqrt{2}}\left[\phi^0 \psi^{Da}_{\Lambda\lambda} + \phi^1 \psi^{Ds}_{\Lambda\lambda}\right]\phi_D(\mathbf{k}_1,\mathbf{k}_2)$$

Integration in $\mathbf{r}$: $\iint \frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3} = \iint \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{d^3\mathbf{r}}{(2\pi)^3} \to \int \frac{d^3\mathbf{k}}{(2\pi)^3}\tilde{\phi}(\mathbf{k})$

# Nucleon wave function: D-states (1)

D-state algebra (rest frame): $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$, $\mathbf{r} = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$

EPJA 36, 329 (2008); PRD 78, 114017 (2008)

$$U_m(\lambda) = \frac{1}{\sqrt{3}} \sigma_m |\tfrac{1}{2}\lambda\rangle \qquad D^{\ell m}(\mathbf{k}) = \mathbf{k}^\ell \mathbf{k}^m + \frac{1}{3}\mathbf{k}^2 \delta_{\ell m}$$

D-state function

$$\Theta^D_{\Lambda\lambda}(\mathbf{k}_i) = \frac{3}{\sqrt{2}} (\varepsilon^*_\Lambda)_\ell D^{\ell m}(\mathbf{k}_i) U_m(\lambda)$$

$$\begin{aligned}
\psi^{Da}_{\Lambda\lambda} &= \frac{1}{\sqrt{2}} \left[\Theta^D_{\Lambda\lambda}(\mathbf{k}_1) - \Theta^D_{\Lambda\lambda}(\mathbf{k}_2)\right] \phi_D(\mathbf{k}^2, \mathbf{r}^2) \\
&= \frac{3}{2} (\varepsilon^*_\Lambda)_\ell G^{\ell m}(\mathbf{k}, \mathbf{r}) U_m(\lambda)\, \phi_D(\mathbf{k}^2, \mathbf{r}^2) \\
\psi^{Ds}_{\Lambda\lambda} &\simeq \left[\cos\phi\ \Theta^D_{\Lambda\lambda}(\mathbf{k}) + \sin\phi\ \Theta^D_{\Lambda\lambda}(\mathbf{r})\right] \phi_D(\mathbf{k}^2, \mathbf{r}^2), \qquad \cos\phi = \frac{1}{5}
\end{aligned}$$

# Nucleon wave function: D-states (2)

$$G^{\ell m}(\mathbf{k},\mathbf{r}) = \mathbf{k}^\ell \mathbf{r}^m + \mathbf{r}^\ell \mathbf{k}^m + \frac{2}{3}(\mathbf{k}\cdot\mathbf{r})\delta_{\ell m}$$

$\mathbf{r}^\ell$: diquark with internal P-state; define spin-1 vector $\zeta_\nu^\ell$

$\Theta^D_{\Lambda\lambda}(\mathbf{r})$: diquark with internal D-state; define spin-1 vector $\varepsilon^m_{D\Lambda}$

$$\psi^{Da}_{\Lambda\lambda(\nu)} \quad \rightarrow \quad \frac{3}{\sqrt{20}} \overbrace{(\varepsilon^*_\Lambda)_\ell G^{\ell m}(\mathbf{k},\zeta_\nu) U_m(\lambda)}^{\ell=1} \; \psi_D(P,k)$$

$$\psi^{Ds}_{\Lambda\lambda} \quad \rightarrow \quad \frac{3\sqrt{2}}{\sqrt{5}} \overbrace{(\varepsilon_\Lambda)_\ell D^{\ell m}(\mathbf{k}) U_m(\lambda)}^{\ell=0} \; \psi_D(P,k) + \frac{1}{\sqrt{5}} \overbrace{\underbrace{\mathbf{k}^2 (\epsilon^*_{D\Lambda})^m}_{\ell=0} U_m(\lambda)} \; \psi_D(P,k)$$

# Nucleon wave function: summary

$$\Psi^L_{\Lambda\lambda}(P,k) = \mathcal{O}^L_\Lambda\, u(P,\lambda)$$

Covariant notation

$$\tilde{k}^\alpha = k^\alpha - \frac{P\cdot k}{M^2}P^\alpha$$

$$\tilde{\gamma}^\alpha = \gamma^\alpha - \frac{P^\alpha}{M^2}\not{P}$$

$$\tilde{g}^{\alpha\beta} = g^{\alpha\beta} - \frac{P^\alpha P^\beta}{M^2}$$

$$D^{\alpha\beta}(P,k) = \tilde{k}^\alpha\tilde{k}^\beta - \frac{1}{3}\tilde{k}^2\tilde{g}^{\alpha\beta}$$

$$G^{\alpha\beta}(\tilde{k},\zeta_\nu) = \tilde{k}^\alpha\zeta^\beta_\nu + \zeta^\alpha_\nu\tilde{k}^\beta - \frac{2}{3}(\tilde{k}\cdot\zeta_\nu)\tilde{g}^{\alpha\beta}$$

Replacements:

$\mathbf{k} \to -\tilde{k}$, $\mathbf{k}^2 \to -\tilde{k}^2$

$\delta_{\ell m} \to \tilde{g}^{\alpha\beta}$

- $S$-state

$$\mathcal{O}^{S,0}_\Lambda = \frac{1}{\sqrt{2}}\phi^0\psi_S(P,k)\mathbb{1}$$

$$\mathcal{O}^{S,1}_\Lambda = \frac{1}{\sqrt{2}}\phi^1\psi_S(P,k)(\varepsilon^*_\Lambda)_\alpha\gamma_5\tilde{\gamma}^\alpha$$

- $P$-state

$$\mathcal{O}^{P,0}_\Lambda = \frac{1}{\sqrt{2}}\phi^0\psi_P(P,k)\tilde{\not{k}}$$

$$\mathcal{O}^{P,1}_\Lambda = \frac{1}{\sqrt{2}}\phi^1\psi_P(P,k)\tilde{\not{k}}(\varepsilon^*_\Lambda)_\alpha\gamma_5\tilde{\gamma}^\alpha$$

- $D$-state

$$\mathcal{O}^{D,0}_\Lambda = \frac{\sqrt{3}}{\sqrt{2\sqrt{10}}}\phi^0|\tilde{k}|\psi_D(P,k)(\varepsilon^*_\Lambda)_\alpha \overbrace{G^{\alpha\beta}(\tilde{k},\zeta_\nu)}^{\ell=1}\gamma_5\tilde{\gamma}_\beta$$

$$\mathcal{O}^{D,1}_\Lambda = -\frac{1}{\sqrt{30}}\phi^1\tilde{k}^2\psi_D(P,k)\overbrace{(\epsilon^*_{D\Lambda})_\alpha}^{\ell=2}\gamma_5\tilde{\gamma}^\alpha$$

$$\mathcal{O}^{D,2}_\Lambda = \sqrt{\frac{3}{5}}\phi^1\psi_D(P,k)\overbrace{(\varepsilon^*_\Lambda)_\alpha D^{\alpha\beta}(P,k)}^{\ell=0}\gamma_5\tilde{\gamma}_\beta$$

# Nucleon wave function: summary (2)

- **Covariant** wave function consistent with **isospin** and **angular momentum**

$$\Psi_{\Lambda\lambda} = n_S \Psi^S_{\Lambda\lambda} + n_P \Psi^P_{\Lambda\lambda} + n_D \Psi^D_{\Lambda\lambda}$$

- $L$-states normalized

$$n_S^2 + n_P^2 + n_D^2 = 1$$

- We can also consider **Isospin breaking** $u \neq d$
- Radial wf $\psi_L(P,k)$ ?
  - **Not** determined by a dynamical equation
  - **Determined by DIS phenomenology**

- $S$-state

$$\mathcal{O}^{S,0}_\Lambda = \frac{1}{\sqrt{2}} \phi^0 \psi_S(P,k) \mathbb{1}$$

$$\mathcal{O}^{S,1}_\Lambda = \frac{1}{\sqrt{2}} \phi^1 \psi_S(P,k) (\varepsilon^*_\Lambda)_\alpha \gamma_5 \tilde{\gamma}^\alpha$$

- $P$-state

$$\mathcal{O}^{P,0}_\Lambda = \frac{1}{\sqrt{2}} \phi^0 \psi_P(P,k) \tilde{\not{k}}$$

$$\mathcal{O}^{P,1}_\Lambda = \frac{1}{\sqrt{2}} \phi^1 \psi_P(P,k) \tilde{\not{k}} (\varepsilon^*_\Lambda)_\alpha \gamma_5 \tilde{\gamma}^\alpha$$

- $D$-state

$$\mathcal{O}^{D,0}_\Lambda = \frac{\sqrt{3}}{\sqrt{2\sqrt{10}}} \phi^0 |\tilde{k}| \psi_D(P,k) (\varepsilon^*_\Lambda)_\alpha \overbrace{G^{\alpha\beta}(\tilde{k},\zeta_\nu)}^{\ell=1} \gamma_5 \tilde{\gamma}_\beta$$

$$\mathcal{O}^{D,1}_\Lambda = -\frac{1}{\sqrt{30}} \phi^1 \tilde{k}^2 \psi_D(P,k) \overbrace{(\epsilon^*_{D\Lambda})_\alpha}^{\ell=2} \gamma_5 \tilde{\gamma}^\alpha$$

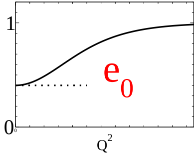
$$\mathcal{O}^{D,2}_\Lambda = \sqrt{\frac{3}{5}} \phi^1 \psi_D(P,k) \overbrace{(\varepsilon^*_\Lambda)_\alpha D^{\alpha\beta}(P,k)}^{\ell=0} \gamma_5 \tilde{\gamma}_\beta$$

# Normalization (1)

Wave function normalized by the nucleon charge ($Q^2 = 0$); $P = (M, 0, 0, 0)$

$$
\begin{aligned}
J^0 &= 3 \sum_{\Lambda} \int_k \overline{\Psi}_{\Lambda\lambda}(P,k) j_q(0) \gamma^0 \Psi_{\Lambda\lambda}(P,k) \\
&= \frac{1}{2}(1+\tau_3) e_0 \underbrace{\int_k \bar{|}\psi_N|^2}_{1}
\end{aligned}
$$

$$
\begin{aligned}
j_q(+\infty) &= \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3}, \quad Q_d = -\frac{1}{3} \\
j_q(0) &= e_0 \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3} e_0, \quad Q_d = -\frac{1}{3} e_0
\end{aligned}
$$




Wave functions normalization $Q^2 = 0$: (rest frame $\tilde{k}^2 = -\mathbf{k}^2$)

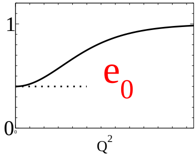
$$
e_0 \int_k |\psi_S|^2 = e_0 \int_k (-\tilde{k}^2)|\psi_P|^2 = e_0 \int_k \tilde{k}^4 |\psi_D|^2 = 1
$$

# Normalization (1)

Wave function normalized by the nucleon charge ($Q^2 = 0$); $P = (M, 0, 0, 0)$

$$\begin{aligned} J^0 &= 3 \sum_{\Lambda} \int_k \overline{\Psi}_{\Lambda\lambda}(P,k) j_q(0) \gamma^0 \Psi_{\Lambda\lambda}(P,k) \\ &= \frac{1}{2}(1+\tau_3) e_0 \underbrace{\int_k |\bar{\psi}_N|^2}_{1} \end{aligned}$$

$$\begin{aligned} j_q(+\infty) &= \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3}, \quad Q_d = -\frac{1}{3} \\ j_q(0) &= e_0 \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3} e_0, \quad Q_d = -\frac{1}{3} e_0 \end{aligned}$$




Wave functions normalization $Q^2 = 0$: (rest frame $\tilde{k}^2 = -\mathbf{k}^2$) $u \neq d$, $e_0 \to e_q^0$

$$e_q^0 \int_k |\psi_q^S|^2 = e_q^0 \int_k (-\tilde{k}^2) |\psi_q^P|^2 = e_q^0 \int_k \tilde{k}^4 |\psi_q^D|^2 = 1$$

# Breaking isospin symmetry

Write wave function in terms of $u$ and $d$ isospin states

$\psi_L$ **deppendent of the flavor of the quark 3**

Proton: $\chi^{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = u$; Neutron: $\chi^{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = d$

**Isospin-0 component:**

$$\Phi_I^0 \psi_L = \phi^0 \chi^I \psi_L = \frac{1}{\sqrt{2}}(ud - du) \begin{pmatrix} u \\ d \end{pmatrix} \psi_L \longrightarrow \phi^0 \chi^I \psi^L_{u(d)}$$

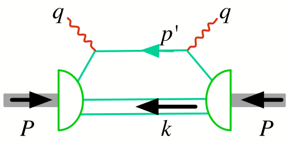
**Isospin-1 component:**

$$\Phi_I^1 \psi_L = \underbrace{-\frac{1}{\sqrt{6}}(ud + du) \begin{pmatrix} u \\ d \end{pmatrix}}_{\ell=0} \psi_L + \underbrace{\sqrt{\frac{2}{3}} \begin{pmatrix} (uu)d \\ -(dd)u \end{pmatrix}}_{\ell=\mp 1} \psi_L$$

$$\rightarrow \left(\phi^1_{\ell=0}\right) \chi^I \psi^L_{u(d)} + \left(\phi^1_{\ell=\mp 1}\right) \chi^I \psi^L_{d(u)}$$

$\psi_L \to \psi_q^L$: different distributions for $u$ and $d$

# DIS in a quark model (2)




$$(J_{s_q,\lambda,\lambda_n})^\mu = -\bar{u}(p', s_q) j^\mu(q) \Psi_{\Lambda\lambda}(P,k)$$

**Hadronic tensor:** $\Psi_{\Lambda\lambda} = \mathcal{O}_\Lambda u(P,\lambda)$, $S^\mu$ spin operator

$$W^{\mu\nu} = 3 \sum_{\Lambda, s_q} \iint_{p'k} \frac{1}{2} \mathrm{tr} \left\{ \mathcal{O}_\Lambda^\dagger j^\mu(q) \underbrace{(m_q + \not{p}')}_{\text{quark p}} j^\nu(q^2) \mathcal{O}_\Lambda \underbrace{(M + \not{P})(1 + \gamma_5 \not{S})}_{N \text{ spin-}S \text{ proj}} \right\}$$

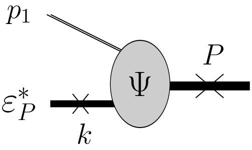
Integration: $E_s = \sqrt{m_s^2 + \mathbf{k}^2}$, $z = \cos\theta = \frac{k_z}{|\mathbf{k}|}$, $\quad p'^2 = m_q^2$

$|\mathbf{k}| = M\kappa$, $E_s = ME_\kappa$

$$\iint_{p'k} = \int \frac{d^4k}{(2\pi)^2} \delta_+(m_q^2 - p'^2)\delta_+(m_s^2 - k^2) = \int \frac{d^3\mathbf{k}}{(2\pi)^2(2E_s)} \overbrace{\delta\left(\frac{Q^2}{Mx}\left[(1-x) - E_s + |\mathbf{k}|z\right]\right)}^{\text{DIS condition}}$$

$$= \frac{M^2 x}{Q^2} \int_0^{+\infty} \frac{\kappa d\kappa}{4\pi E_\kappa} \int_{-1}^{1} dz \delta(z - z_0) = \frac{M^2 x}{Q^2} \int_{\kappa_{\min}}^{+\infty} \frac{\kappa d\kappa}{4\pi E_\kappa} = \frac{\pi x}{Q^2} \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi$$

$\psi_L(P,k)$ can be represented usind the covariant variable




$$\chi = \frac{(M-m_s)^2 - (P-k)^2}{Mm_s} = 2\frac{P\cdot k}{Mm_s} - 2$$

because $P^2 = M^2$ and $k^2 = m_s^2$.

Then, with $r = \frac{m_s}{M}$

$$\iint_{p'k} \psi^2(\chi) = \frac{\pi x}{Q^2}\frac{Mm_s}{16\pi^2}\int_\xi^{+\infty} d\chi\psi^2(\chi), \qquad \xi = \frac{r}{1-x} + \frac{1-x}{r} - 2$$

# DIS: wave functions and quark distribution functions

- Case $r = 1$: $M = m_s$

$$\xi = \frac{x^2}{(1-x)}$$

Wave functions will have only 1 singularity at $x = 0$

# DIS: wave functions and quark distribution functions

- Case $r = 1$: $M = m_s$

$$\xi = \frac{x^2}{(1-x)}$$

Wave functions will have only 1 singularity at $x = 0$ Good

# DIS: wave functions and quark distribution functions

- Case $r = 1$: $M = m_s$

$$\xi = \frac{x^2}{(1-x)}$$

Wave functions will have only 1 singularity at $x = 0$ Good

- We can calculate $f_q$ (S-state, Normalized to 1) PRC 77,015202 (2008)

$$f_q^S(x) = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

# DIS: wave functions and quark distribution functions

- Case $r = 1$: $M = m_s$

$$\xi = \frac{x^2}{(1-x)}$$

Wave functions will have only 1 singularity at $x = 0$ Good

- We can calculate $f_q$ (S-state, Normalized to 1) PRC 77,015202 (2008)

$$f_q^S(x) = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

- Inverse relation

$$\frac{df_q^S}{dx} = -\frac{x(2-x)}{(1-x)^2} \frac{M m_s}{16\pi^2} [\psi_q^S(\xi)]^2$$

# DIS: wave functions and quark distribution functions

- Case $r = 1$: $M = m_s$

$$\xi = \frac{x^2}{(1-x)}$$

Wave functions will have only 1 singularity at $x = 0$ Good

- We can calculate $f_q$ (S-state, Normalized to 1) PRC 77,015202 (2008)

$$f_q^S(x) = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

- Inverse relation

$$\frac{df_q^S}{dx} = -\frac{x(2-x)}{(1-x)^2} \frac{M m_s}{16\pi^2} [\psi_q^S(\xi)]^2$$

- If $f_q^S \approx x^\alpha (1-x)^\gamma$, we should have

$$[\psi_q^S(\xi)]^2 \approx \frac{1}{\xi^{1-\alpha/2}(\beta+\xi)^{1+\alpha/2}} \approx x^{\alpha-2}(1-x)^{\gamma+1}$$

$\beta$ dimessionless parameters (wf momentum scale in $M$ units)

# DIS: wave functions and normalization

$$f_q^S(x) = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

DIS normalization

WF normalization ($Q^2 = 0$)

$$\int_0^1 dx f_q^S(x) = 1,$$

$$e_q^0 \int_k |\psi_q^S(\chi)|^2 \big|_{Q^2=0} = e_q^0 \underbrace{\int_{-\infty}^1 dx f_q^S(x)}_{N} = 1$$

How to fix $e_q^0$ ?

$$e_q^0 = \frac{1}{N} = \frac{\int_0^1 dx f_q^S(x)}{\int_{-\infty}^0 dx f_q^S(x) + \int_0^1 dx f_q^S(x)} < 1$$

# DIS: wave functions and normalization

$$f_q^S(x) = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

DIS normalization

WF normalization ($Q^2 = 0$)

$$\int_0^1 dx f_q^S(x) = 1,$$

$$e_q^0 \int_k |\psi_q^S(\chi)|^2 |_{Q^2=0} = e_q^0 \underbrace{\int_{-\infty}^1 dx f_q^S(x)}_{N} = 1$$

How to fix $e_q^0$ ?

$$e_q^0 = \frac{1}{N} = \frac{\int_0^1 dx f_q^S(x)}{\int_{-\infty}^0 dx f_q^S(x) + \int_0^1 dx f_q^S(x)} < 1$$

- DIS define $\psi_q^L$ for $0 < x < 1$

# DIS: wave functions and normalization

$$f_q^S(x) = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

DIS normalization

$$\int_0^1 dx f_q^S(x) = 1,$$

WF normalization ($Q^2 = 0$)

$$e_q^0 \int_k |\psi_q^S(\chi)|^2 \Big|_{Q^2=0} = e_q^0 \underbrace{\int_{-\infty}^1 dx f_q^S(x)}_{N} = 1$$

How to fix $e_q^0$ ?

$$e_q^0 = \frac{1}{N} = \frac{\int_0^1 dx f_q^S(x)}{\int_{-\infty}^0 dx f_q^S(x) + \int_0^1 dx f_q^S(x)} < 1$$

- DIS define $\psi_q^L$ for $0 < x < 1$
- $f_q^S$ for $x < 0 \Rightarrow$ determine quark charge ($e_q^0$) at $Q^2 = 0$

# DIS: gluon effects

Gluon contributions are not considered in this model
Only valence quarks (no sea quarks):

$$\int_0^1 dx f_u^S(x) = \int_0^1 dx f_d^S(x) = 1,$$
$$\frac{1}{3}\int_0^1 dx \left[2 f_u^S(x) + f_d^S(x)\right] = 1 \qquad \text{[proton charge]}$$

**Proton momentum sum rule:** $N_g$ gluon contribution, $N_g \approx 0.5$

$$2\int_0^1 dx x f_u^S(x) + \int_0^1 dx x f_d^S(x) + N_g = 1.$$

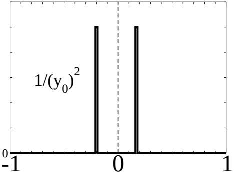
$S$-state approximation $(n_P = n_D = 0)$ and $f_u = f_d \equiv f_q$:

$$\int_0^1 dx x f_q^S(x) = 0.167, \qquad e_q^0 \int_{-\infty}^1 dx f_q^S(x) = 1$$

# DIS: Toy model

Quark distribution function given by Dirac function (symmetry of $\xi$ invariance in the transformation $1-x \to \frac{1}{1-x}$):

$$f_q^S(x) = \underbrace{\delta(1-x-y_0)}_{\text{pole } x=1-y_0} + \underbrace{\delta\left(\frac{1}{1-x} - y_0\right)}_{\text{pole } x=\frac{y_0-1}{y_0}<0}$$




**Result:**

$$\int_0^1 dx\, x f_q^S(x) = 1 - y_0, \qquad e_q^0\left(1+\frac{1}{y_0^2}\right) = 1$$

$$\Rightarrow y_0 = 0.833,\ e_q^0 = 0.41$$

# Structure functions (1)

Use elementary structure functions $\int_\chi = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi$

$$f_q^L(x) = \int_\chi k^{2L} \left[\psi_q^L(\chi)\right]^2 \qquad L = 0,1,2 \quad (S,P,D)$$

$$g_q^L(x) = \int_\chi P_2(z_0) k^{2L} \left[\psi_q^L(\chi)\right]^2 \qquad L = 1,2 \quad (P,D)$$

$$d_q(x) = \int_\chi P_2(z_0) k^2 \psi_q^S(\chi) \psi_q^D(\chi) \qquad (SD \text{ interference})$$

$$h_q^0(x) = \int_\chi z_0 k \, \psi_q^S(\chi) \psi_q^P(\chi) \qquad (SP \text{ interference})$$

$$h_q^2(x) = \int_\chi z_0 k^3 \, \psi_q^P(\chi) \psi_q^D(\chi) \qquad (PD \text{ interference})$$

$$h_q^1(x) = \int_\chi (1 - z_0^2) \frac{k^2}{4Mx} \, \psi_q^S(\chi) \psi_q^P(\chi) \qquad (SP \text{ interference})$$

$$h_q^3(x) = \int_\chi (1 - z_0^2) \frac{k^4}{4Mx} \, \psi_q^P(\chi) \psi_q^D(\chi) \qquad (PD \text{ interference})$$

# Structure functions (2) [Using charge symmetry]

$f_u(x) = u$-distribution in the proton [$d$-distribution in the neutron]
$f_d(x) = d$-distribution in the proton [$u$-distribution in the neutron]

$g_i^u(x)$: $u$-contribution of $g_i$ in the proton [$d$-contribution of $g_i$ in the neutron]
$g_i^d(x)$: $d$-contribution of $g_i$ in the proton [$u$-contribution of $g_i$ in the neutron]

**Proton:**

$$f_p(x) = \sum_q e_q^2 f_q(x) \qquad g_i^p(x) = \frac{1}{2}\sum_q e_q^2 g_i^q(x)$$

**Neutron:** $e_u \leftrightarrow e_d$ $\qquad f_q = n_S^2 f_q^S + n_P^2 f_q^P + n_D^2 f_q^D - 2 n_S n_P h_q^0$

$$\begin{aligned}
g_1^u &= \frac{2}{3} f_u - n_D^2 f_u^D - \frac{8}{9} n_P^2 d_u + \frac{8}{9} n_P^2 g_u^P + \frac{29}{60} n_D^2 g_u^D - \frac{2}{9} a_{SD} d_u + \frac{2}{9} a_{PD} h_u^2 \\
g_1^d &= -\frac{1}{3} f_d + \frac{8}{15} n_D^2 g_d^D + \frac{4}{9} n_P^2 f_d^P - \frac{4}{9} n_P^2 g_d^P - \frac{8}{9} a_{SD} d_d + \frac{8}{9} a_{PD} h_d^2 \\
g_2^u &= -\frac{4}{3} n_P^2 g_u^P - \frac{29}{40} n_D^2 g_u^D + \frac{1}{3} a_{SD} d_u - \frac{4}{3} n_S n_P (h_u^1 - h_u^0) + \frac{2}{9} a_{PD} (h_u^3 - h_u^2) \\
g_2^d &= +\frac{2}{3} n_P^2 g_d^P - \frac{4}{5} n_D^2 g_d^D + \frac{4}{3} a_{SD} d_d + \frac{2}{3} n_S n_P (h_d^1 - h_d^0) + \frac{2}{9} a_{PD} (h_d^3 - h_d^2)
\end{aligned}$$

# Structure functions (2) [Using charge symmetry]

$f_u(x) = u$-distribution in the proton [$d$-distribution in the neutron] $a_{SD} = -3\sqrt{\frac{2}{5}} n_S n_D$

$f_d(x) = d$-distribution in the proton [$u$-distribution in the neutron] $a_{PD} = -3\sqrt{\frac{2}{5}} n_P n_D$

$g_i^u(x)$: $u$-contribution of $g_i$ in the proton [$d$-contribution of $g_i$ in the neutron]

$g_i^d(x)$: $d$-contribution of $g_i$ in the proton [$u$-contribution of $g_i$ in the neutron]

**Proton:**

$$f_p(x) = \sum_q e_q^2 f_q(x) \qquad g_i^p(x) = \frac{1}{2} \sum_q e_q^2 g_i^q(x)$$

**Neutron:** $e_u \leftrightarrow e_d$ $\qquad f_q = n_S^2 f_q^S + n_P^2 f_q^P + n_D^2 f_q^D - 2 n_S n_P h_q^0$

$$g_1^u = \frac{2}{3} f_u - n_D^2 f_u^D - \frac{8}{9} n_P^2 d_u + \frac{8}{9} n_P^2 g_u^P + \frac{29}{60} n_D^2 g_u^D - \frac{2}{9} a_{SD} d_u + \frac{2}{9} a_{PD} h_u^2$$

$$g_1^d = -\frac{1}{3} f_d + \frac{8}{15} n_D^2 g_d^D + \frac{4}{9} n_P^2 f_d^P - \frac{4}{9} n_P^2 g_d^P - \frac{8}{9} a_{SD} d_d + \frac{8}{9} a_{PD} h_d^2$$

$$g_2^u = -\frac{4}{3} n_P^2 g_u^P - \frac{29}{40} n_D^2 g_u^D + \frac{1}{3} a_{SD} d_u - \frac{4}{3} n_S n_P (h_u^1 - h_u^0) + \frac{2}{9} a_{PD} (h_u^3 - h_u^2)$$

$$g_2^d = +\frac{2}{3} n_P^2 g_d^P - \frac{4}{5} n_D^2 g_d^D + \frac{4}{3} a_{SD} d_d + \frac{2}{3} n_S n_P (h_d^1 - h_d^0) + \frac{2}{9} a_{PD} (h_d^3 - h_d^2)$$

# Functional form for the wave functions

**Wave functions for** $L = S, P, D$**:** $\theta \neq 0$ $\quad \kappa^2 = \frac{1}{4}\chi(\chi + 4)$

$$\psi_q^S(\chi) = \frac{1}{N_q^S}\frac{1}{m_s}\frac{\beta \cos\theta + \chi \sin\theta}{\chi^{n_0}(\beta + \chi)^{n_1 - n_0}}$$

$$\psi_q^P(\chi) = \frac{1}{N_q^P}\frac{1}{M\kappa}\frac{1}{m_s}\frac{\beta \cos\theta + \chi \sin\theta}{\chi^{n_0}(\beta + \chi)^{n_1 - n_0}}$$

$$\psi_q^D(\chi) = \frac{1}{N_q^D}\frac{1}{(M\kappa)^2}\frac{1}{m_s}\frac{\beta \cos\theta + \chi \sin\theta}{\chi^{n_0}(\beta + \chi)^{n_1 - n_0}}$$

$\beta, \theta, n_0, n_1$ deppend of $L$ and $q$

**Normalization:**

$$e_q^0 \int_\chi (-\tilde{k}^2)^L |\psi_q^L(\chi)|^2 = 1$$

**Data:**
SMS, SLAC, HERMES, Jlab and COMPASS
obtained for several regions of $Q^2$ (not only large $Q^2$)
**Fits to the data:** $Q^2 = 1$ GeV$^2$

- Unpolarized $$\int_0^1 dx\, f_q^{\exp}(x) = 1$$

  Martin, Roberts, Stirling and Thorne, PLB 531, 216 (2002)-(MRST02)

$$x\, f_u^{\exp}(x) = \mathbf{0.130}\, x^{0.31}(1-x)^{3.50}(1+3.83\sqrt{x}+37.65x)$$
$$x\, f_d^{\exp}(x) = \mathbf{0.061322}\, x^{0.35}(1-x)^{4.03}(1+49.05\sqrt{x}+8.65x)$$

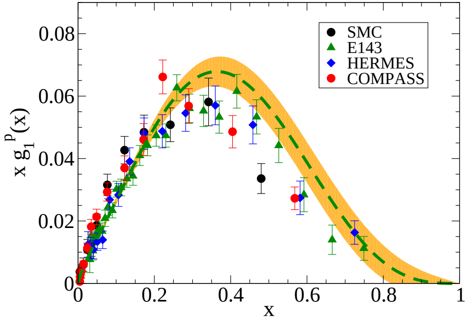
- Polarized: Leader, Sidorov and Stamenov (LSS10)
  PRD 82. 114018 (2010), $Q^2 = 1$ GeV$^2$

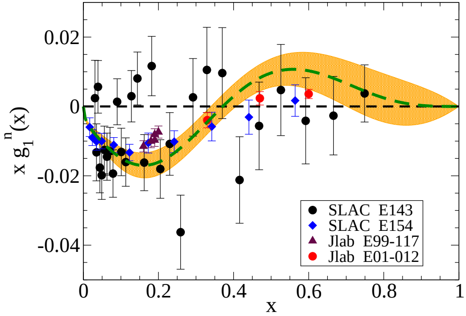
$$g_1^q(x) = \Delta q(x) + \frac{h^q(x)}{Q^2}$$

$$x\Delta u(x) = 0.548\, x^{0.782}(1-x)^{3.335}(1-1.779\sqrt{x}+10.2\, x)$$
$$x\Delta d(x) = -0.394\, x^{0.547}(1-x)^{4.056}(1+6.758\, x)$$

# Data (polarized)

$$g_1^q(x) = \Delta q(x) + \frac{h^q(x)[1 \pm \delta_q(x)]}{Q^2}$$ ← high twist corrections




$$\Gamma_1^p = \int_0^1 dx f_{1p}^{\text{exp}}(x) = 0.128 \pm 0.013$$

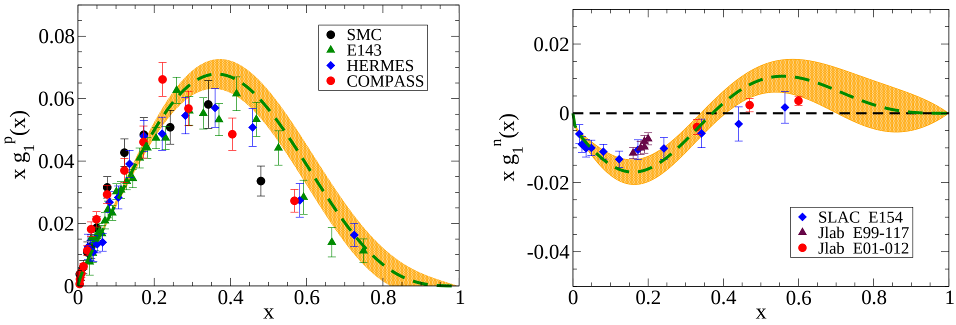
$$\Gamma_1^n = \int_0^1 dx f_{1n}^{\text{exp}}(x) = -0.042 \pm 0.013$$

$$\Gamma_1^u = \frac{3}{5}(4\Gamma_1^p - \Gamma_1^n) = 0.333 \pm 0.039$$

$$\Gamma_1^d = \frac{6}{5}(4\Gamma_1^n - \Gamma_1^p) = -0.355 \pm 0.080$$

# Data (polarized)

$$g_1^q(x) = \Delta q(x) + \frac{h^q(x)[1 \pm \delta_q(x)]}{Q^2}$$ ← high twist corrections




$$\Gamma_1^p = \int_0^1 dx\, f_{1p}^{\rm exp}(x) = 0.128 \pm 0.013$$

$$\Gamma_1^n = \int_0^1 dx\, f_{1n}^{\rm exp}(x) = -0.042 \pm 0.013$$

$$\Gamma_1^u = \frac{3}{5}(4\Gamma_1^p - \Gamma_1^n) = 0.333 \pm 0.039$$

$$\Gamma_1^d = \frac{6}{5}(4\Gamma_1^n - \Gamma_1^p) = -0.355 \pm 0.080$$

**Proton:**

$$f_p(x) = \sum_q e_q^2 f_q(x) \qquad g_i^p(x) = \frac{1}{2}\sum_q e_q^2 g_i^q(x)$$

**Neutron:** $e_u \leftrightarrow e_d$ $\qquad f_q = n_S^2 f_q^S + n_P^2 f_q^P + n_D^2 f_q^D - 2 n_S n_P h_q^0$

$$
\begin{aligned}
g_1^u &= \frac{2}{3} f_u - n_D^2 f_u^D - \frac{8}{9} n_P^2 d_u + \frac{8}{9} n_P^2 g_u^P + \frac{29}{60} n_D^2 g_u^D - \frac{2}{9} a_{SD} d_u + \frac{2}{9} a_{PD} h_u^2 \\
g_1^d &= -\frac{1}{3} f_d + \frac{8}{15} n_D^2 g_d^D + \frac{4}{9} n_P^2 f_d^P - \frac{4}{9} n_P^2 g_d^P - \frac{8}{9} a_{SD} d_d + \frac{8}{9} a_{PD} h_d^2 \\
g_2^u &= -\frac{4}{3} n_P^2 g_u^P - \frac{29}{40} n_D^2 g_u^D + \frac{1}{3} a_{SD} d_u - \frac{4}{3} n_S n_P (h_u^1 - h_u^0) + \frac{2}{9} a_{PD} (h_u^3 - h_u^2) \\
g_2^d &= +\frac{2}{3} n_P^2 g_d^P - \frac{4}{5} n_D^2 g_d^D + \frac{4}{3} a_{SD} d_d + \frac{2}{3} n_S n_P (h_d^1 - h_d^0) + \frac{2}{9} a_{PD} (h_d^3 - h_d^2)
\end{aligned}
$$

# Fits to the data

Spectator QM: very rich structure in the DIS regime

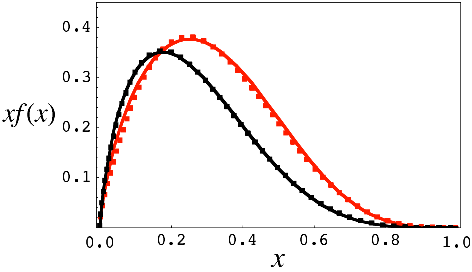
**How can we study the effect of the individual $L$ states ?**

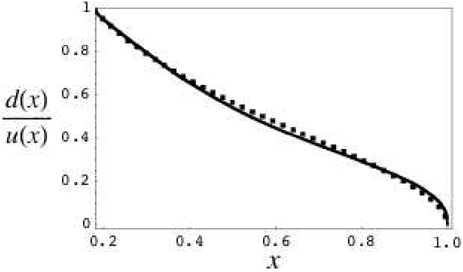
**Fiting process (MRST02 & LSS10 parametrizations):**

- Step 1: S-state component - fited to unpolarized PDFs
  (Adjust $\beta_{Sq}, \theta_{Sq}, n_{0Sq}$ and $n_{1Sq}$)
- Step 2: Estimate the strength of the $P$ and $D$ states $(n_P, n_D)$
  Uses the moments $\Gamma_1^u, \Gamma_1^d$
- Step 3: Global fit
  Improves the description **breaking the symmetry**
  between $S$, $P$ and $D$ radial wave functions

# Fits to the data: step 1 $q(x) \equiv f_q(x)$

| | $\beta_{Sq}$ | $\theta_{Sq}$ | $n_{0Sq}$ | $n_{1Sq}$ | $C_q^S$ | $e_q^0$ |
|---|---|---|---|---|---|---|
| $u$ | **0.9** | **0.4**$\pi$ | **0.51** | 3 | 2.197 | 0.3545 |
| $d$ | **1.25** | $\frac{1}{4}\pi$ | **0.49** | **3.2** | 2.279 | 0.3940 |




$f_u$, $f_d$

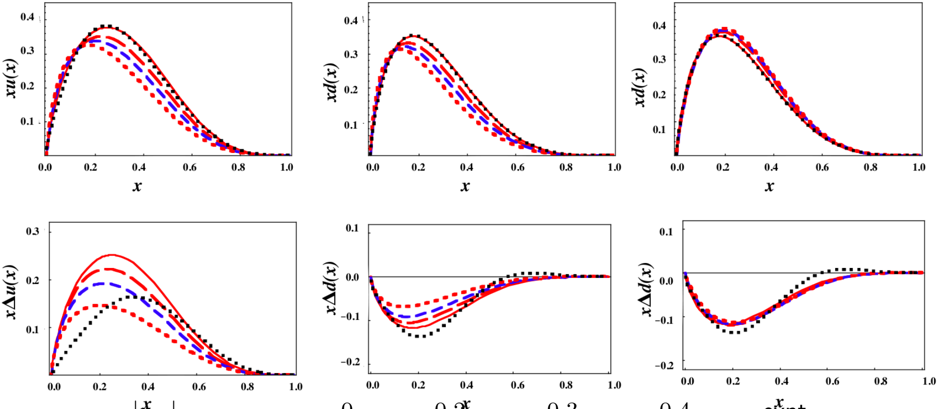
$$x f_u(x) \approx x^{0.20}(1-x)^{3.0} \qquad x f_u^{\exp}(x) \approx x^{0.31}(1-x)^{3.50}$$

$$x f_d(x) \approx x^{0.36}(1-x)^{3.4} \qquad x f_d^{\exp}(x) \approx x^{0.35}(1-x)^{4.03}$$

# Fits to the data: step 2: adjust $n_P$ $\Delta q(x) \equiv g_1^q(x)$

**Effect of the $P$ state:** $f_q \simeq f_q^S - 2n_S n_P h_q^0$, $\psi_q^S \approx (M\kappa)\psi_q^P$

$\cdots$ **Fit**; —— 0.0; — — 0.2; - - - 0.4; $\cdot\cdot\cdot$ 0.6 $n_P < 0$

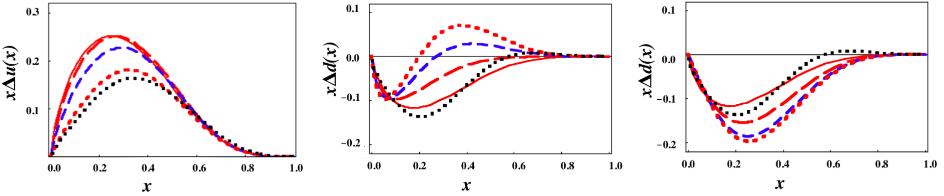



| $\lvert n_P \rvert$ | 0 | 0.2 | 0.3 | 0.4 | expt |
|---|---|---|---|---|---|
| $\Gamma_1^u\ (n_P > 0)$ | 0.667 | 0.627 | 0.563 | 0.448 | 0.333 |
| $\Gamma_1^d\ (n_P > 0)$ | −0.333 | −0.313 | −0.280 | −0.223 | −0.355 |
| $\Gamma_1^d\ (n_P < 0)$ | −0.333 | −0.321 | −0.307 | −0.289 | −0.355 |

# Fits to the data: step 2: adjust $n_D$

**Effect of the** $D$ **state**: $\psi_q^S \approx (M\kappa)^2 \psi_q^D$

· · · **LSS10**: —— 0.0; — — 0.2; - - - 0.4; · · · 0.6 $n_D < 0$




| $\lvert n_D \rvert$ | 0 | 0.2 | 0.4 | 0.6 | expt |
|---|---|---|---|---|---|
| $\Gamma_1^u\,(n_D > 0)$ | 0.667 | 0.643 | 0.544 | 0.367 | 0.333 |
| $\Gamma_1^d\,(n_D > 0)$ | −0.333 | −0.293 | −0.252 | −0.218 | −0.355 |
| $\Gamma_1^d\,(n_D < 0)$ | −0.333 | −0.369 | −0.395 | −0.404 | −0.355 |

# Fits to the data: step 2

Same funcional form for all states:
$\psi_q^S \approx (M\kappa)\psi_q^P \approx (M\kappa)^2\psi_q^D$

**Fit $n_P$ and $n_D$ to the moments $\Gamma_1^q$**

$$\Gamma_1^u = 0.333 \pm 0.039, \qquad \Gamma_1^d = -0.355 \pm 0.080$$

| solution | $n_P(u)$ | $n_D(u)$ | $n_P(d)$ | $n_D(d)$ |
|---|---|---|---|---|
| 1 | 0.43 | 0.18 | −0.43 | −0.18 |
| 2 | 0.08 | 0.59 | 0.08 | −0.59 |

Two possible solutions (equal quality):

- Solution 1: large $P$ state; $n_P(d) < 0$, $n_D(d) < 0$
- Solution 2: large $D$ state; $n_D(d) < 0$, $n_P(q) \approx 0$

# Fits to the data: step 3

$n_P, n_D$ **fixed for models 1 and 2**

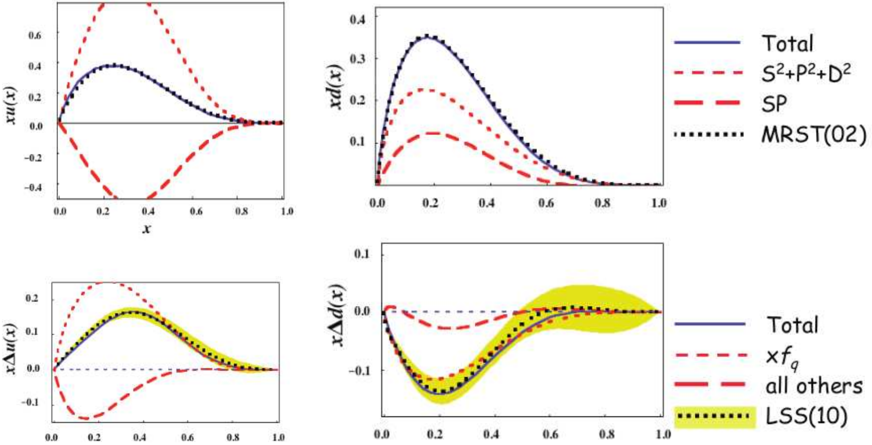
$$\psi_q^L(\chi) \approx \frac{\beta \cos\theta + \chi \sin\theta}{\chi^{n_0}(\beta+\chi)^{n_1-n_0}}$$

**Refit** wave functions

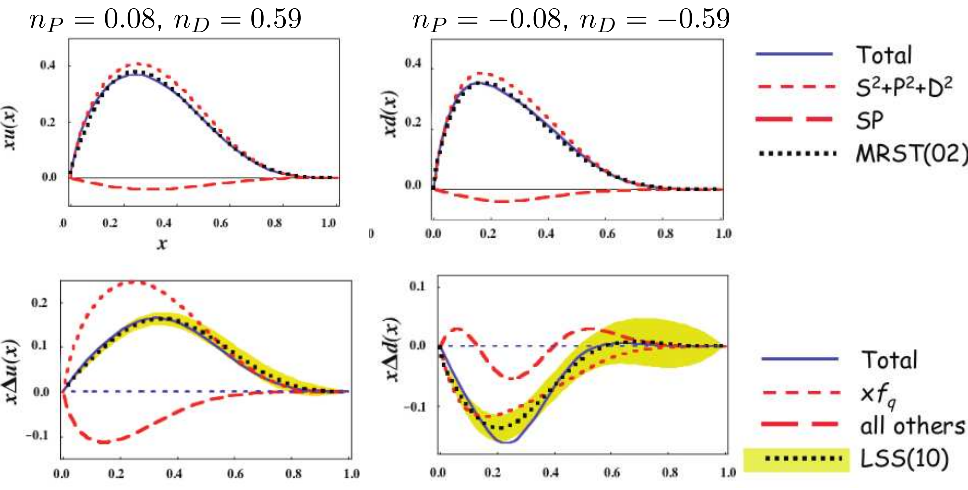
- $\beta_{Lq}$, $n_{0Lq}, n_{1Lq}$ same for **all** $L$
- $\theta_{Lq}$ adustated in same cases

# Fits to the data: step 3 (model 1) [$P$: 18%, $D$: 3%]

$n_P = 0.43$, $n_D = 0.18$ $\qquad$ $n_P = -0.43$, $n_D = -0.18$

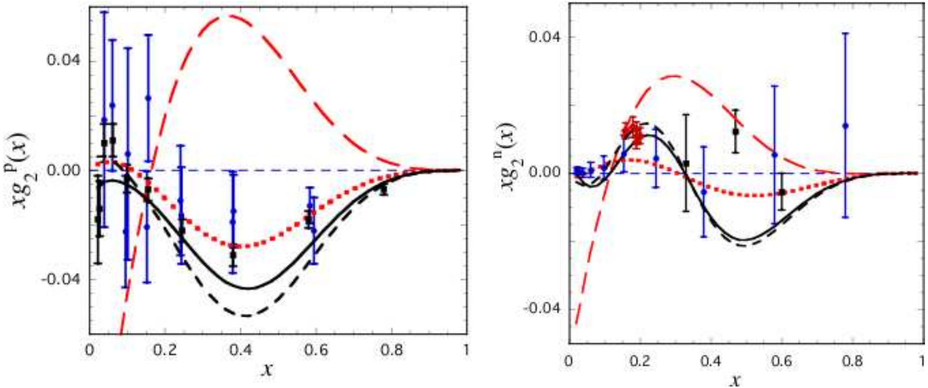


# Fits to the data: step 3 (model 2) [$P$: 0.6%, $D$: 35%]



# Predictions for $g_2$

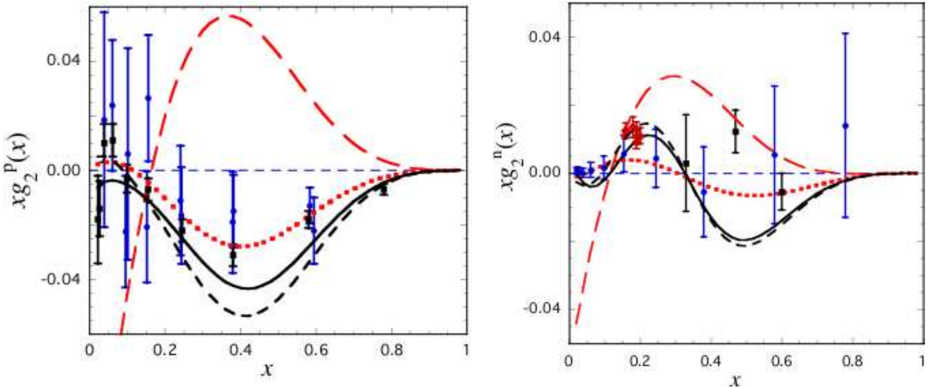
— — Model 1; —- Model 2; · · · Model 1 (P=0); - - - Model 2 (P=0)




**Data:** $g_2^p$: SLAC-E143, SLAC-E155 $g_2^n$: SLAC-E155, Jlab-Kramer, Jlab-Hall A

# Predictions for $g_2$

— — Model 1; —- Model 2; · · · Model 1 (P=0); - - - Model 2 (P=0)




**Data:** $g_2^p$: SLAC-E143, SLAC-E155 $g_2^n$: SLAC-E155, Jlab-Kramer, Jlab-Hall A

Only **Model 2** gives a good result (35% D-state)

# Conclusions

- Spectator Quark Model formalism appied to the Deep Inelastic Scattering
  - No gluon or sea quark effects considered
  - Formalism used to constrain the shape of wave functions
- Good description of the $f_q$ and $g_1^q$ data with model 1 and model 2
  $\Rightarrow$ **consistent with $J_g \approx 0$**
- Model 2 gives a good description of the $g_2$ data
- The model requires a large $D$-state mixture
  (phenomenologic calibration of the model)
- No systematic fit performed
  (we cannot exclude a model with smaller $D$-state mixture)

# Discussion

- **D-state mixture**
  - What is the physical source of that effect (larger than other models) ?
  - Without a explicit interaction model
    it is not possible to explain the effect
  - We can however look for signs of that effect in other processes like the nucleon elastic form factors or $\gamma^* N \to \Delta$ reaction
- **Where is the glue ?**
  - No need of gluon effects to explain nucleon spin ($g_1^N$ data)
    (gluons included in constituent quark structure)
  - Maybe for $g_2^N$ (more precise data nedded)
  - ... gluon effects expeced for larger $Q^2$ (Altareli-Parisi equation)
- **Regime of application of the model ?**
  - Results derived in the large $Q^2, \nu$ limit; but no gluons included
  - To compare with the data we should use $Q_0^2 = 2 - 5$ GeV$^2$
  - For **very** large $Q^2$ use QCD evolution equations (DGLAP);
    gluon effects will emerge for larger $Q^2$
    (even if there are only quarks at the $Q_0^2$ regime)

# Discussion

- **D-state mixture**
  - What is the physical source of that effect (larger than other models) ?
  - Without a explicit interaction model
    it is not possible to explain the effect
  - We can however look for signs of that effect in other processes like the nucleon elastic form factors or $\gamma^* N \to \Delta$ reaction
- **Where is the glue ?**
  - No need of gluon effects to explain nucleon spin ($g_1^N$ data)
    (gluons included in constituent quark structure)
  - Maybe for $g_2^N$ (more precise data nedded)
  - ... gluon effects expeced for larger $Q^2$ (Altareli-Parisi equation)
- **Regime of application of the model ?**
  - Results derived in the large $Q^2, \nu$ limit; but no gluons included
  - To compare with the data we should use $Q_0^2 = 2 - 5$ GeV$^2$
  - For **very** large $Q^2$ use QCD evolution equations (DGLAP);
    gluon effects will emerge for larger $Q^2$
    (even if there are only quarks at the $Q_0^2$ regime)

**Thank you** 

# Selected bibliography (part 1)


- **Spin and angular momentum in the nucleon**,
  F. Gross, G. Ramalho and M. T. Pena, Phys. Rev. D **85**, 093006 (2012) [arXiv:1201.6337 [hep-ph]].
- **Covariant nucleon wave function with S, D, and P-state components**,
  F. Gross, G. Ramalho and M. T. Pena, Phys. Rev. D **85**, 093005 (2012) [arXiv:1201.6336 [hep-ph]].
- **A pure S-wave covariant model for the nucleon**,
  F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C **77**, 015202 (2008) [arXiv:nucl-th/0606029].

# Selected bibliography (part 2)


- **A covariant formalism for the $N^*$ electroproduction at high momentum transfer**, G. Ramalho, F. Gross, M. T. Peña and K. Tsushima, **Exclusive Reactions and High Momentum Transfer IV, 287 (2011)** arXiv:1008.0371 [hep-ph].
- **A Covariant model for the nucleon and the $\Delta$**, G. Ramalho, M. T. Peña and F. Gross, Eur. Phys. J. A **36**, 329 (2008) [arXiv:0803.3034 [hep-ph]].
- **D-state effects in the electromagnetic $N\Delta$ transition**, G. Ramalho, M. T. Peña and F. Gross, Phys. Rev. D **78**, 114017 (2008) [arXiv:0810.4126 [hep-ph]].

# Nucleon wave function: D-states (1)

D-state algebra (rest frame): $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$, $\mathbf{r} = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$

EPJA 36, 329 (2008); PRD 78, 114017 (2008)

$$U_m(\lambda) = \frac{1}{\sqrt{3}}\sigma_m|\tfrac{1}{2}\lambda\rangle \qquad D^{\ell m}(\mathbf{k}) = \mathbf{k}^\ell \mathbf{k}^m + \frac{1}{3}\mathbf{k}^2\delta_{\ell m}$$

D-state function

$$\Theta^D_{\Lambda\lambda}(\mathbf{k}_i) = \frac{3}{\sqrt{2}}(\varepsilon^*_\Lambda)_\ell D^{\ell m}(\mathbf{k}_i) U_m(\lambda)$$

$$\begin{aligned}
\psi^{Da}_{\Lambda\lambda} &= \frac{1}{\sqrt{2}}\left[\Theta^D_{\Lambda\lambda}(\mathbf{k}_1) - \Theta^D_{\Lambda\lambda}(\mathbf{k}_2)\right]\phi_D(\mathbf{k}^2, \mathbf{r}^2) \\
&= \frac{3}{2}(\varepsilon^*_\Lambda)_\ell G^{\ell m}(\mathbf{k}, \mathbf{r}) U_m(\lambda)\, \phi_D(\mathbf{k}^2, \mathbf{r}^2) \\
\psi^{Ds}_{\Lambda\lambda} &= \frac{1}{\sqrt{6}}\left[2\Theta^D_{\Lambda\lambda}(\mathbf{k}_3) - \Theta^D_{\Lambda\lambda}(\mathbf{k}_1) - \Theta^D_{\Lambda\lambda}(\mathbf{k}_2)\right]\phi_D(\mathbf{k}^2, \mathbf{r}^2) \\
&\simeq \left[\cos\phi\, \Theta^D_{\Lambda\lambda}(\mathbf{k}) + \sin\phi\, \Theta^D_{\Lambda\lambda}(\mathbf{r})\right]\phi_D(\mathbf{k}^2, \mathbf{r}^2), \qquad \cos\phi = \frac{1}{5}
\end{aligned}$$

# Nucleon wave function: D-states (2)

$$G^{\ell m}(\mathbf{k},\mathbf{r}) = \mathbf{k}^\ell \mathbf{r}^m + \mathbf{r}^\ell \mathbf{k}^m + \frac{2}{3}(\mathbf{k}\cdot\mathbf{r})\delta_{\ell m}$$

$\mathbf{r}^\ell$: diquark with internal P-state; define spin-1 vector $\zeta_\nu^\ell$

$$\mathbf{r}^\ell \phi_D \to \frac{2}{\sqrt{3}} c_P |\mathbf{k}| \zeta_\nu^\ell \psi_D$$

$\Theta_{\Lambda\lambda}^D(\mathbf{r})$: diquark with internal D-state; define spin-1 vector $\varepsilon_{D\Lambda}^m$

$$\Theta_{\Lambda\lambda}^D(\mathbf{r}) = (\varepsilon_\Lambda^*)_\ell D^{\ell m}(\mathbf{r})\phi_D \to \frac{2\sqrt{2}}{3} c_D (\epsilon_{D\Lambda}^*)^m \mathbf{k}^2 \psi_D$$

$c_P, c_D$ determined by the exact integration in $\mathbf{r}$ in the rest frame

$$\begin{aligned}
\psi_{\Lambda\lambda(\nu)}^{Da} &\to \frac{3}{\sqrt{20}} (\varepsilon_\Lambda^*)_\ell G^{\ell m}(\mathbf{k},\zeta_\nu) U_m(\lambda)\, \psi_D(P,k) \\
\psi_{\Lambda\lambda}^{Ds} &\to \frac{3\sqrt{2}}{\sqrt{5}} (\varepsilon_\Lambda)_\ell D^{\ell m}(\mathbf{k}) U_m(\lambda)\, \psi_D(P,k) \\
& \quad + \frac{1}{\sqrt{5}} \mathbf{k}^2 (\epsilon_{D\Lambda}^*)^m U_m(\lambda)\, \psi_D(P,k)
\end{aligned}$$

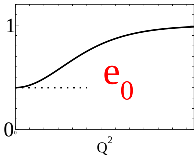
# Normalization

Wave function normalized by the nucleon charge ($Q^2 = 0$); $P = (M, 0, 0, 0)$

$$J^0 = 3 \sum_\Lambda \int_k \overline{\Psi}_{\Lambda\lambda}(P,k) j_q(0) \gamma^0 \Psi_{\Lambda\lambda}(P,k) \equiv \frac{1}{2}(1+\tau_3)$$

$$j_q(+\infty) = \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3}, \quad Q_d = -\frac{1}{3}$$

$$j_q(0) = e_0 \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3} e_0, \quad Q_d = -\frac{1}{3} e_0$$




Wave functions normalization $Q^2 = 0$: (rest frame $\tilde{k}^2 = -\mathbf{k}^2$)

$$e_0 \int_k |\psi_S|^2 = e_0 \int_k (-\tilde{k}^2)|\psi_P|^2 = e_0 \int_k \tilde{k}^4 |\psi_D|^2 = 1$$

Then, using $\int_k \overline{|\psi_N|^2} = n_S^2 \int_k |\psi_S|^2 + n_S^2 \int_k (-\tilde{k}^2)|\psi_P|^2 + n_S^2 \int_k \tilde{k}^4 |\psi_D|^2$

$$J^0 = \frac{1}{2}(1+\tau_3)\, e_0 \int_k \overline{|\psi_N|^2} \equiv \frac{1}{2}(1+\tau_3), \text{ if } n_S^2 + n_P^2 + n_D^2 = 1$$

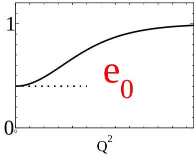
# Normalization

Wave function normalized by the nucleon charge ($Q^2 = 0$); $P = (M, 0, 0, 0)$

$$J^0 = 3 \sum_\Lambda \int_k \overline{\Psi}_{\Lambda\lambda}(P,k) j_q(0) \gamma^0 \Psi_{\Lambda\lambda}(P,k) \equiv \frac{1}{2}(1+\tau_3)$$

$$j_q(+\infty) = \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3}, \quad Q_d = -\frac{1}{3}$$

$$j_q(0) = e_0 \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3}e_0, \quad Q_d = -\frac{1}{3}e_0$$




Wave functions normalization $Q^2 = 0$: (rest frame $\tilde{k}^2 = -\mathbf{k}^2$) $u \neq d$, $e_0 \to e_q^0$

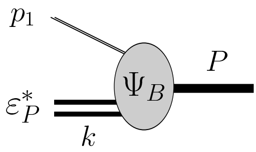
$$e_q^0 \int_k |\psi_q^S|^2 = e_q^0 \int_k (-\tilde{k}^2) |\psi_q^P|^2 = e_q^0 \int_k \tilde{k}^4 |\psi_q^D|^2 = 1$$

Then, using $\int_k \overline{|\psi_N|^2} = n_S^2 \int_k |\psi_S|^2 + n_S^2 \int_k (-\tilde{k}^2)|\psi_P|^2 + n_S^2 \int_k \tilde{k}^4 |\psi_D|^2$

$$J^0 = \frac{1}{2}(1+\tau_3)\, e_0 \int_k \overline{|\psi_N|^2} \equiv \frac{1}{2}(1+\tau_3), \text{ if } n_S^2 + n_P^2 + n_D^2 = 1$$

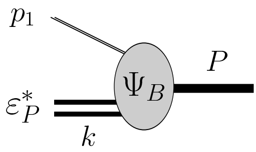
- Baryon wave functions: $B =$ **diquark** $\oplus$ **quark**
  Combination of **diquark** (12) and single **quark** (3) states, using $SU(6) \otimes O(3)$:

$$\Psi_B = \sum \text{(flavor)} \otimes \text{(spin)} \otimes \text{(orbital)} \otimes \underbrace{\psi_B(P,k)}_{\text{radial}}$$



- Baryon wave functions: $B =$ **diquark** $\oplus$ **quark**
  Combination of **diquark** (12) and single **quark** (3) states, using $SU(6) \otimes O(3)$:

$$\Psi_B = \sum \text{(flavor)} \otimes \text{(spin)} \otimes \text{(orbital)} \otimes \underbrace{\psi_B(P,k)}_{\text{radial}}$$

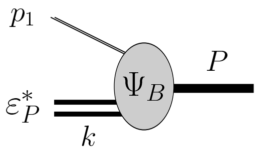



  - $\Psi_B$ in **rest frame** using quark states

# Spectator QM: Baryon wave functions (3)

- Baryon wave functions: $B =$ **diquark** $\oplus$ **quark**
  Combination of **diquark** (12) and single **quark** (3) states, using $SU(6) \otimes O(3)$:

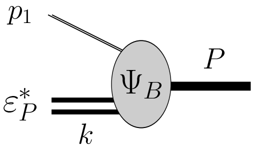
$$\Psi_B = \sum \text{(flavor)} \otimes \text{(spin)} \otimes \text{(orbital)} \otimes \underbrace{\psi_B(P,k)}_{\text{radial}}$$




  - $\Psi_B$ in **rest frame** using quark states
  - **Covariant** generalization of $\Psi_B$ in terms **baryon** properties

- Baryon wave functions: $B =$ **diquark** $\oplus$ **quark**
  Combination of **diquark** (12) and single **quark** (3) states, using $SU(6) \otimes O(3)$:

$$\Psi_B = \sum \text{(flavor)} \otimes \text{(spin)} \otimes \text{(orbital)} \otimes \underbrace{\psi_B(P,k)}_{\text{radial}}$$




  - $\Psi_B$ in **rest frame** using quark states
  - **Covariant** generalization of $\Psi_B$ in terms **baryon** properties
  - $\Psi_B$ can be used on **any** frame and/or $Q^2$ regime

# DIS: integration

DIS condition is CST: $p'^2 - m_q^2 = 0$: $\Rightarrow \frac{Mx}{Q^2}\delta(E_s - M(1-x) - |\mathbf{k}|z)$

Change of variable $k = M\kappa$, $E_\kappa = r\sqrt{1+\frac{\kappa}{r}}$, using $r = \frac{m_s}{M}$

$$\iint_{p'k} \psi_{L'}(P,k)\psi_L(P,k) = \frac{M^2 x}{Q^2}\int_{\kappa_{\min}}^{+\infty} \frac{\kappa d\kappa}{4\pi E_\kappa}\psi_{L'}(P,k)\psi_L(P,k)$$

where $\kappa_{\min} = \frac{r^2-(1-x)^2}{2(1-x)}$

**Wave functions:**

$\psi_L(P,k)$ can be represented usind the covariant variable

$$\chi = \frac{(M-m_s)^2 - (P-k)^2}{Mm_s} = 2\frac{P\cdot k}{Mm_s} - 2$$

$$\iint_{p'k} \psi^2(\chi) = \frac{M^2 x}{Q^2}\int_{\kappa_{\min}}^{+\infty} \frac{\kappa d\kappa}{4\pi E_\kappa}\psi^2(\chi) \overset{\kappa\to\chi}{=} \frac{\pi x}{Q^2}\,\frac{Mm_s}{16\pi^2}\int_{\xi}^{+\infty} d\chi\psi^2(\chi),$$

where

$$\xi = \frac{r}{1-x} + \frac{1-x}{r} - 2$$

# Nucleon wave function: S-state

S-state in quark-diquark [PRC 77,015202 (2008)]: $k =$ diquark momentum

$$\Psi_S(P,k) = \frac{1}{\sqrt{2}}\left[\Phi_I^0\Phi_S^0 + \Phi_I^1\Phi_S^1\right]\psi_S(P,k)$$

$\Phi^0_{I,S}$ : anti-symmetric in the exchange of quark states $(12)$ - $M_A$
$\Phi^1_{I,S}$ : symmetric in the exchange of quark states $(12)$ - $M_S$

Example $|p\uparrow\rangle$: Isospin states

$[M_A]: \quad \Phi_I^0 = \frac{1}{\sqrt{2}}\left[ud - du\right]u$

$[M_S]: \quad \Phi_I^1 = \frac{1}{\sqrt{6}}\left[2uud - (ud + du)u\right]$

# Nucleon wave function: S-state; spin part

Example $|p \uparrow\rangle$: $\quad \uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \chi_S = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Spin-0: $$\Phi_S^0 = \overbrace{\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)}^{\varepsilon^s} \uparrow \;=\; \varepsilon^s \chi_S$$

Spin-1: $$\Phi_S^1 = \frac{1}{\sqrt{6}} \left[2 \uparrow\uparrow\downarrow - (\uparrow\downarrow + \downarrow\uparrow) \uparrow\right] \;=\; -\frac{1}{\sqrt{3}} (\sigma \cdot \varepsilon_P^*) \chi_S$$

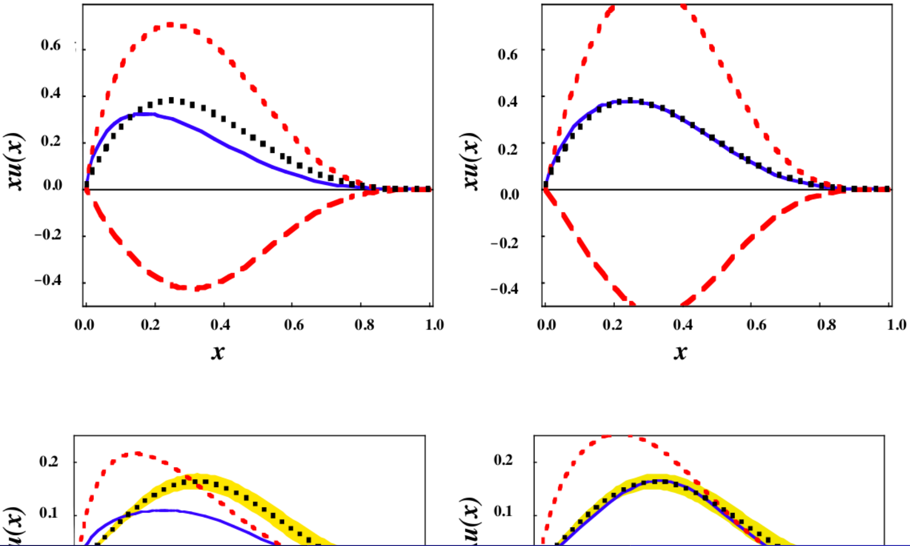
**Relativistic generalization:**

$$\Phi_S^0 \to u(P, \uparrow) \qquad \Phi_S^1 \to -(\varepsilon_P^*)_\alpha U^\alpha(P, \uparrow)$$

- Dirac nucleon spinor $u(P, \uparrow)$; Diquark polarization vector: $\varepsilon_P^\alpha$ [**rest frame**-fixed-axis base PRC 77, 015202 (2008)]
- Vector spin 1/2 S-state $\left[1 \oplus \frac{1}{2} \to \frac{1}{2}\right]$:
$$U^\alpha(P, \lambda_n) = \frac{1}{\sqrt{3}} \gamma_5 \left(\gamma^\alpha - \frac{P^\alpha}{M}\right) u(P, \lambda_n)$$

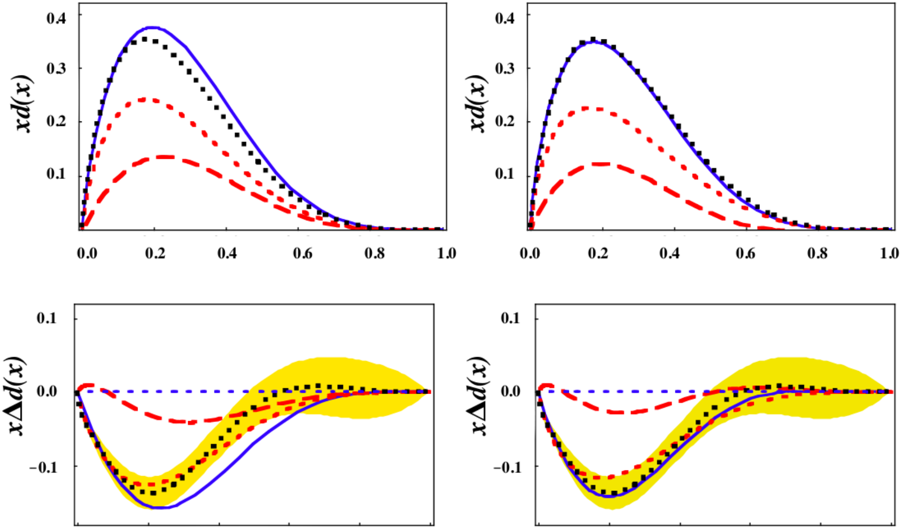
# Fits to the data: step 3 (model 1) quark $u$

$n_P = 0.43$, $n_D = 0.18$



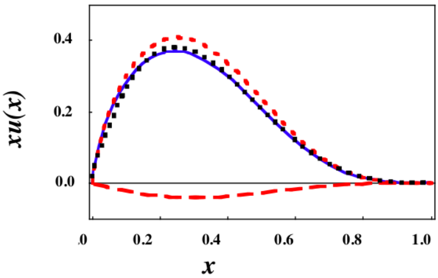
# Fits to the data: step 3 (model 1) quark $d$

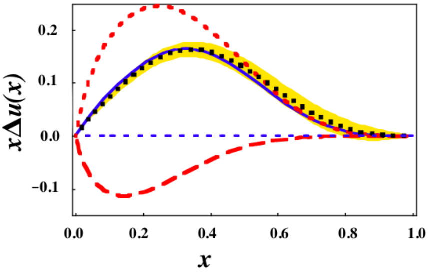
$n_P = -0.43$, $n_D = -0.18$



# Fits to the data: step 3 (model 2) quark $u$

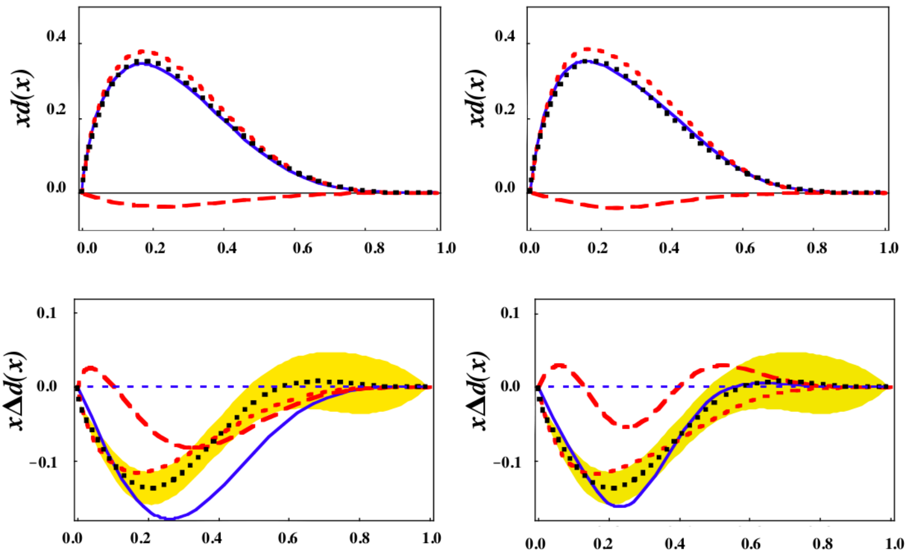
$n_P = 0.08$, $n_D = 0.59$



# Fits to the data: step 3 (model 2) quark $d$

$n_P = -0.08$, $n_D = -0.59$



# DIS: wave functions and normalization

$$f_q^S(x) = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

DIS normalization

WF normalization ($Q^2 = 0$)

$$\int_0^1 dx f_q^S(x) = 1,$$

$$e_q^0 \int_k |\psi_q^S(\chi)|^2 \big|_{Q^2=0} = e_q^0 \overbrace{\int_{-\infty}^1 dx f_q^S(x)}^{N} = 1$$

How to fix $e_q^0$ ?

$$N = \int_{-\infty}^1 dx f_q^S(x), \qquad e_q^0 = \frac{1}{N} = \frac{\overbrace{\int_0^1 dx f_q^S(x)}^{=1}}{\underbrace{\int_{-\infty}^0 dx f_q^S(x)}_{>0} + \underbrace{\int_0^1 dx f_q^S(x)}_{=1}} < 1$$

# DIS: wave functions and normalization

$$f_q^S(x) = \frac{Mm_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

DIS normalization

WF normalization ($Q^2 = 0$)

$$\int_0^1 dx f_q^S(x) = 1,$$

$$e_q^0 \int_k |\psi_q^S(\chi)|^2\Big|_{Q^2=0} = e_q^0 \overbrace{\int_{-\infty}^1 dx f_q^S(x)}^{N} = 1$$

How to fix $e_q^0$ ?

$$N = \int_{-\infty}^1 dx f_q^S(x), \qquad e_q^0 = \frac{1}{N} = \frac{\overbrace{\int_0^1 dx f_q^S(x)}^{=1}}{\underbrace{\int_{-\infty}^0 dx f_q^S(x)}_{>0} + \underbrace{\int_0^1 dx f_q^S(x)}_{=1}} < 1$$

- DIS define $\psi_q^L$ for $0 < x < 1$

# DIS: wave functions and normalization

$$f_q^S(x) = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

DIS normalization — WF normalization ($Q^2 = 0$)

$$\int_0^1 dx f_q^S(x) = 1, \qquad e_q^0 \int_k |\psi_q^S(\chi)|^2\big|_{Q^2=0} = e_q^0 \overbrace{\int_{-\infty}^1 dx f_q^S(x)}^{N} = 1$$

**How to fix $e_q^0$ ?**

$$N = \int_{-\infty}^1 dx f_q^S(x), \qquad e_q^0 = \frac{1}{N} = \frac{\overbrace{\int_0^1 dx f_q^S(x)}^{=1}}{\underbrace{\int_{-\infty}^0 dx f_q^S(x)}_{>0} + \underbrace{\int_0^1 dx f_q^S(x)}_{=1}} < 1$$

- DIS define $\psi_q^L$ for $0 < x < 1$

# Breaking isospin symmetry

Write wave function in terms of $u$ and $d$ isospin states

$\psi_L$ **deppendent of the flavor of the quark 3**

Proton: $\chi^{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = u$; Neutron: $\chi^{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = d$

**Isospin-0 component:**

$$\Phi_I^0 \psi_L = \phi^0 \chi^I \psi_L = \frac{1}{\sqrt{2}}(ud - du) \begin{pmatrix} u \\ d \end{pmatrix} \psi_L \longrightarrow \phi^0 \chi^I \psi^L_{u(d)}$$

**Isospin-1 component:**

$$\begin{aligned} \Phi_I^1 \psi_L &= \phi^1 \chi^I \psi_L = -\frac{1}{\sqrt{3}} \sum_{\ell=0,\mp 1} (\sigma \cdot \xi_\ell^*) \chi^I \psi_L \\ &= \underbrace{-\frac{1}{\sqrt{6}}(ud + du) \begin{pmatrix} u \\ d \end{pmatrix}}_{\ell=0} \psi_L + \underbrace{\sqrt{\frac{2}{3}} \begin{pmatrix} (uu)d \\ -(dd)u \end{pmatrix}}_{\ell=\mp 1} \psi_L \\ &\rightarrow (\phi^1_{\ell=0})\, \chi^I \psi^L_{u(d)} + (\phi^1_{\ell=\mp 1})\, \chi^I\, \psi^L_{d(u)} \end{aligned}$$

$\psi_L \rightarrow \psi_q^L$: different distributions for $u$ and $d$