Energy correlators in heavy ion collisions: jet substructure and color coherence

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Seminar
LIP, Lisbon
February 9th, 2023

C. Andres, FD, R. K. Elayavalli, J. Holguin, C. Marquet, I. Moult, arXiv:2209.11236

IGFAE
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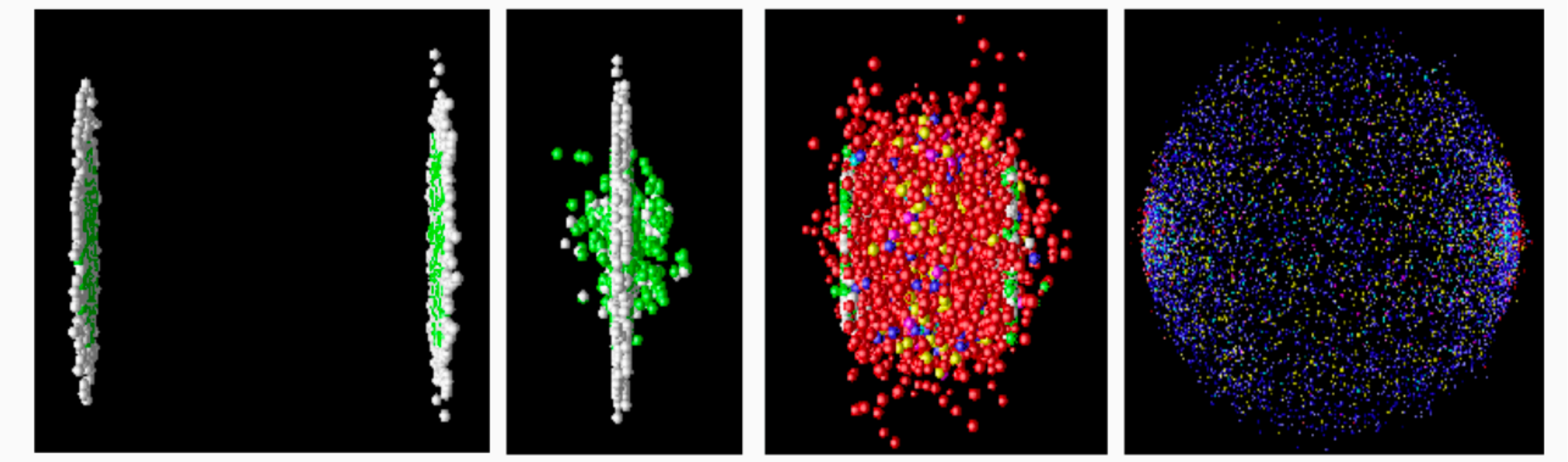
XUNTA DE GALICIA

Heavy-ion collisions



- High-energy nuclei collide producing thousands of particles

- Most of the particles are soft and reach thermal equilibrium, creating a Quark-Gluon Plasma (QGP)

- The QGP is a unique opportunity to study QCD in its deconfined phase

- QGP properties can be studied either through the bulk degrees of freedom which are well described by hydrodynamics, or through hard probes created at the moment of the collision

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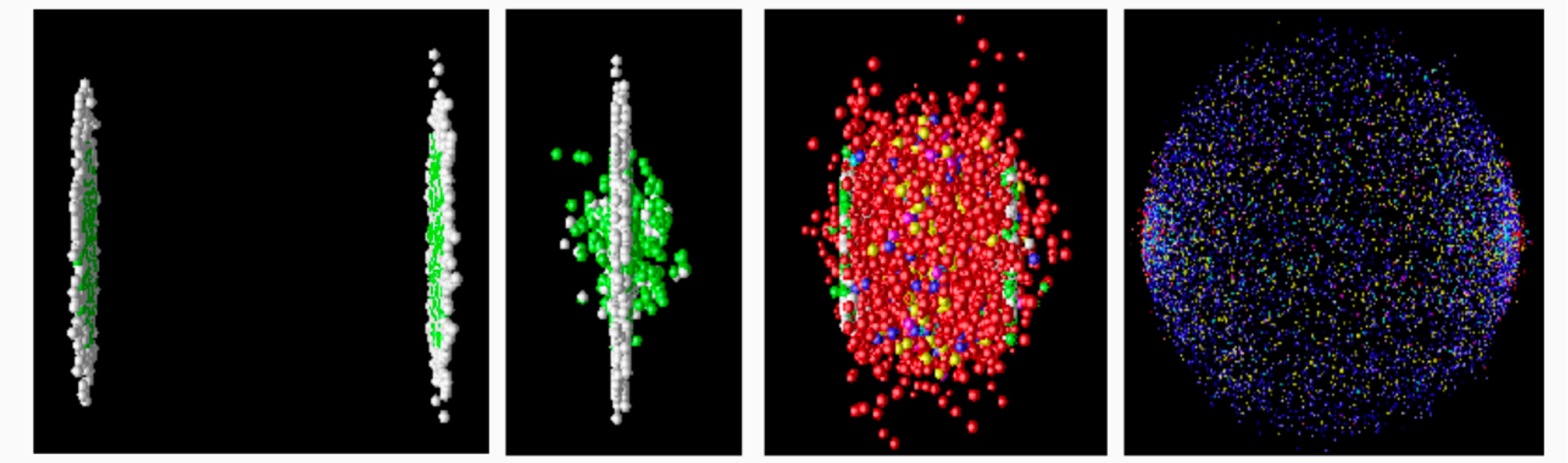

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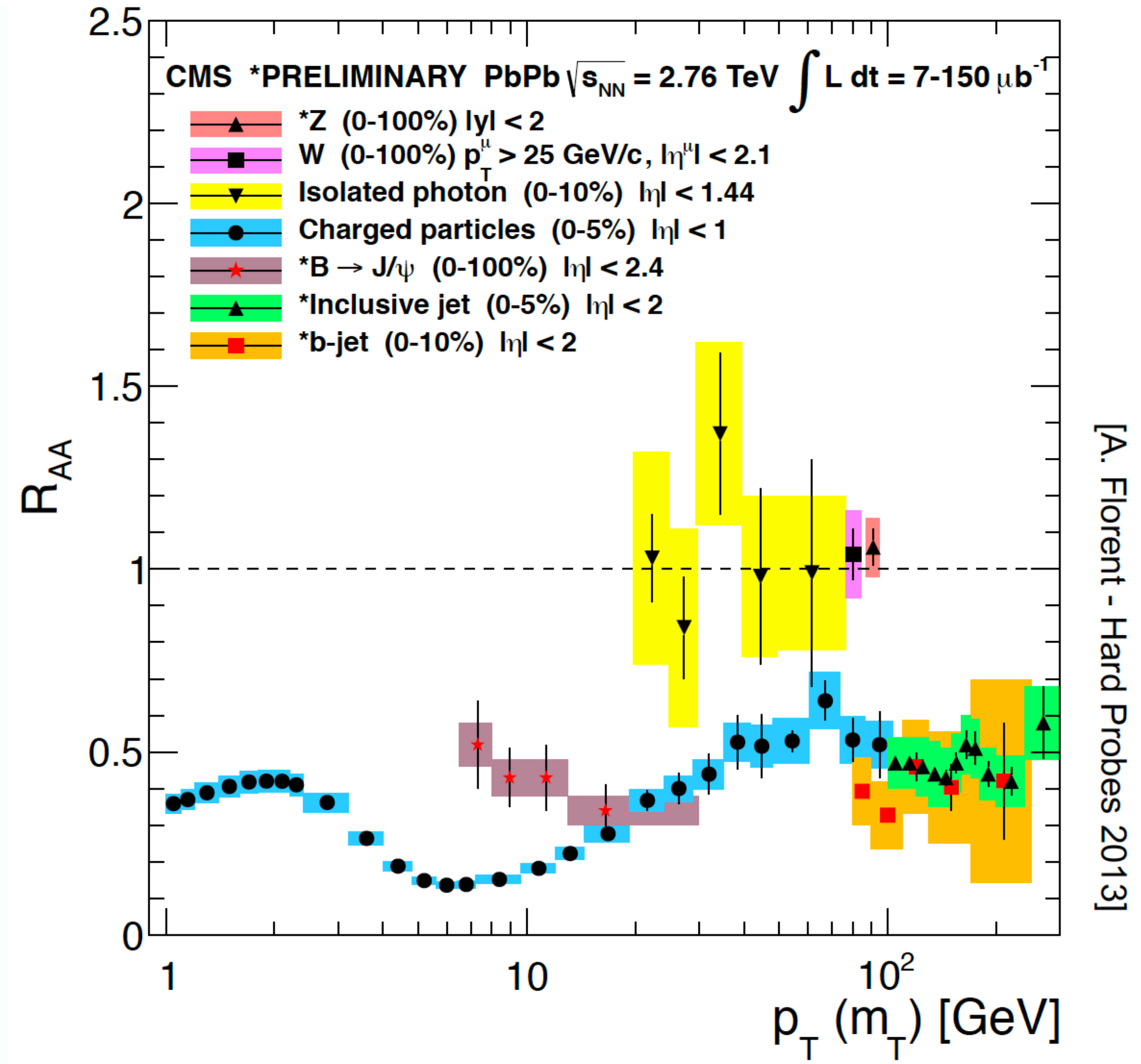
Hard probes in HIC

$$R_{AA} = \frac{dN_{AA}/d^2p_T\,dy}{\langle N_{coll}\rangle\, dN_{pp}/d^2p_T\,dy}$$



- Colorless probes are not affected (photons, Z, W)
- Hadrons and jets lose energy when interacting with the QGP, thus their spectra is suppressed with respect to independent nucleon collisions

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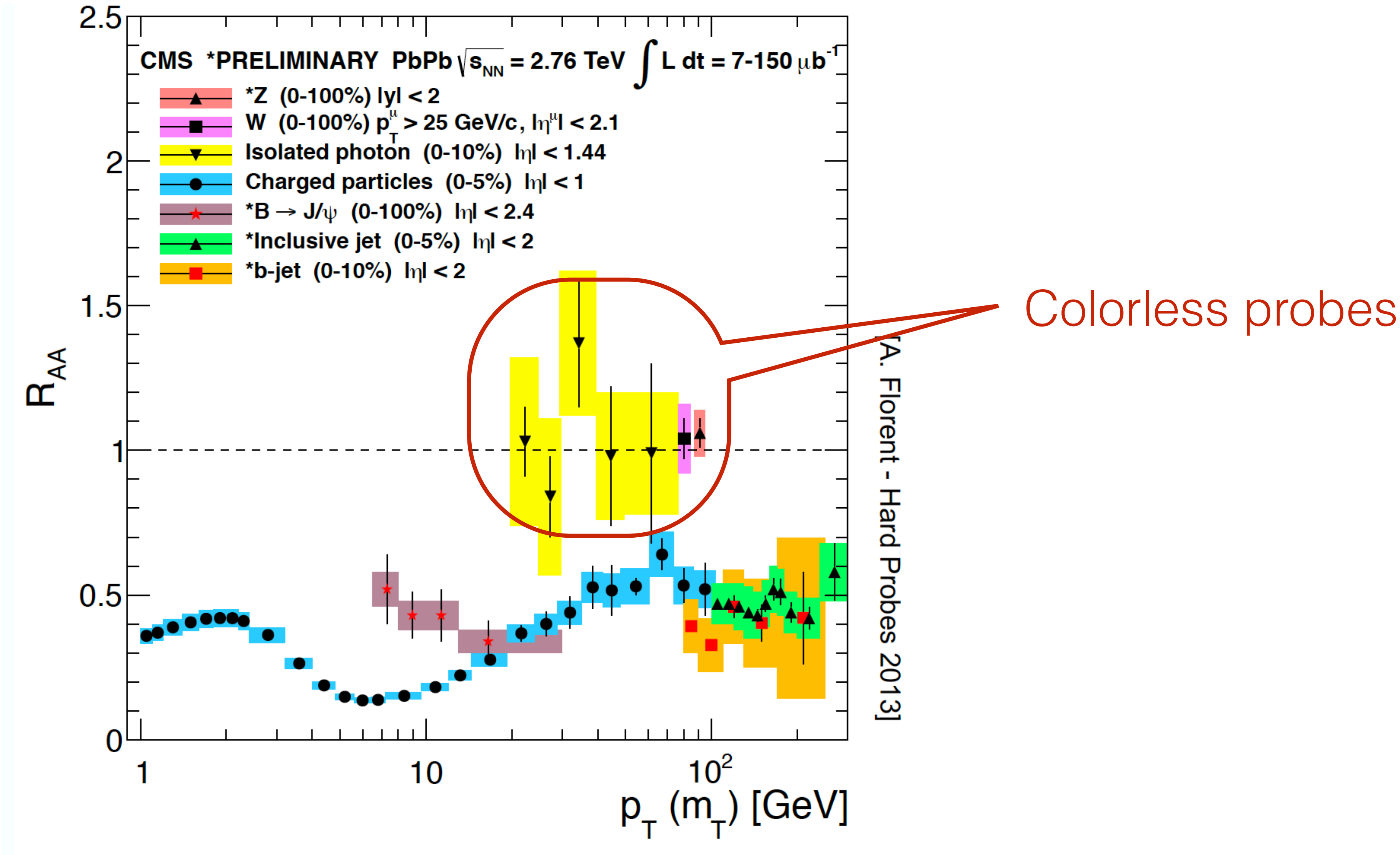

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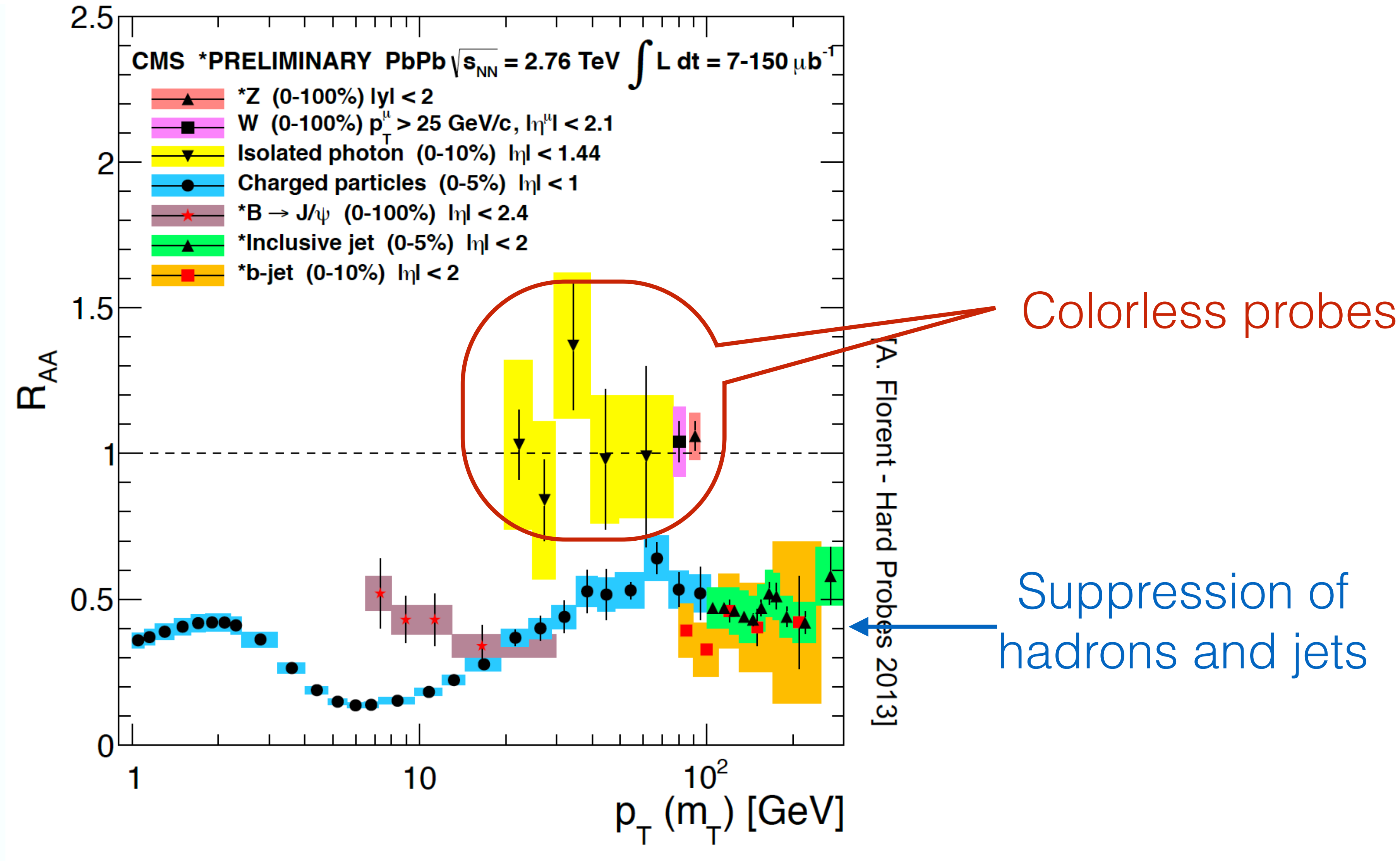

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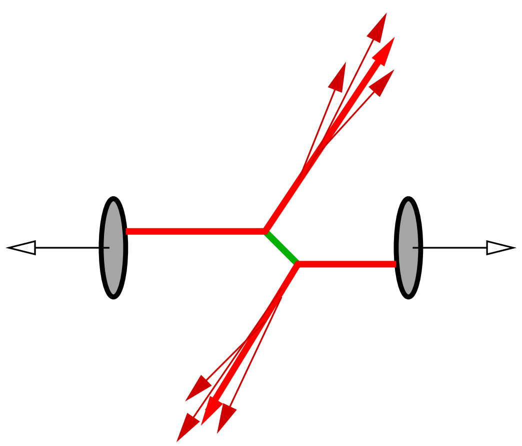
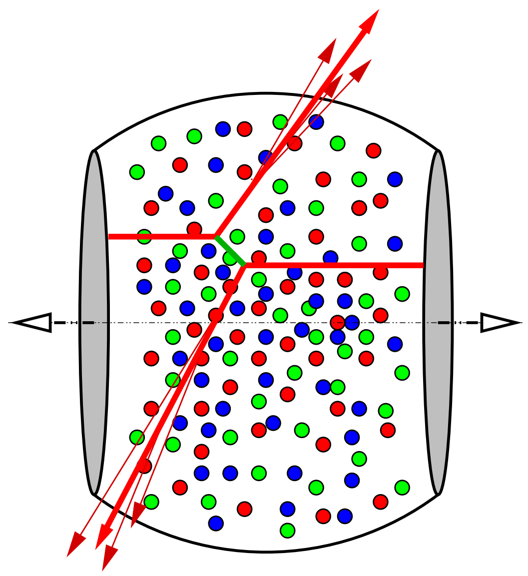
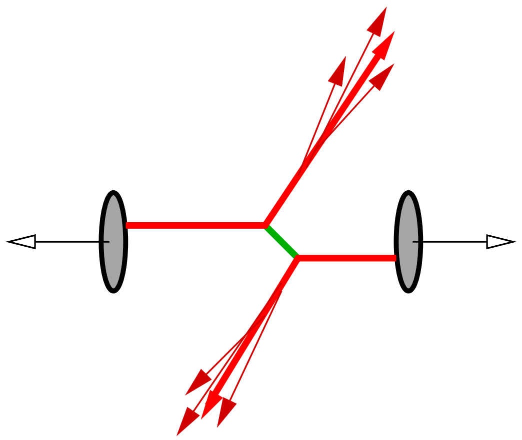
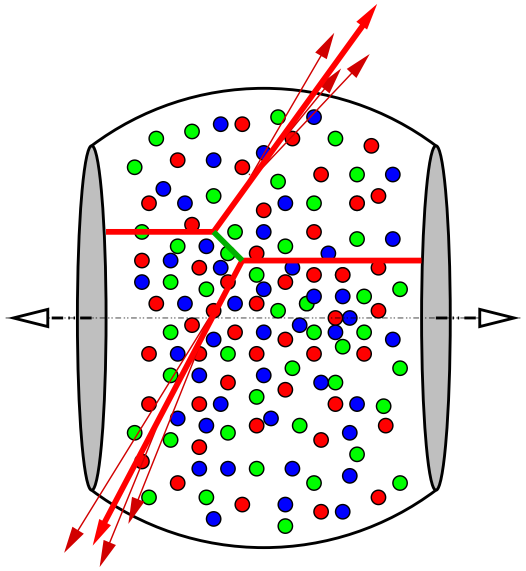
Jet quenching

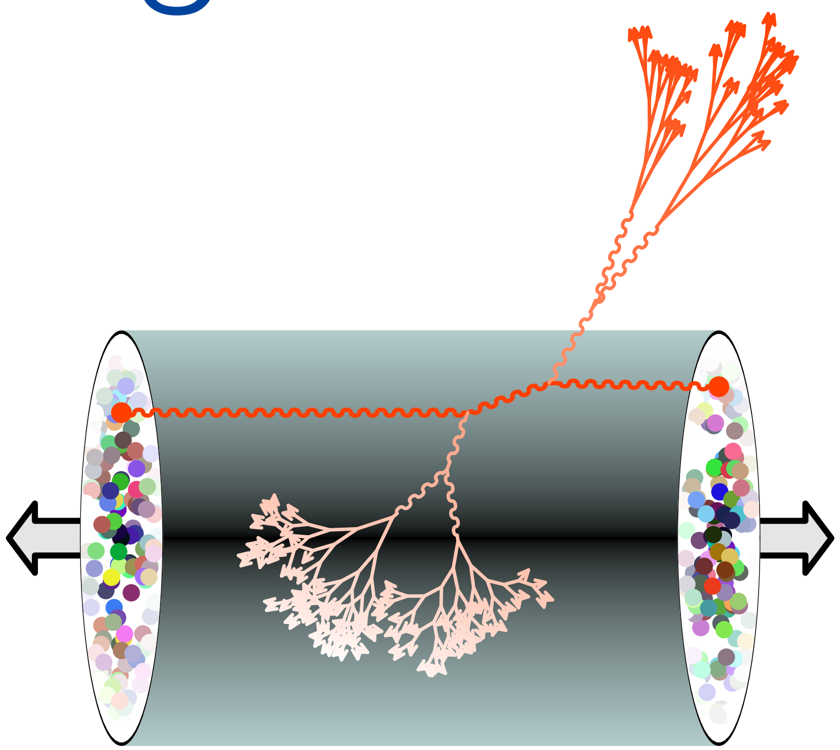




Jet quenching







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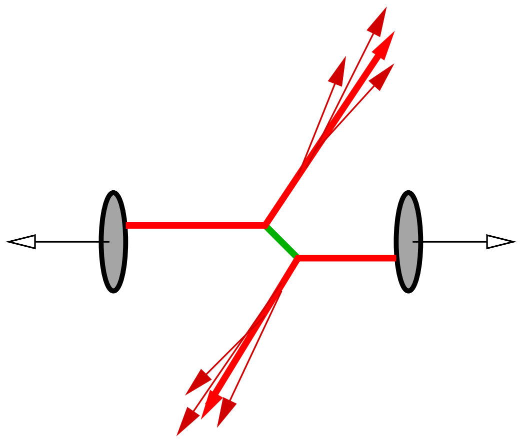
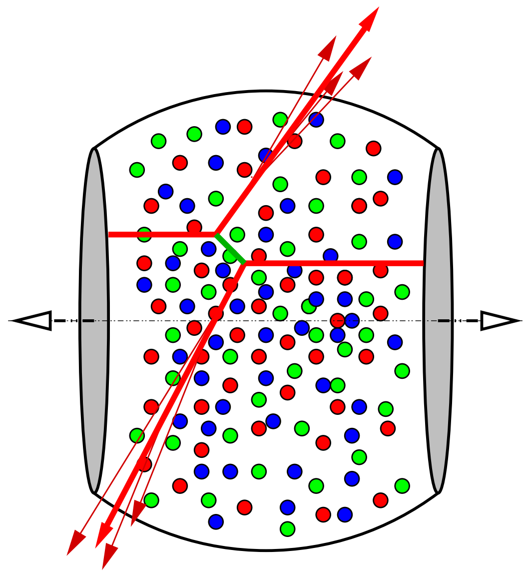
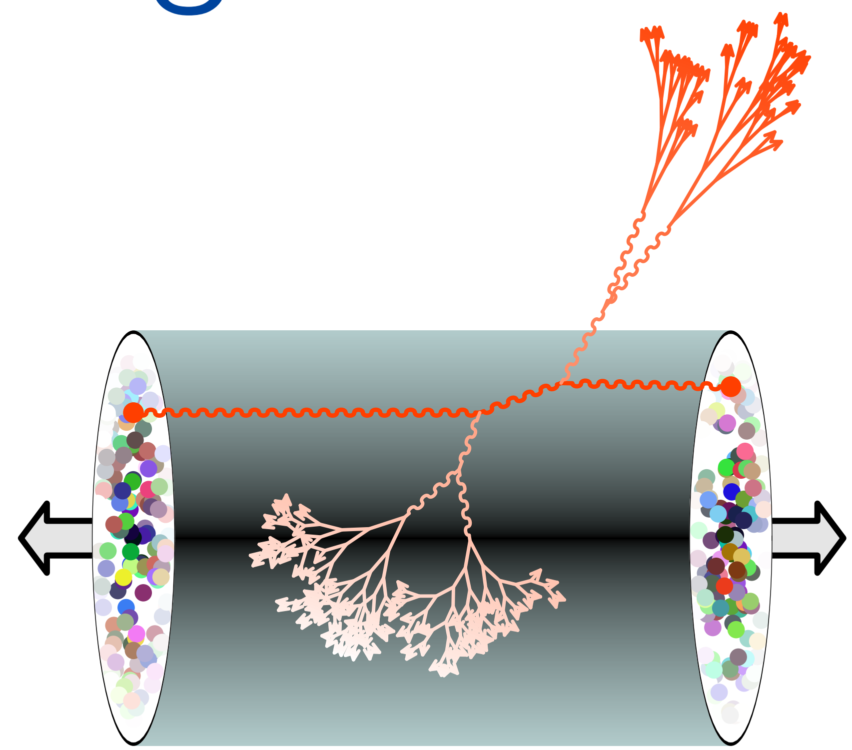
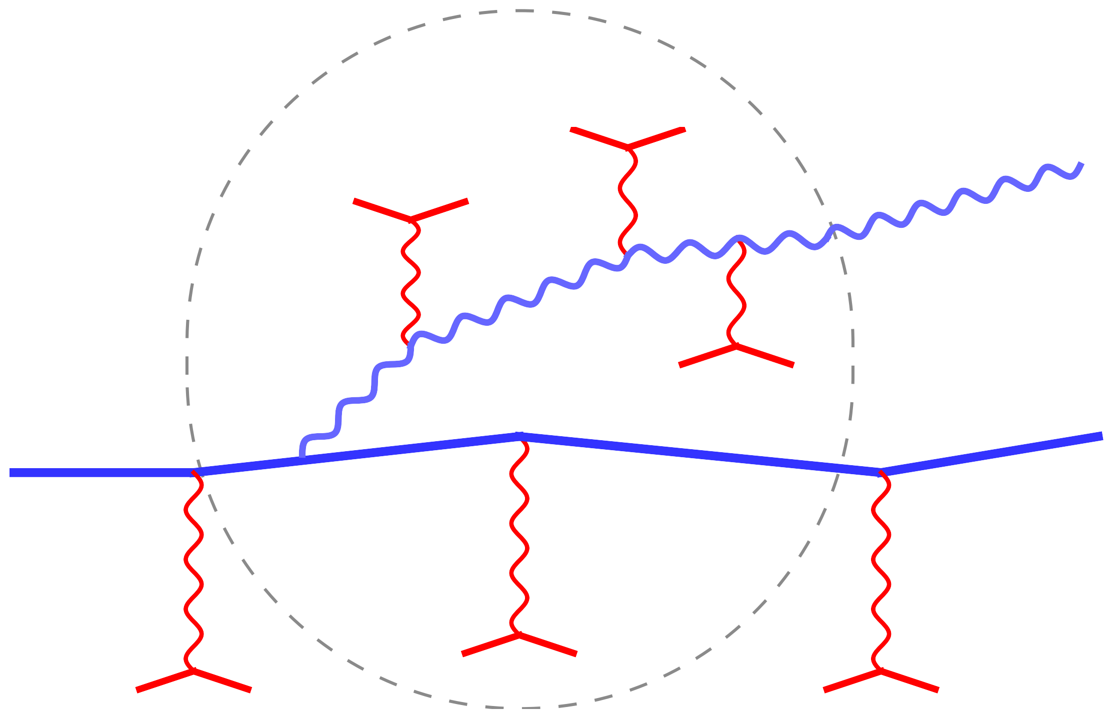






- Basic principle: colored particles lose energy and slow down when going through the plasma
- Radiative energy loss: stimulated emissions (dominant for light quarks and gluons)
- Back-to-back jets lose different amounts of energy

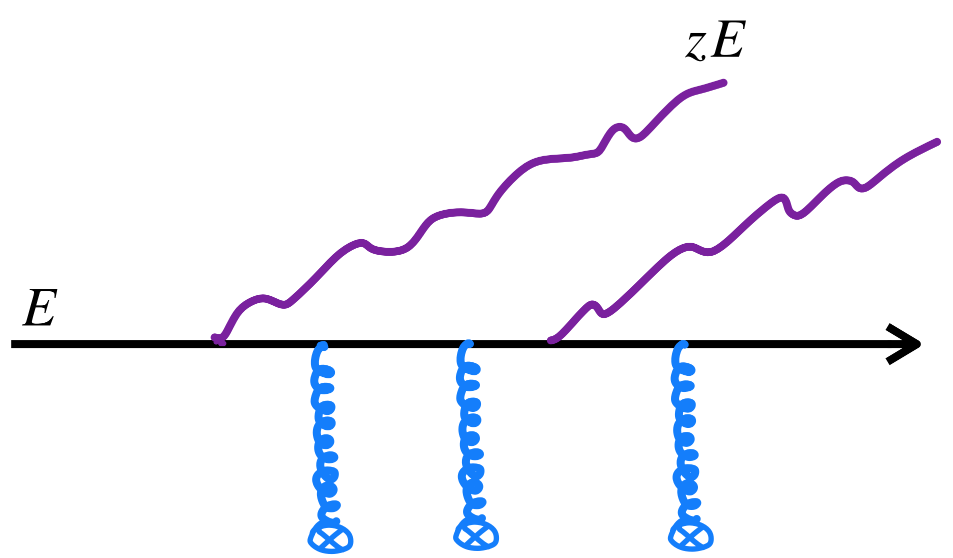
Medium-induced radiation



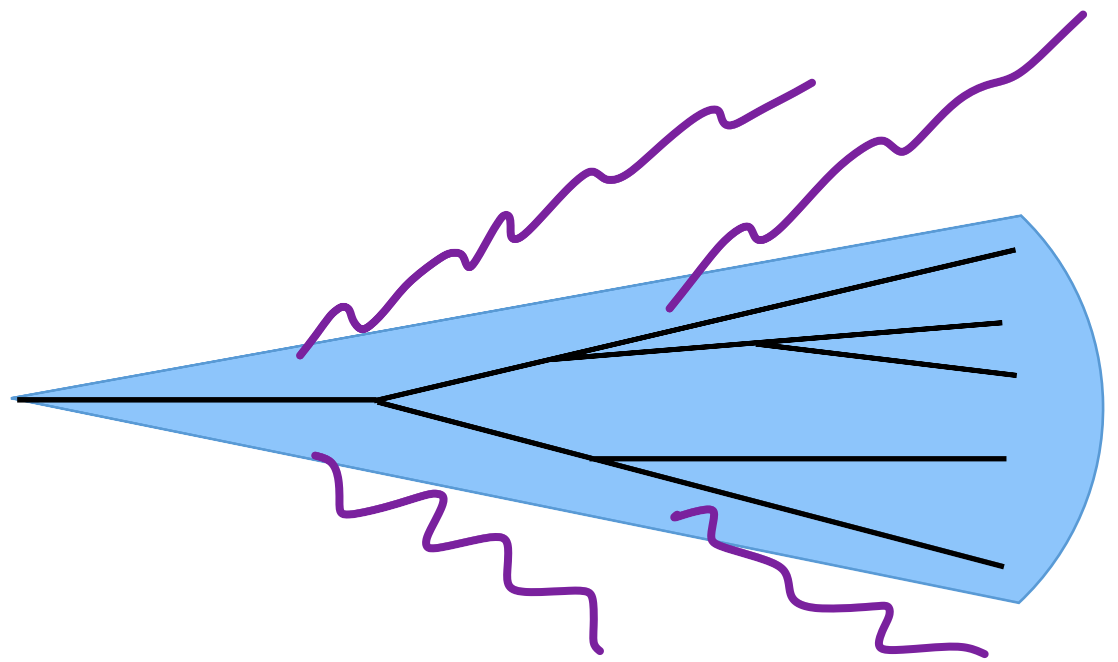
- Theoretical advances have focused on understanding how radiation is enhanced by the medium
- Energy lost by a high-energy parton can be understood in terms of soft medium-induced radiation
- Single gluon spectrum understood very well in the soft limit

From energy loss to jet substructure

- For energy loss calculation we only need the soft limit $z \ll 1$
  - ✦ Soft divergence of the vacuum vertex



- For jet substructure
  - ✦ Emissions from multiple sources
  - ✦ Harder vertices



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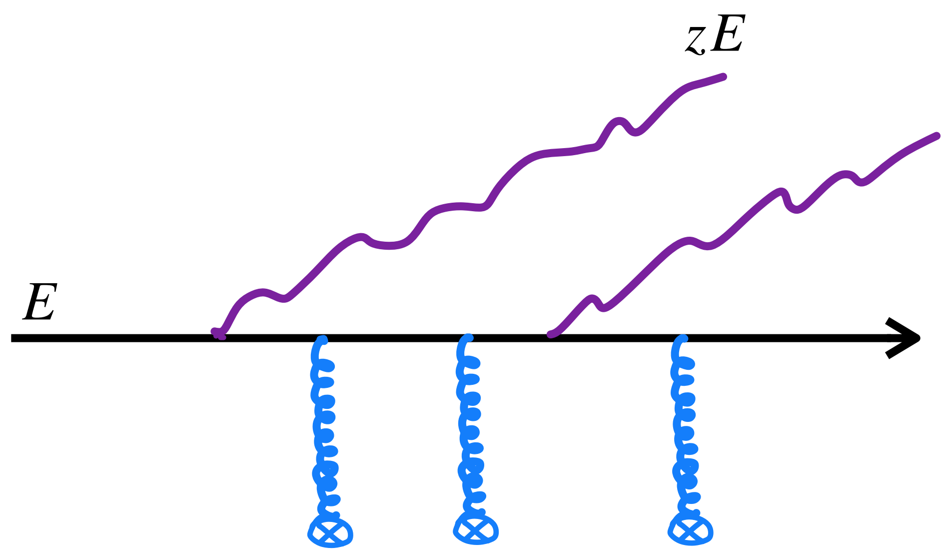

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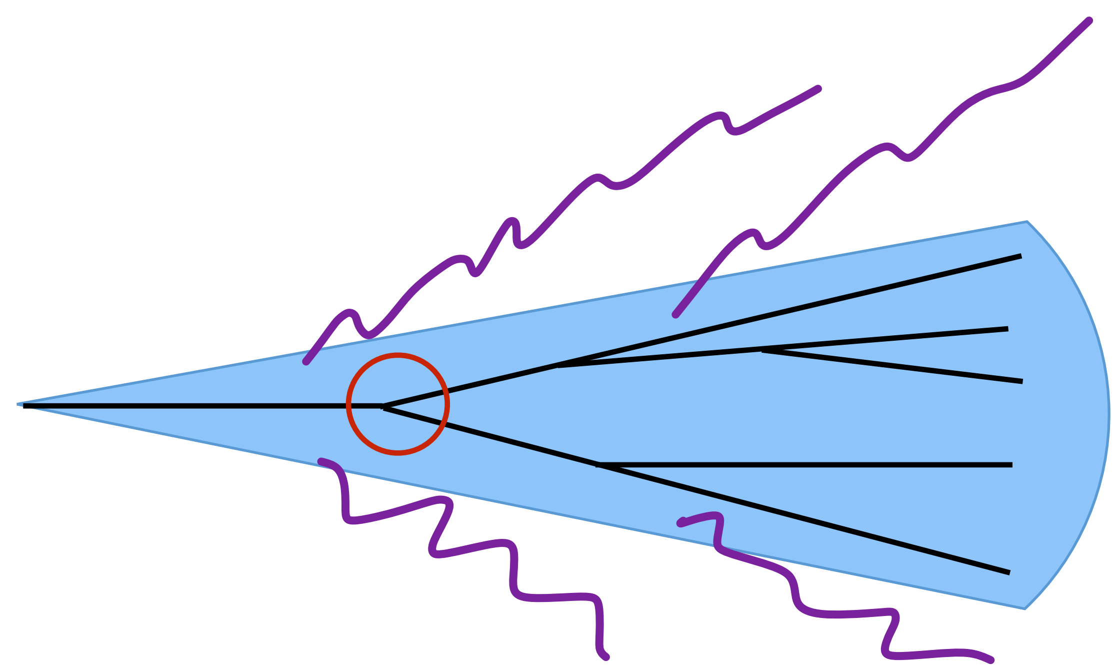

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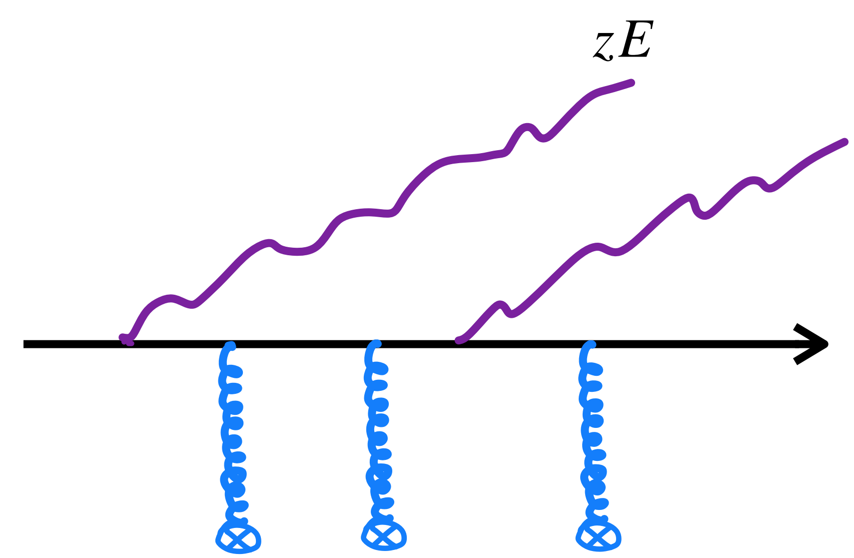

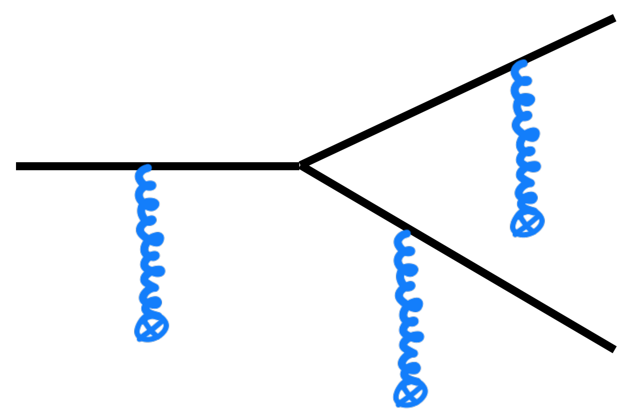

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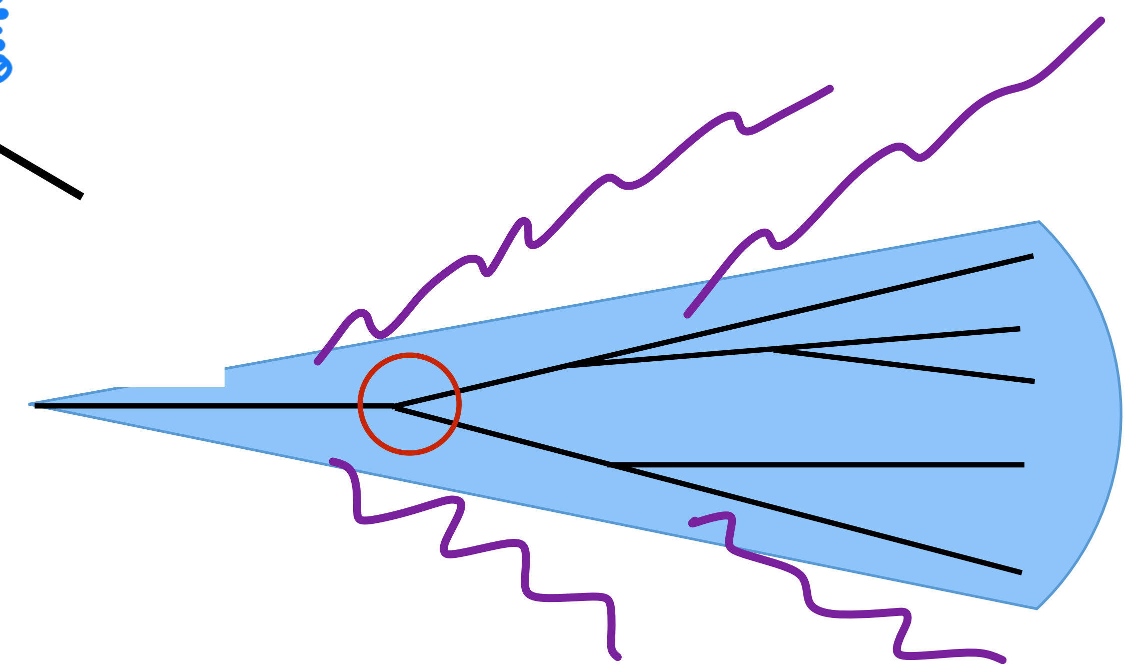
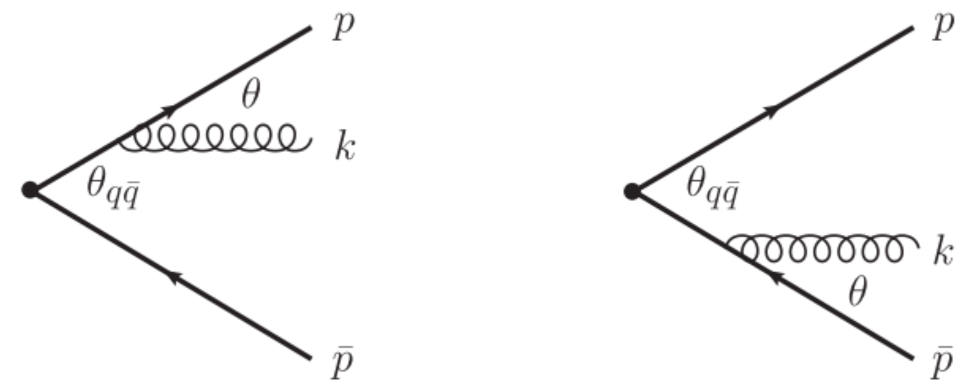
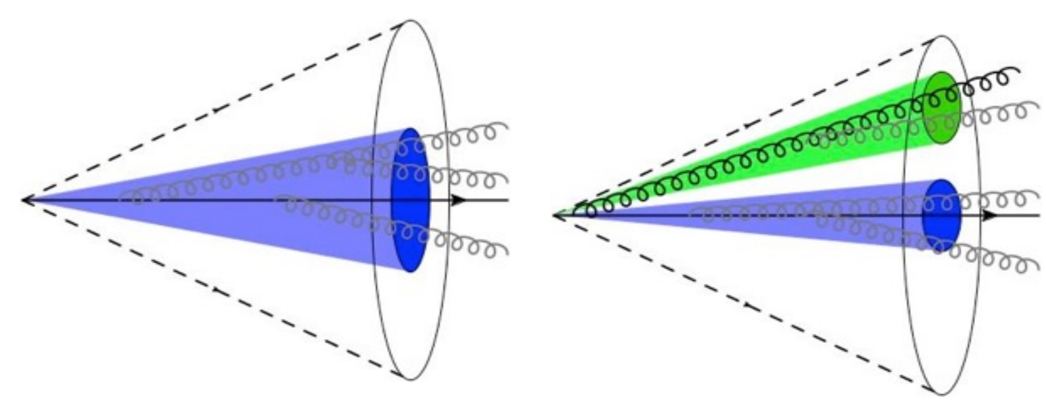


Color coherence in jet quenching

- Antenna calculations show that medium interactions can break angular ordering

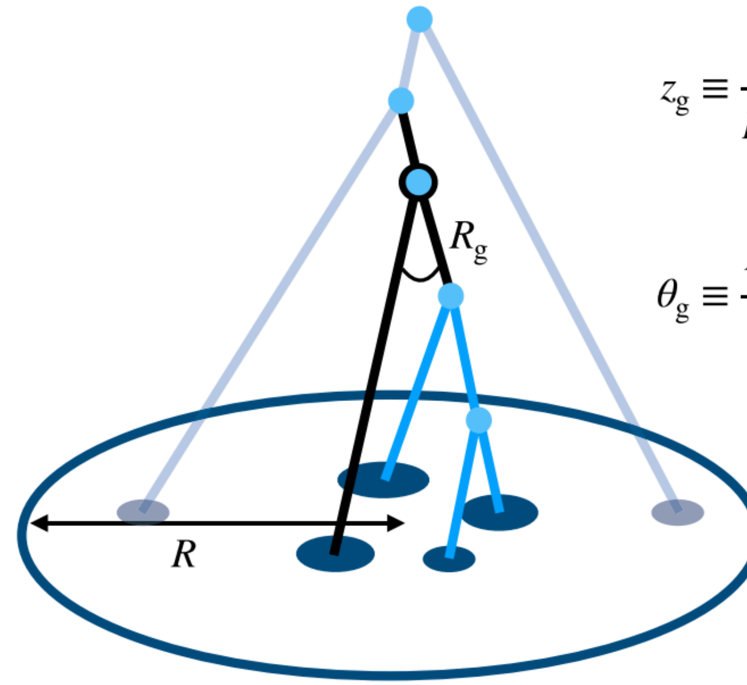


- Emergence of a resolution scale



- Can this be seen at the level of one splitting?

Grooming



$$z_g \equiv \frac{p_{T,\text{subleading}}}{p_{T,\text{leading}} + p_{T,\text{subleading}}}$$

$$\theta_g \equiv \frac{R_g}{R} \equiv \frac{\sqrt{\Delta y^2 + \Delta \varphi^2}}{R}$$

- Procedure in which reconstructed jets are reclustered while removing soft radiation to get access to the hardest splitting

- Widely used for jet substructure studies in pp collisions

- Measurement of the splitting function

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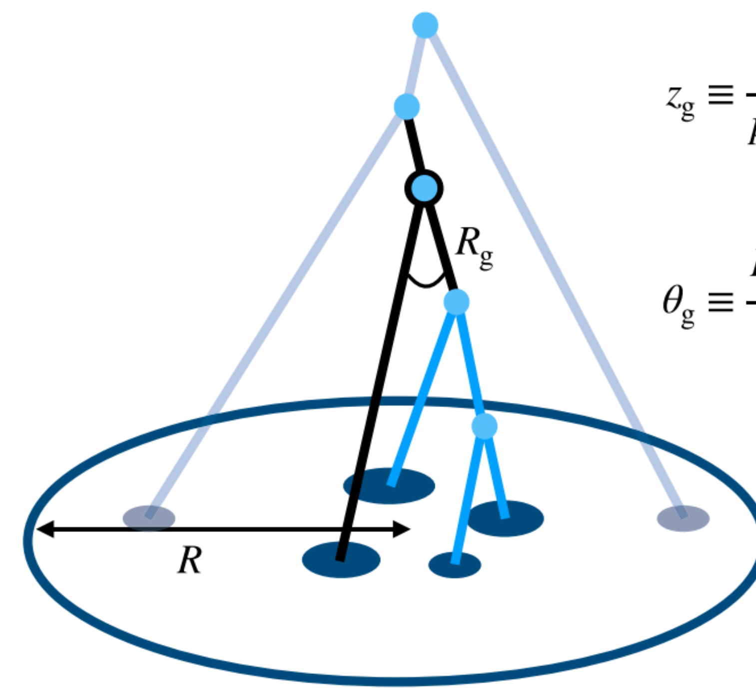

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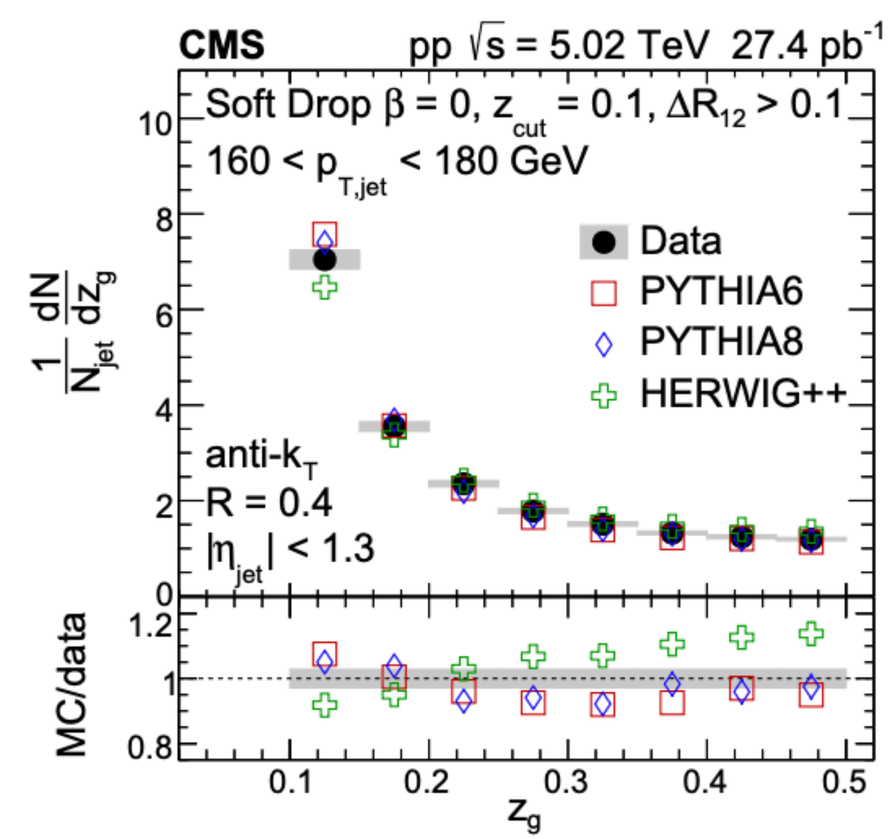
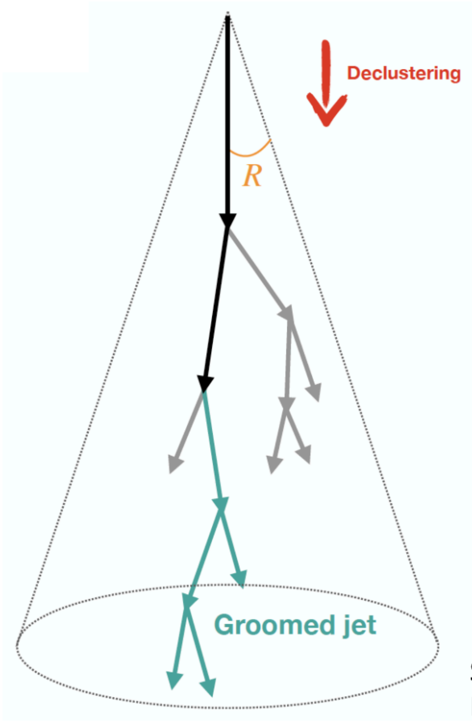


Grooming in heavy ion collisions



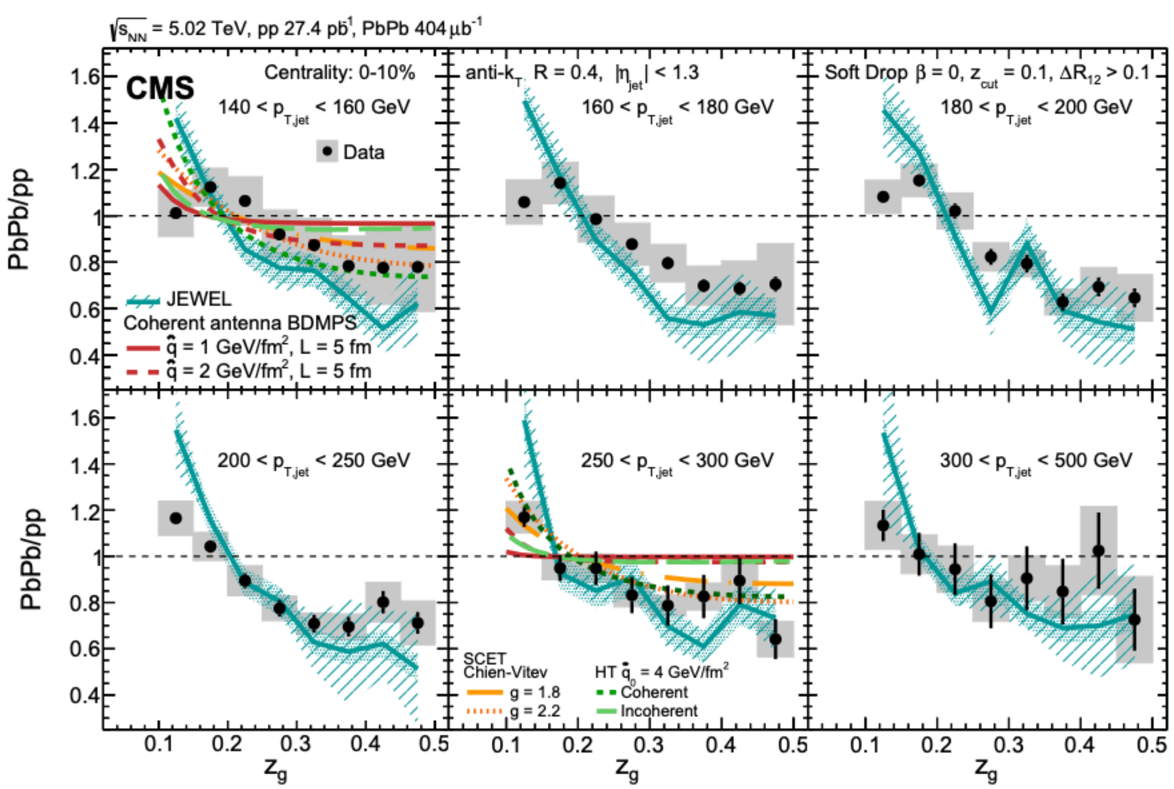
- Extract angle and energy fraction of the hardest splitting θ_g, z_g and look for modification of its distributions

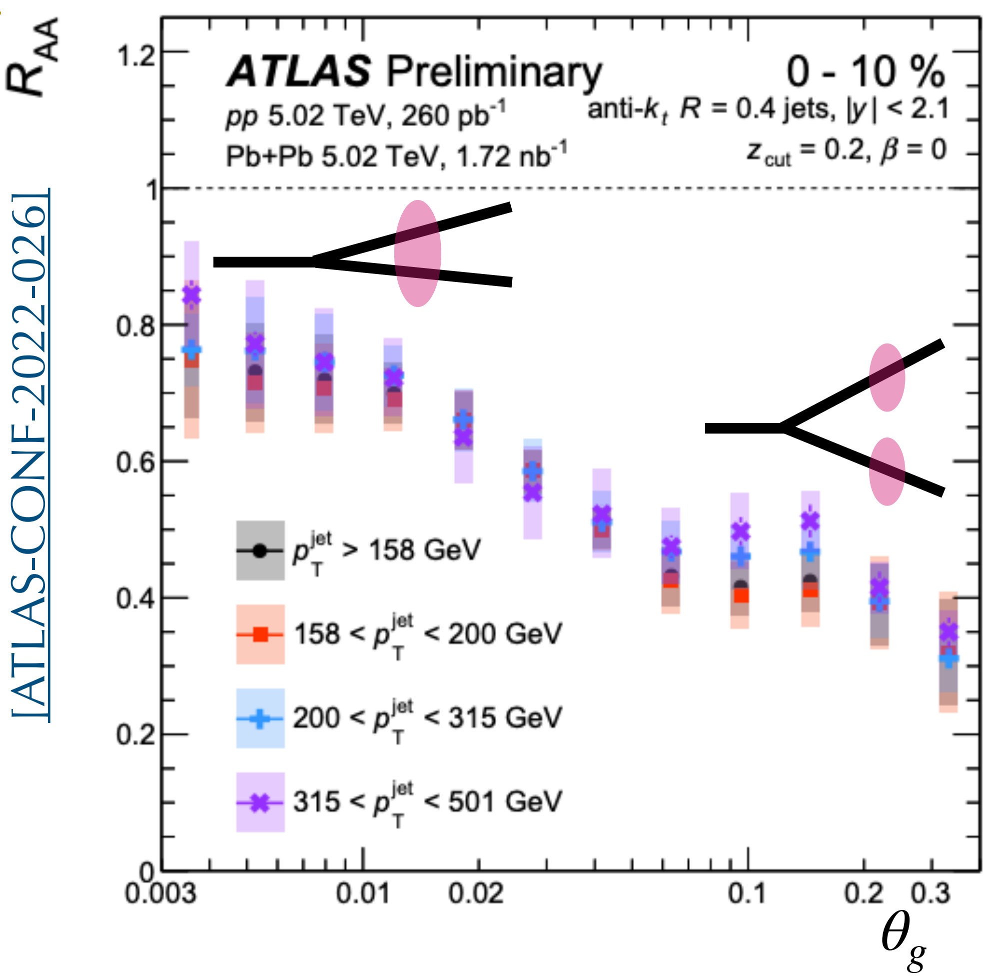
- Issues with having a robust angular variable from grooming

J. Mulligan, M. Ploskon 2006.01812

- Proposed grooming procedure for HIC

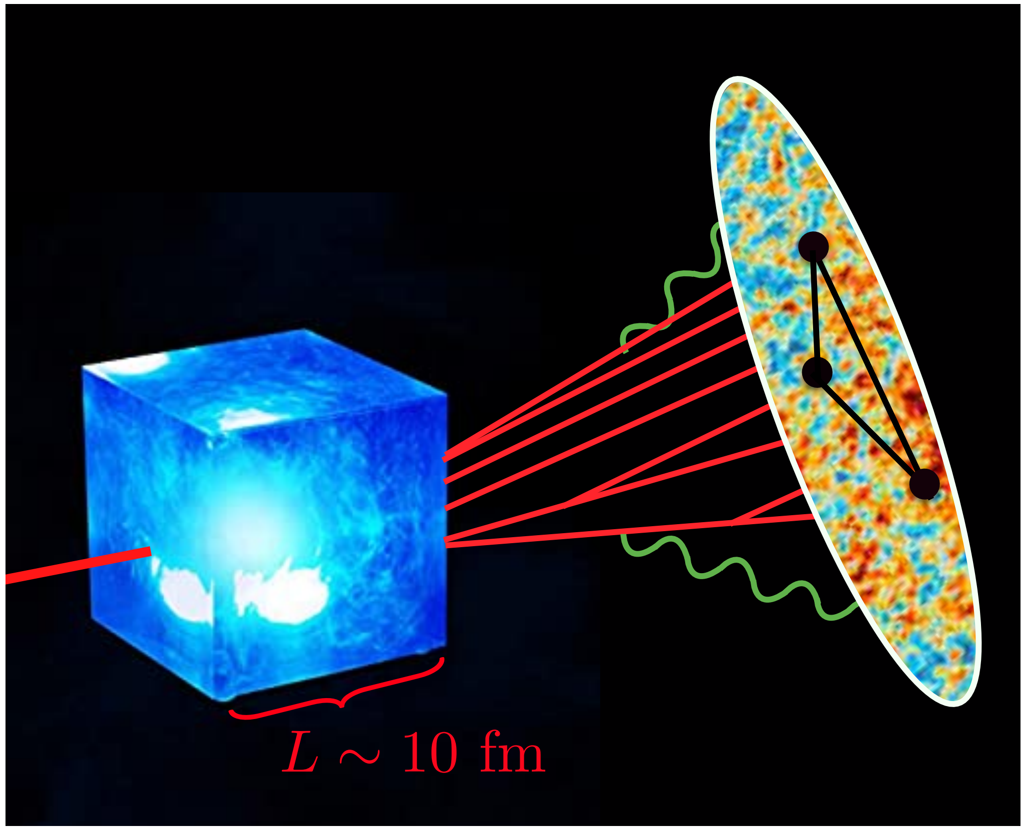
Y. Mehtar-Tani, A. Soto-Ontoso, K. Tywoniuk 1911.00375

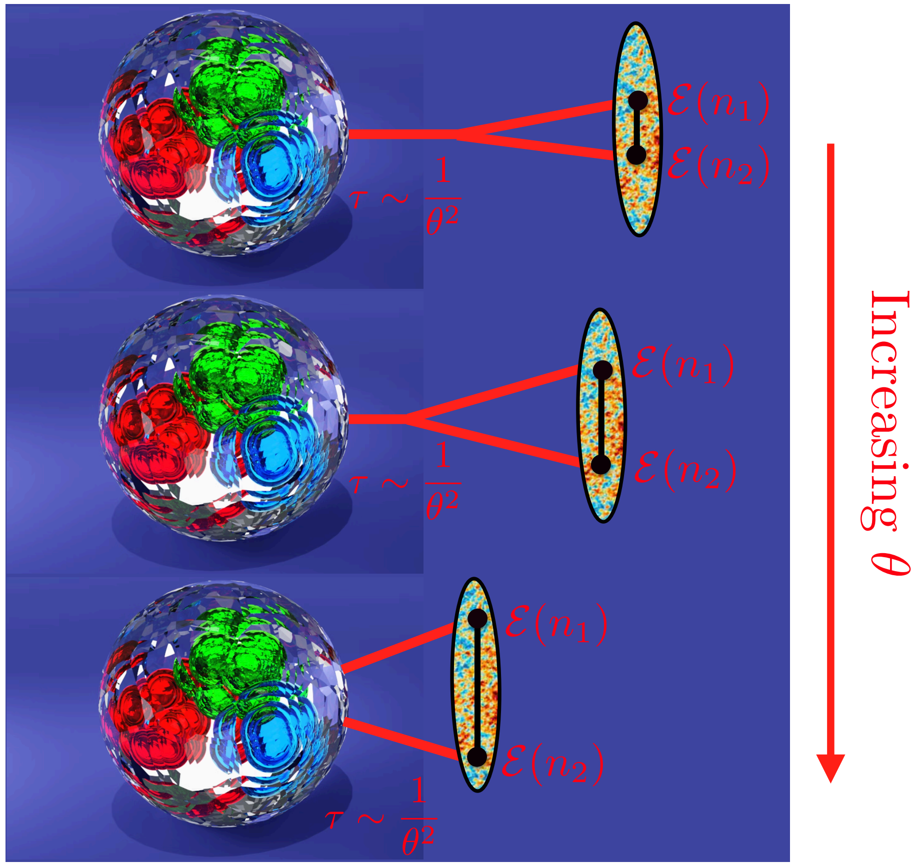




[ATLAS-CONF-2022-026]

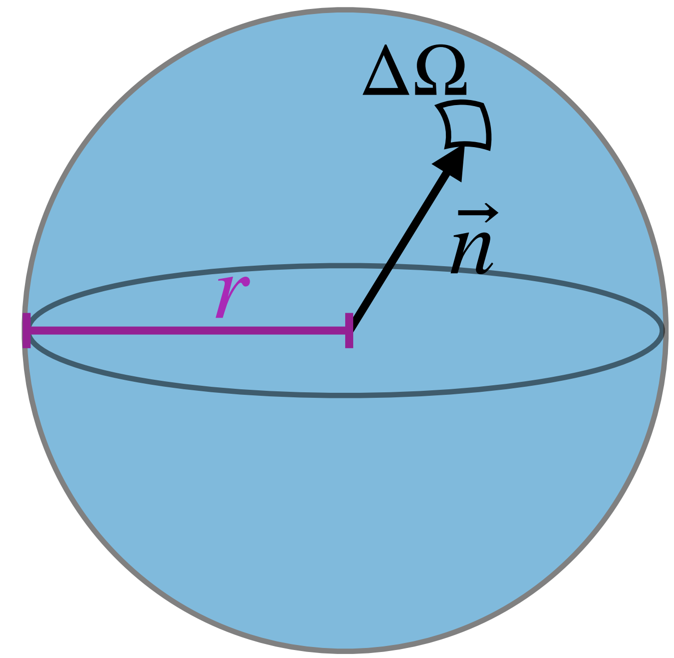
Energy Correlators





Energy flux operators

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\, r^2 n^i T_{0i}(t, r\vec{n})$$



- The 1-point function measures the total energy flux through an area element

$$\langle \mathcal{E}(\vec{n}) \rangle \propto \sum_i E_i$$

Sum over all particles going through $\Delta\Omega$

- Energy weighting naturally removes soft physics without grooming

D. Hoffman, J. Maldacena 0803.1467

Energy correlators

$$\frac{\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle}{Q^{2n}} = \frac{1}{\sigma}\sum_{ij}\int \frac{d\sigma_{ij}}{d\vec{n}_i d\vec{n}_j}\frac{E_i^n E_j^n}{Q^{2n}}\delta^{(2)}(\vec{n}_i - \vec{n}_1)\delta^{(2)}(\vec{n}_j - \vec{n}_2)$$

Energy correlators

- 2-point function

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- As a function of the relative angle only

$$\frac{d\Sigma^{(n)}}{d\theta} = \int d\vec{n}_{1,2}\frac{\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle}{Q^{2n}}\delta(\vec{n}_2\cdot\vec{n}_1 - \cos\theta)$$

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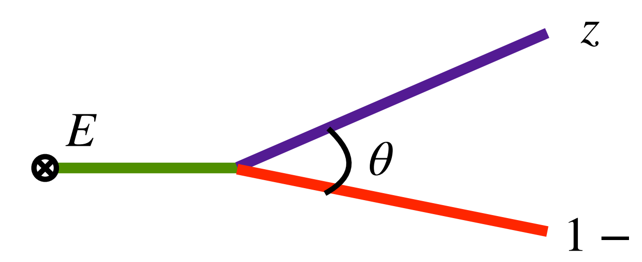
- ✦ Infrared and collinear safe for $n = 1$

Energy correlators



- For a quark jet at first order, $Q = E$ the energy of the jet

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma_{qg}} \int dz \frac{d\sigma_{qg}}{d\theta dz} z^n (1-z)^n + \mathcal{O}\left(\frac{\mu_s}{E}\right)$$

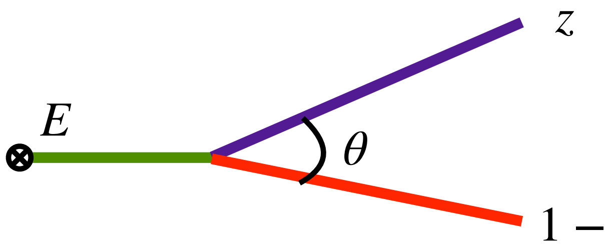
μ_s a softer scale over which the cross section is inclusive

- qq and gg contributions are higher order
- Additional energy loss ($E_q + E_g \neq E$) is also subleading

$$z = \frac{E_g}{E}$$

Energy correlators in vacuum

D. Hoffman, J. Maldacena 0803.1467
H. Chen , I. Moult, J. Sandor, H. X. Zhu 2202.04085



- At leading order

$$\frac{d\sigma_{qg}^{\mathrm{vac}}}{d\theta dz} = \frac{\alpha_s C_{\mathrm{F}} \sigma}{\pi} \frac{1+(1-z)^2}{z\ \theta} \quad \Rightarrow \quad \frac{d\Sigma^{(1)}}{d\theta} \sim \frac{1}{\theta}$$

- Collinear emissions can be resummed using CFT techniques changing the scaling only by an anomalous dimension

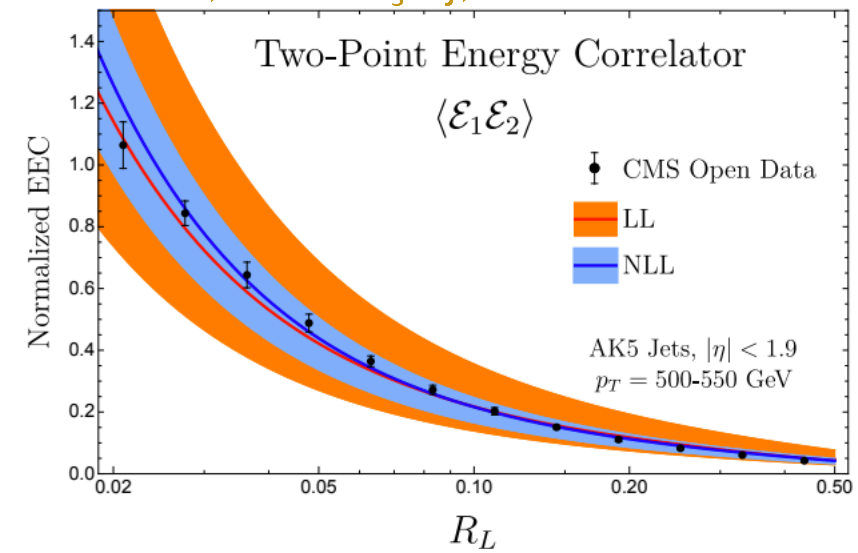
$$\frac{d\Sigma^{(1)}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

$\gamma(3)$ is the twist-2 spin-3 QCD anomalous dimension

- Higher-orders, soft physics, quark/gluon ratios can change the overall normalization but not the power-law behavior

Energy correlators in vacuum

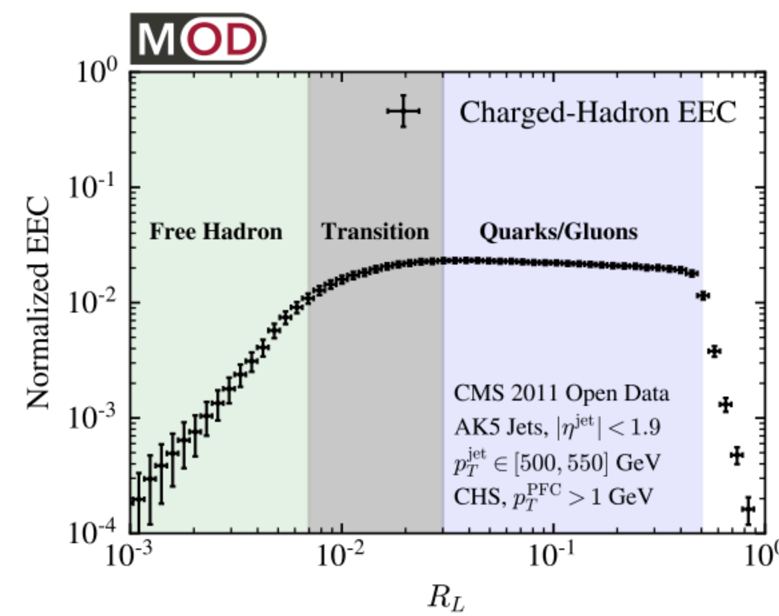
K. Lee, B. Meçaj, I. Moult 2205.03414



- Have not yet been measured

- Analyses done by theorist with CMS open data

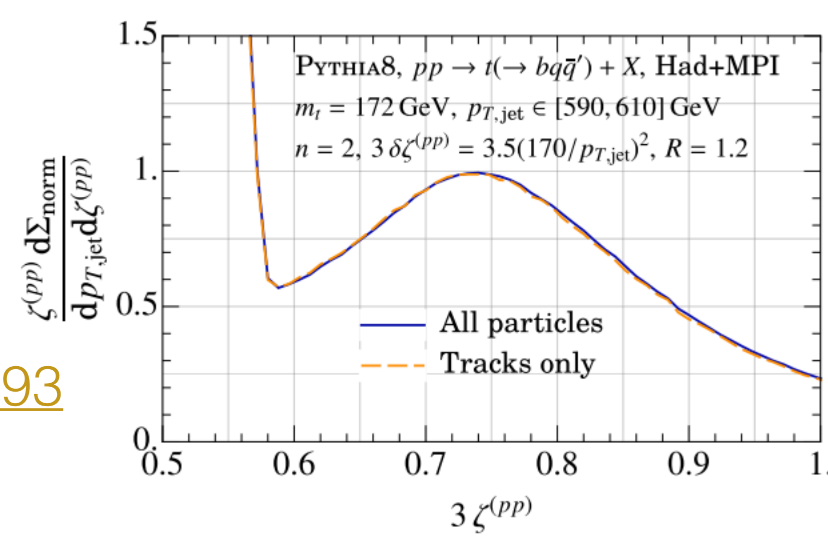
P. T. Komiske, I. Moult, J. Thaler, H. X. Zhu 2201.07800



- Sensitivity to hadronization transition

- Sensitivity to top mass in the 3-point function

J. Holguin, I. Moult, A. Pathak, M. Procura 2201.08393



Energy correlators in HIC

- Background is expected to be less of an issue
  - Energy weighting removes most of the soft physics, specially if one increases the power in the energy weighting
  - Uncorrelated background does not affect the shape of the correlations, only the normalization

- Observables are not event-by-event
  - Fluctuations are less important
  - Requires large statistics
  - Cannot be used to tag events

Energy correlators in HIC

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- Calculation of inclusive two-particle cross-section in HIC is very challenging
- It is reasonably well understood in the soft limit $z \to 0$ or when all transverse momenta are integrated over, thus losing the angle dependence
- For the energy correlator calculation is is crucial to keep z finite and also the angle dependence
- Some additional assumptions/approximations must be made to evaluate the cross section

Evaluation of in-medium splittings

- Full evaluation keeping z and θ not yet implemented

- Two available approximations:
  - Opacity expansion $(N = 1)$ Sievert, Vitev 1807.03799
    - Unitarity problems can lead to negative cross sections
    - Recursive formulas to generate all orders (not yet implemented numerically)

  - Semi-hard approximation
    - Resums multiple scatterings in the eikonal approximation through Wilson lines in straight-line trajectories
    - Assumes semi-hard splittings (z not too small)
    - Neglects effects coming from broadening of transverse momenta of produced particles

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653
Isaksen, Tywoniuk 2107.02542

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FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653
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Evaluation of in-medium splittings

- Medium is assumed uniform, with length L
- The strength of the interactions is encoded in the jet quenching parameter $\hat{q}$, which measures the average transverse momentum transferred per unit length
- Emissions with a long formation time are not sensitive to the medium and therefore are emitted as in vacuum
- Multiple medium scatterings destroy the color coherence between the daughter partons

Time and angular scales

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653

- (Vacuum) formation time:

$$t_f = \frac{2}{z(1-z)E\theta^2}$$

$$\theta_L \sim (EL)^{-1/2}$$

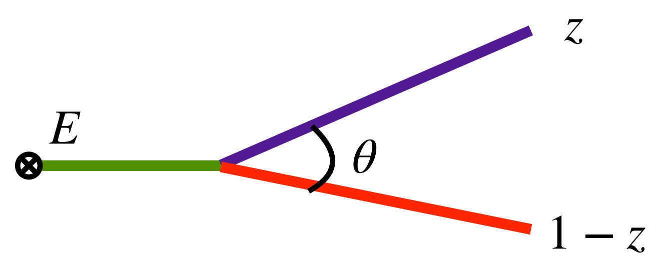
Below θ_L all emissions have a formation time larger than L

- Decoherence time:

$$t_d \sim (\hat{q}\theta^2)^{-1/3}$$

$$\theta_c \sim (\hat{q}L^3)^{-1/2}$$

Below θ_c splittings do not color decohere and the medium does not resolve them



If $\theta_L > \theta_c$ then θ_c becomes irrelevant

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FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653

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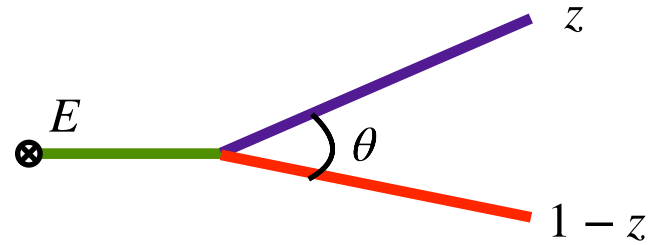
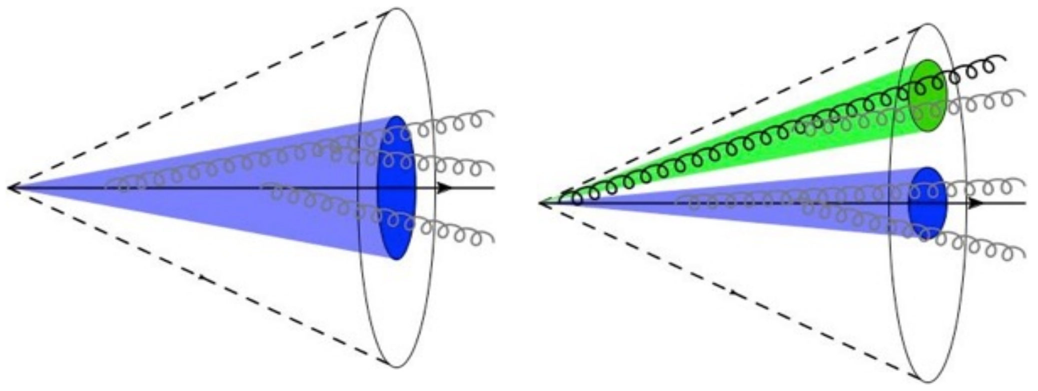




If $\theta_L > \theta_c$ then θ_c becomes irrelevant

Time and angular scales

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653

- (Vacuum) formation time:

$$t_f = \frac{2}{z(1-z)E\theta^2}$$

$$\theta_L \sim (EL)^{-1/2}$$

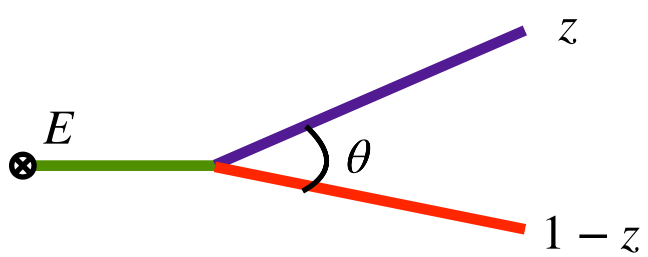
Below θ_L all emissions have a

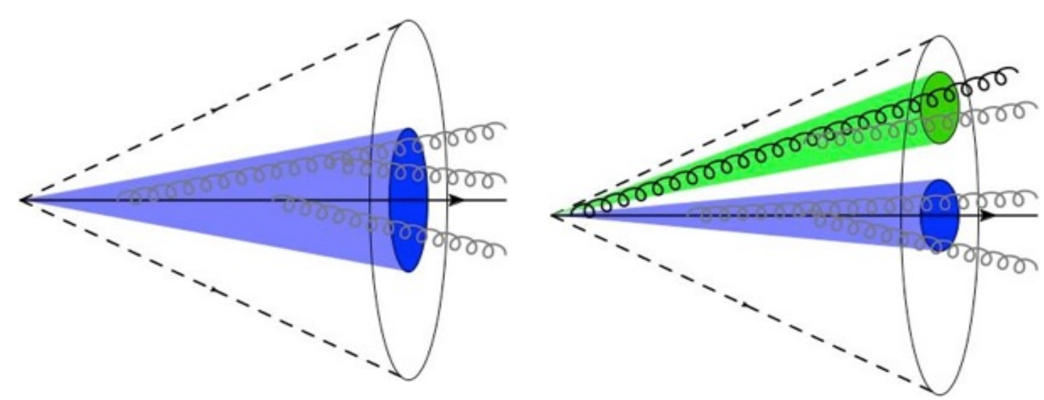
Can be extended to include a more realistic interaction or expanding media, but then we would not know the scales directly from the equations

$$t_d \sim (\hat{q}\theta^2)^{-1/3}$$

$$\theta_c \sim (\hat{q}L^3)^{-1/2}$$

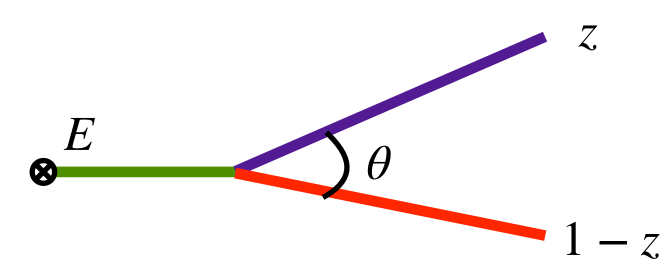
Below θ_c splittings do not color decohere and the medium does not resolve them





If $\theta_L > \theta_c$ then θ_c becomes irrelevant

Angular scales



- Three parameters $E, \hat{q}, L$

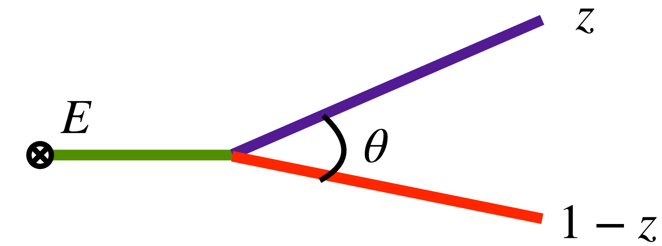
- Two competing angular scales

$$\theta_L \sim (EL)^{-1/2} \qquad\qquad \theta_c \sim (\hat{q}L^3)^{-1/2}$$

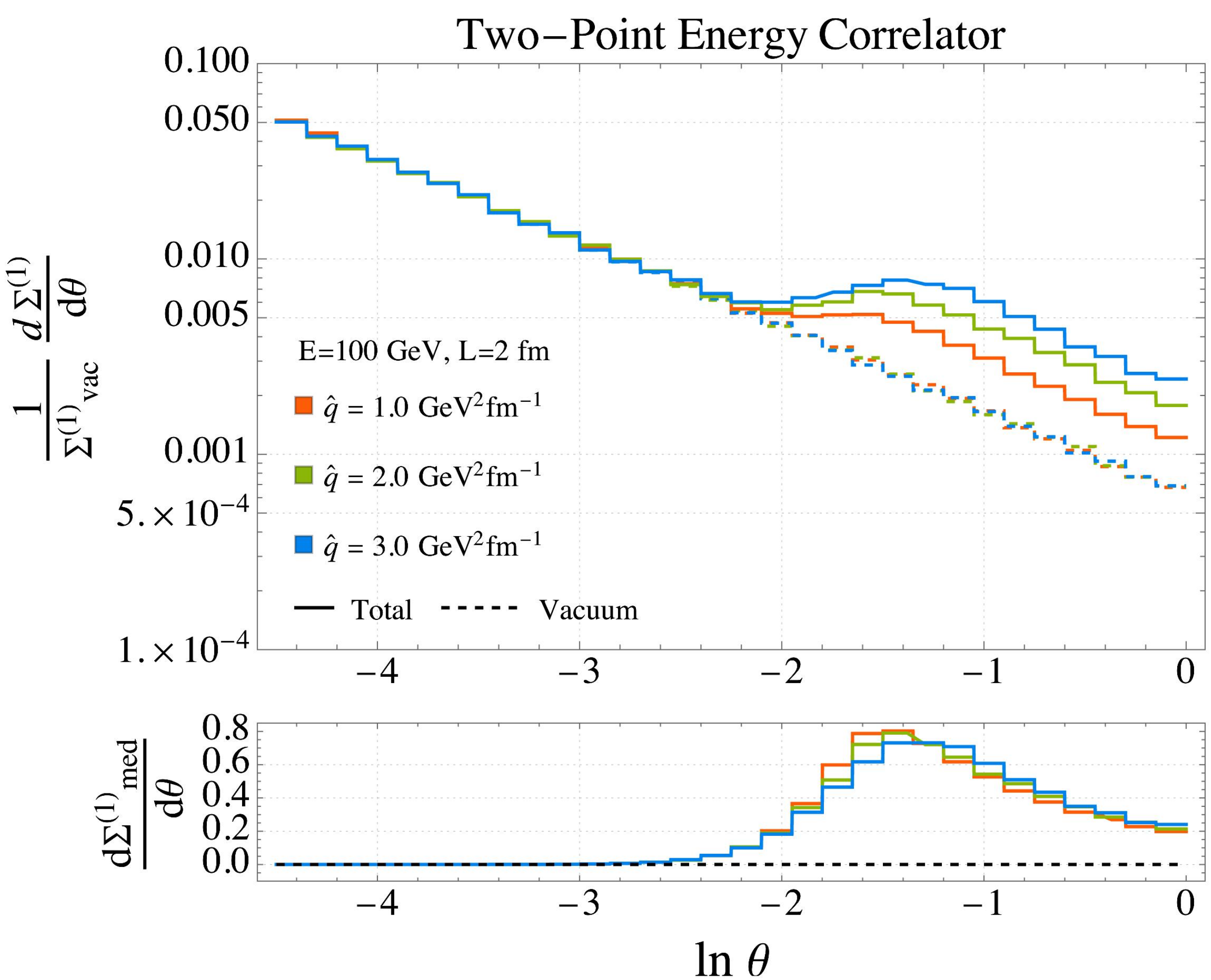
- For $\theta < \theta_L$, splitting occurs outside of the medium, no medium modification is expected

- For $\theta < \theta_c$, the medium does not resolve the splitting, small medium-modification expected

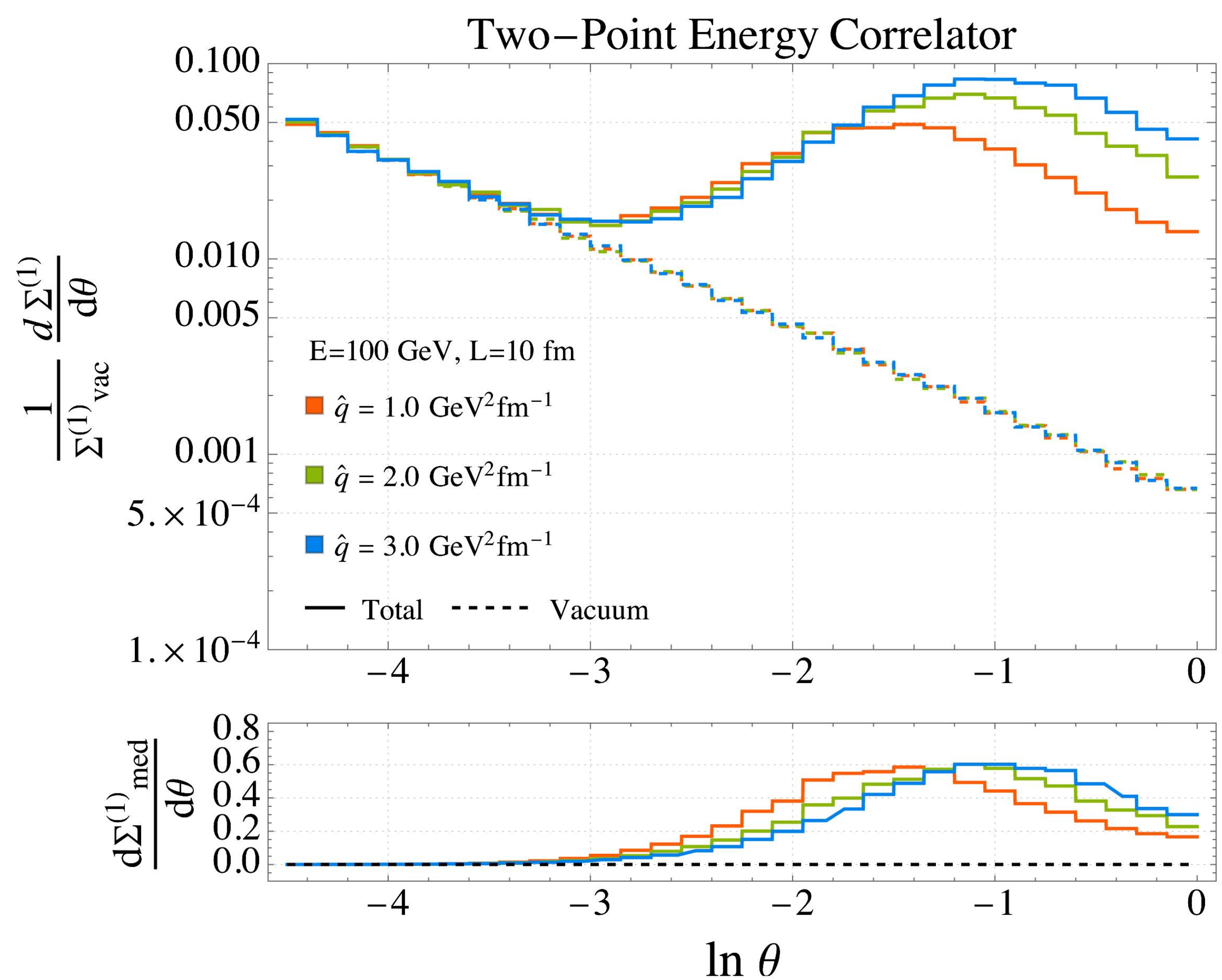
Results



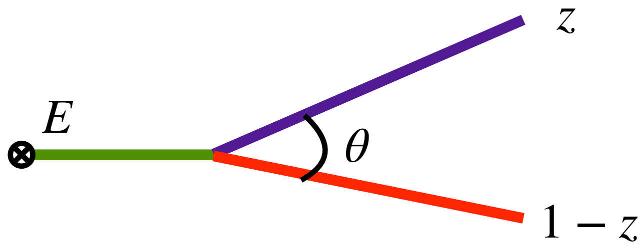
$\theta_c < \theta_L$



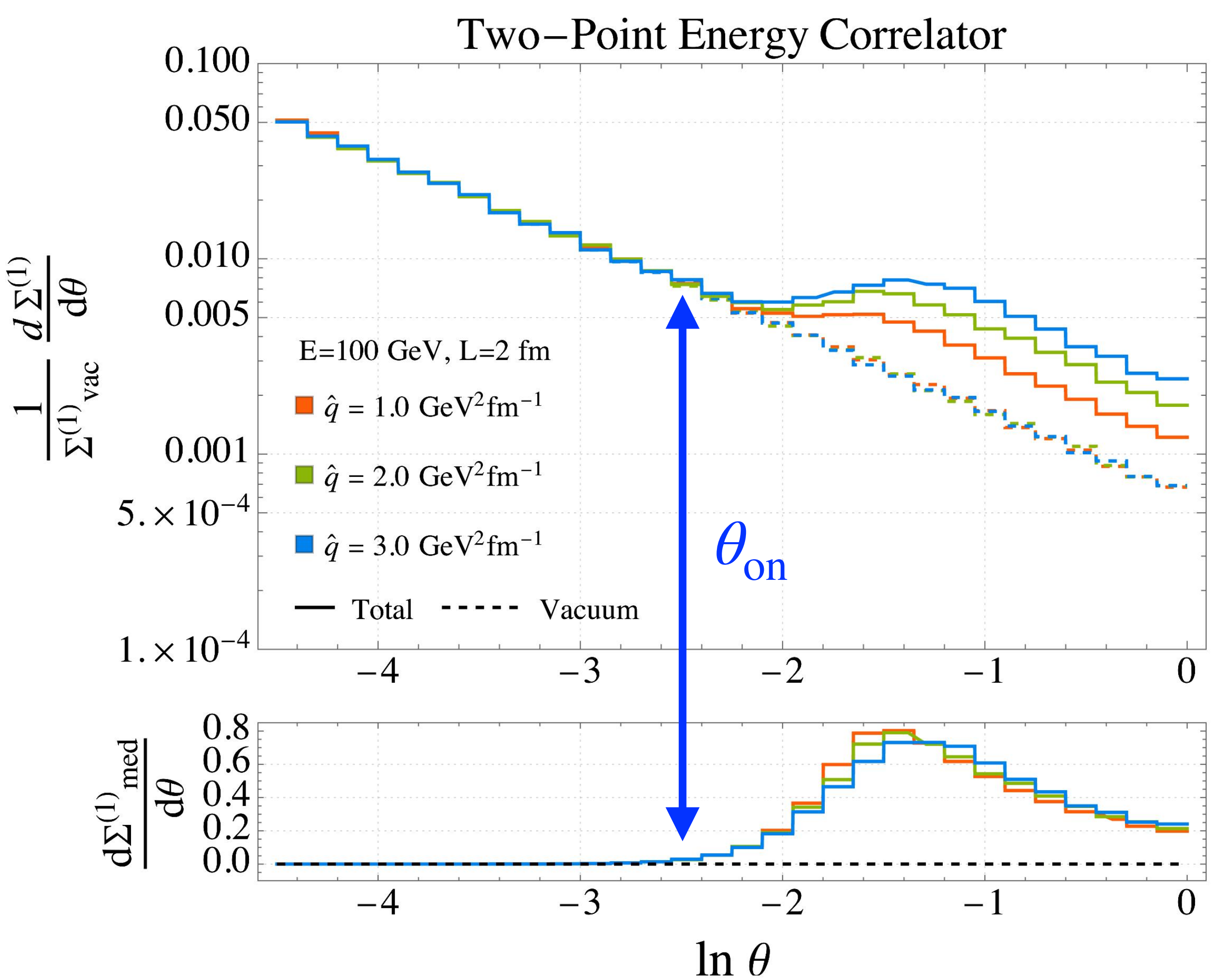
$\theta_c > \theta_L$



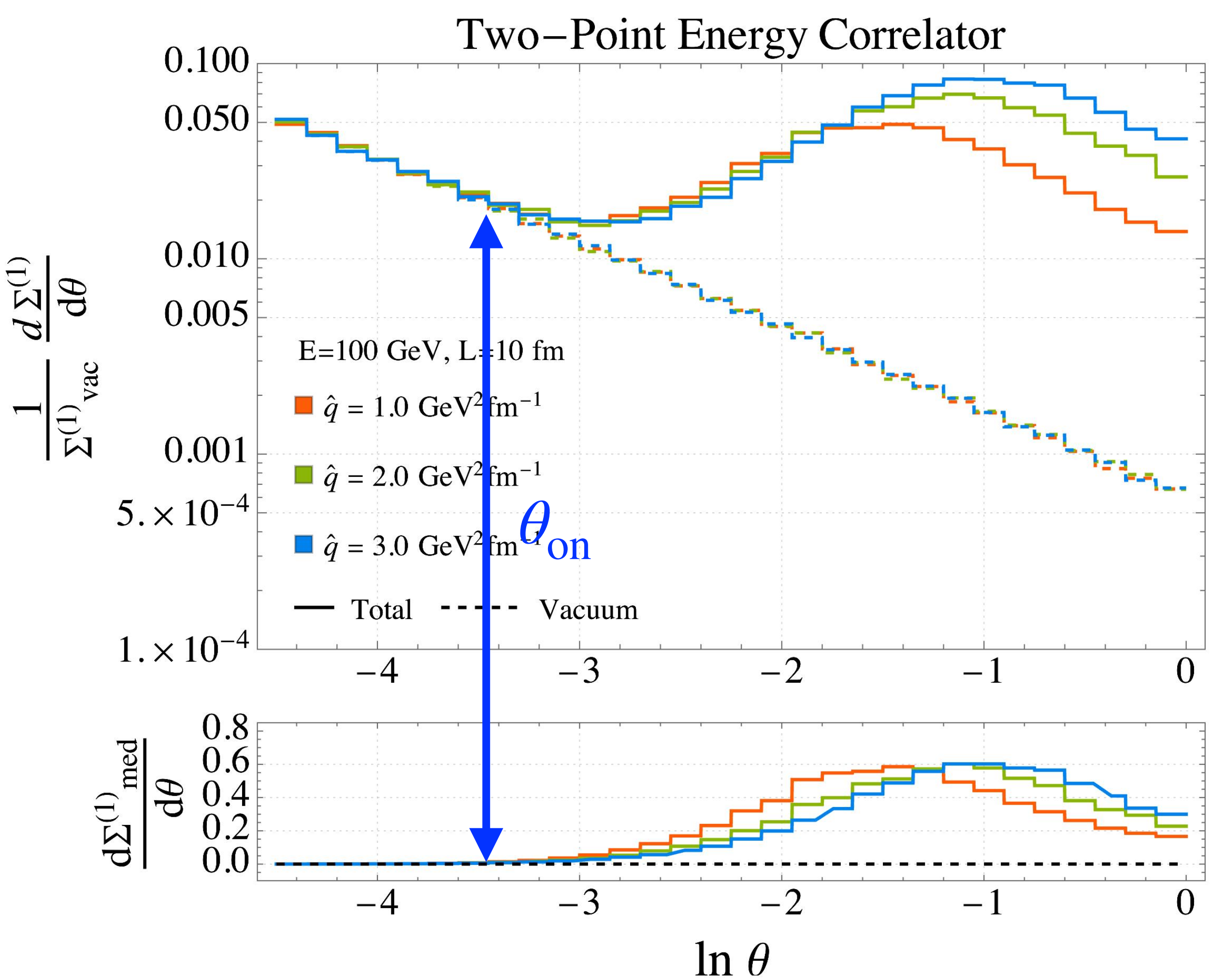
Results



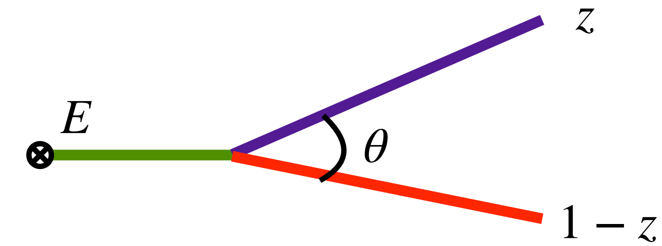
$\theta_c < \theta_L$



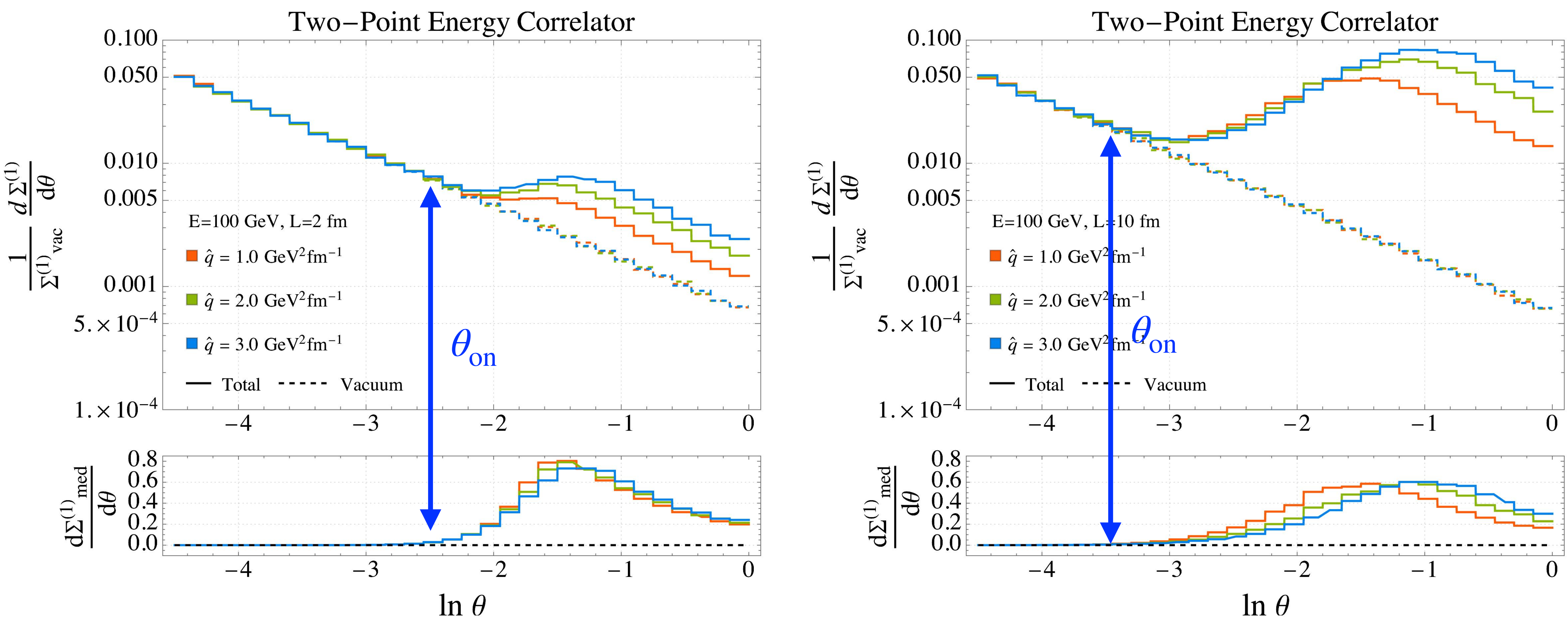
$\theta_c > \theta_L$



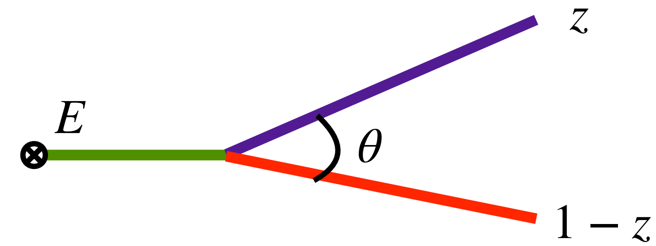
Results



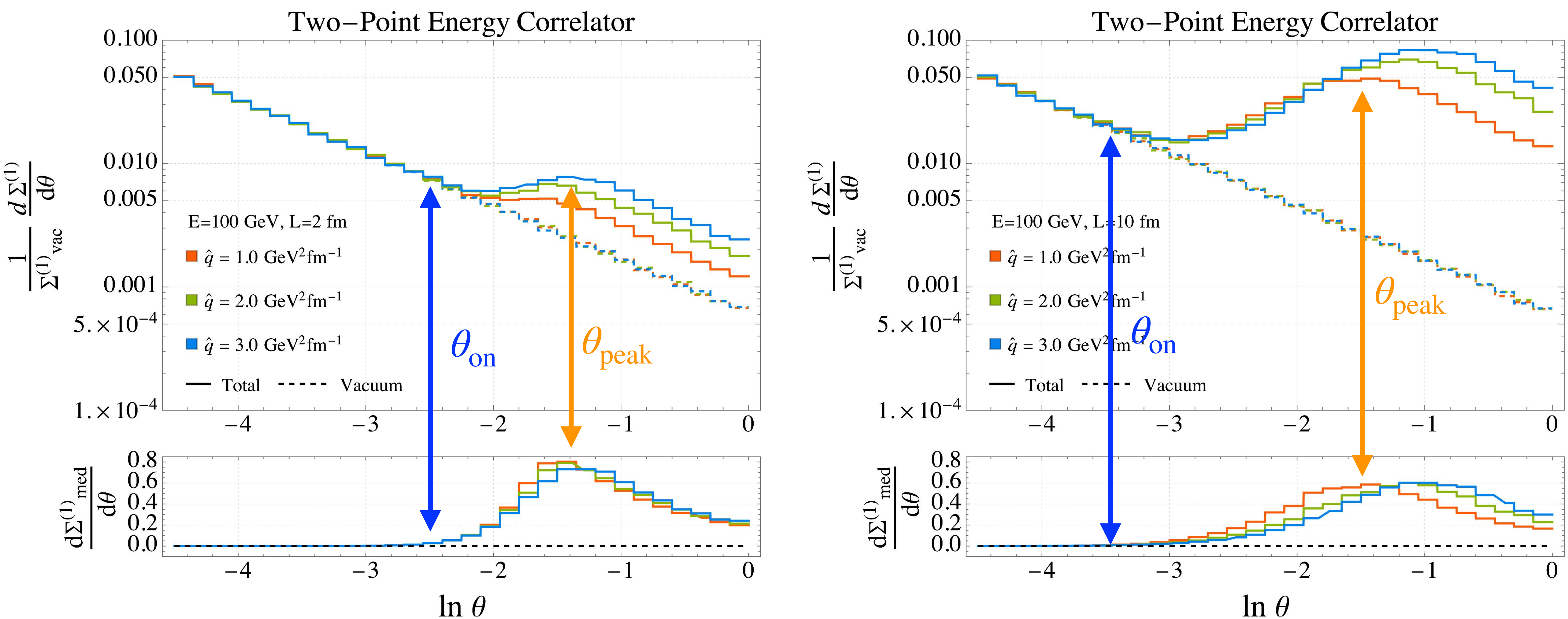
$\theta_c <$ θ_{on} does not depend on $\hat{q}$ $> \theta_L$



Results



$\theta_c <$ θ_{on} does not depend on $\hat{q}$ $> \theta_L$



Extracting the behavior of $\theta_{\rm on}$ and $\theta_{\rm peak}$

- Generated the EEC for 248 sets of parameters with $E \in [50{,}700]$ GeV, $L \in [0.2{,}10]$ fm, $\hat{q} \in [1{,}3]$ GeV^2/fm

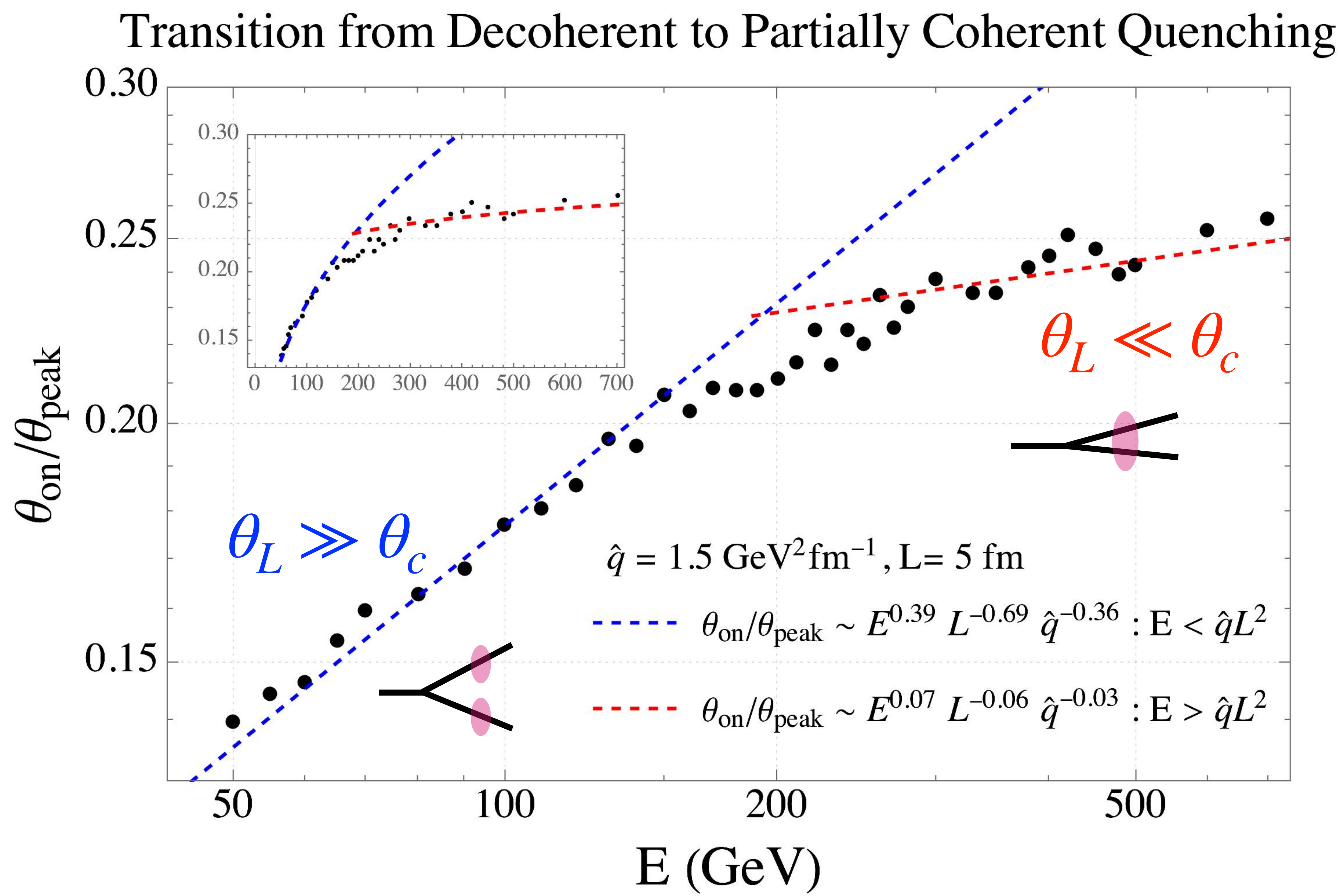
- Extracted scaling behavior of $\theta_{\rm on}$ and $\theta_{\rm peak}$ in terms of the three parameters

- In all regions the onset angle exhibits the same behavior

$$\theta_{\rm on} \sim \theta_L^{1\pm0.1}$$

- The peak angle has different behaviors in the two different regimes
  - For $\theta_L > \theta_c$: $\theta_{\rm peak}^{\rm DC} \sim E^{-0.86\pm0.1} L^{0.21\pm0.1} \hat{q}^{0.36\pm0.1} \sim \theta_d^{1.4\pm0.1} \theta_L^{-0.4\pm0.1}$
  - For $\theta_L < \theta_c$: $\theta_{\rm peak}^{\rm PC} \sim E^{-0.54\pm0.1} L^{-0.31\pm0.1} \hat{q}^{0.09\pm0.1} \sim \theta_c^{-0.2\pm0.1} \theta_L^{1.1\pm0.1}$

EECs and color coherence



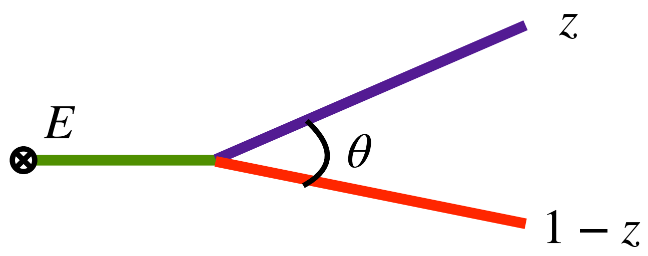
Conclusions

- Energy correlators provide a powerful tool for understanding jets in HIC
  - ✦ Experimentally accesible
  - ✦ Can be calculated perturbatively thanks to insensitivity to soft physics and uncorrelated background
  - ✦ Characteristic features of the calculation for in-medium splittings are clearly imprinted in the observables
- Energy correlators provide a robust angular variable which can be used to probe color coherence in jets in the QGP

Outlook

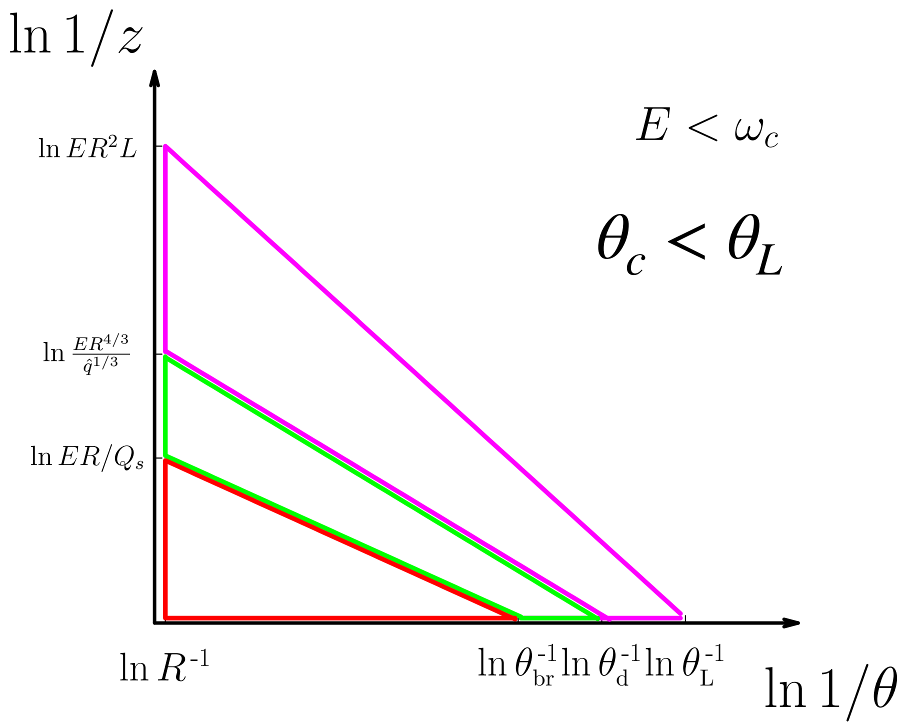
- Lots of new exciting developments!
- Test other models for the in-medium splitting calculation
  - GLV: Onset angle is not defined as sharply as in the multiple scattering case. Could be used to show the importance of the LPM regime
  - Tilted Wilson lines with Yukawa potential: Onset of coherence is NOT a feature of the harmonic approximation
- Expanding media
  - Using energy correlators to find the relevant angular scales
- Heavy quarks
  - Can be used to measure the dead-cone (calculation in pp coming out very soon)
- Monte Carlo studies
  - Test resilience to background
  - Test the effects of having the full parton shower

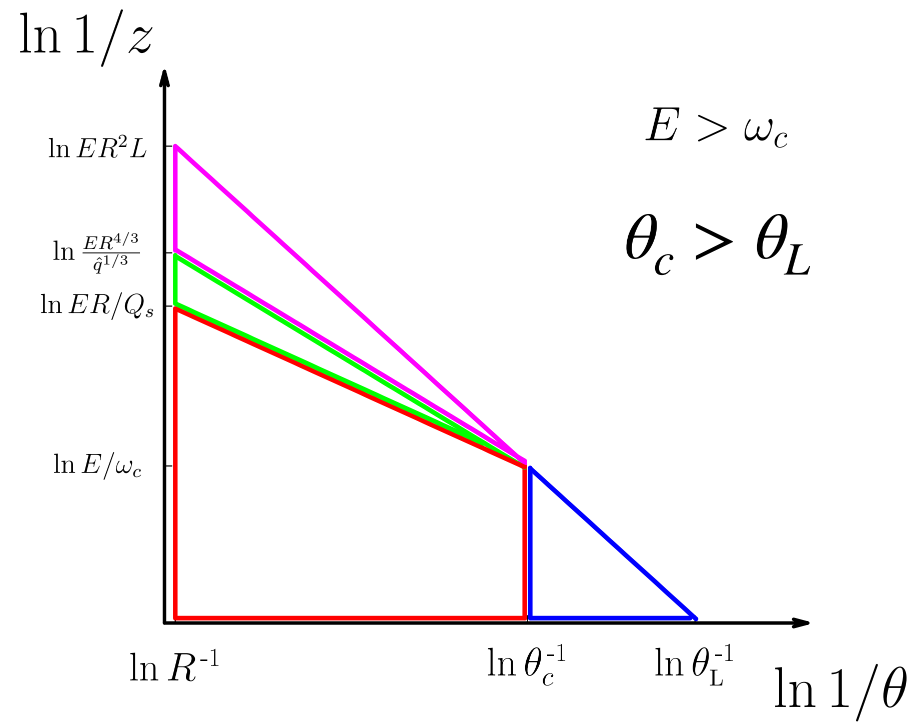
Thank you!



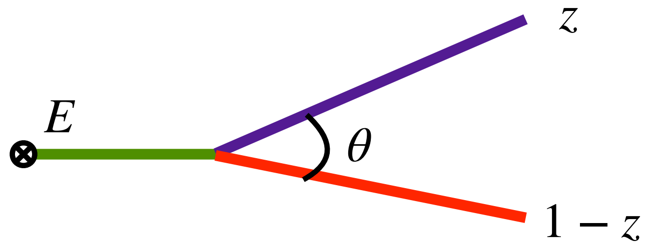
Lund plane

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653





$\omega_c \sim \hat{q} L^2$



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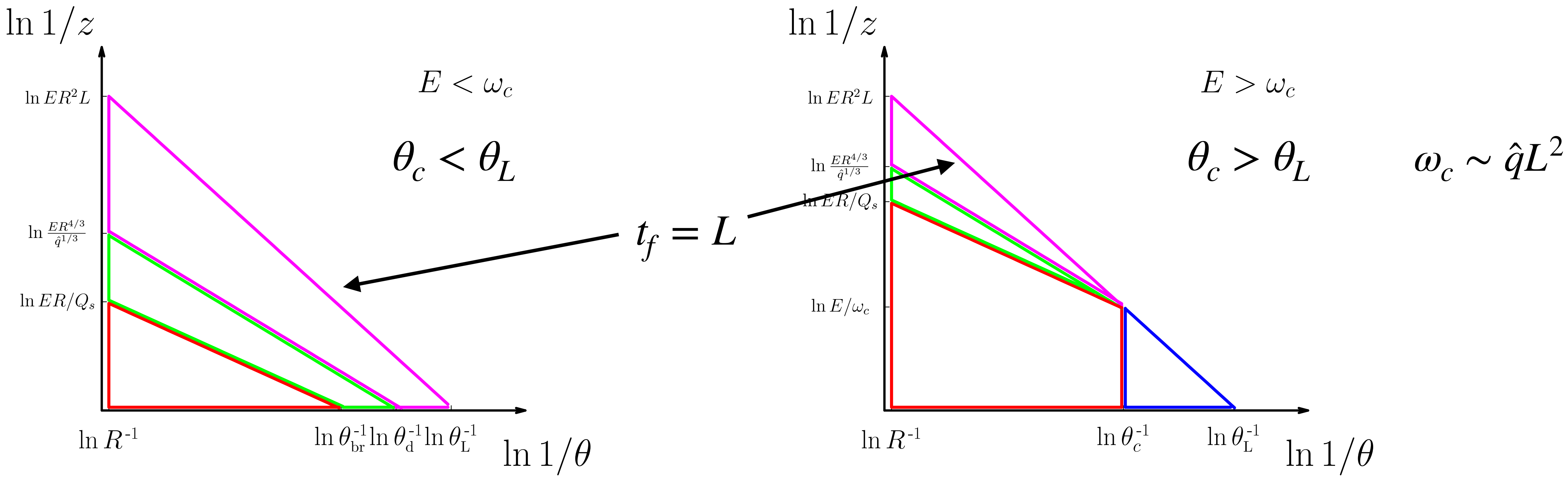

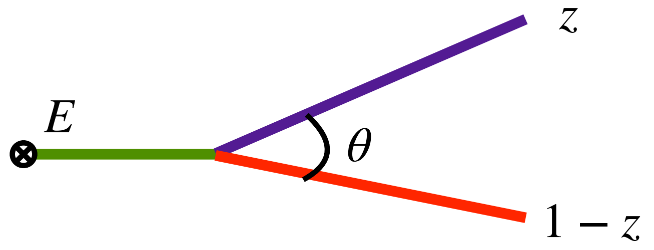

Lund plane

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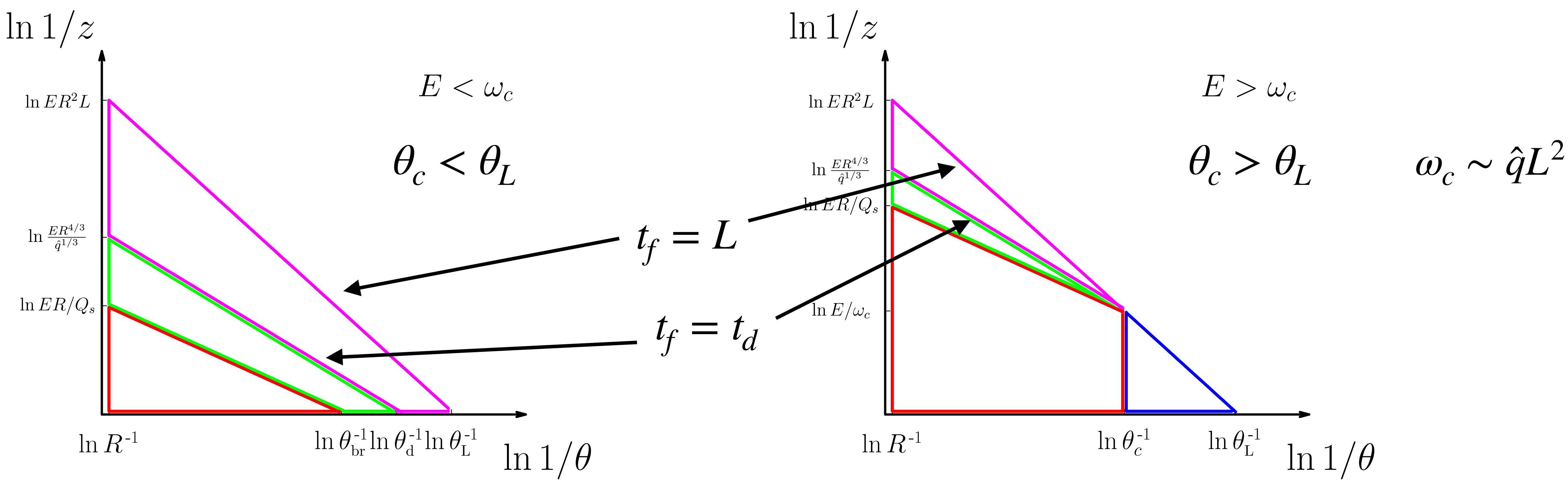

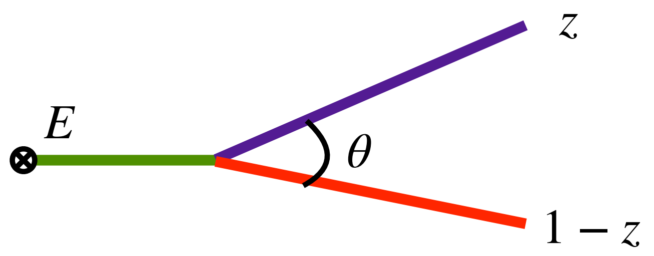

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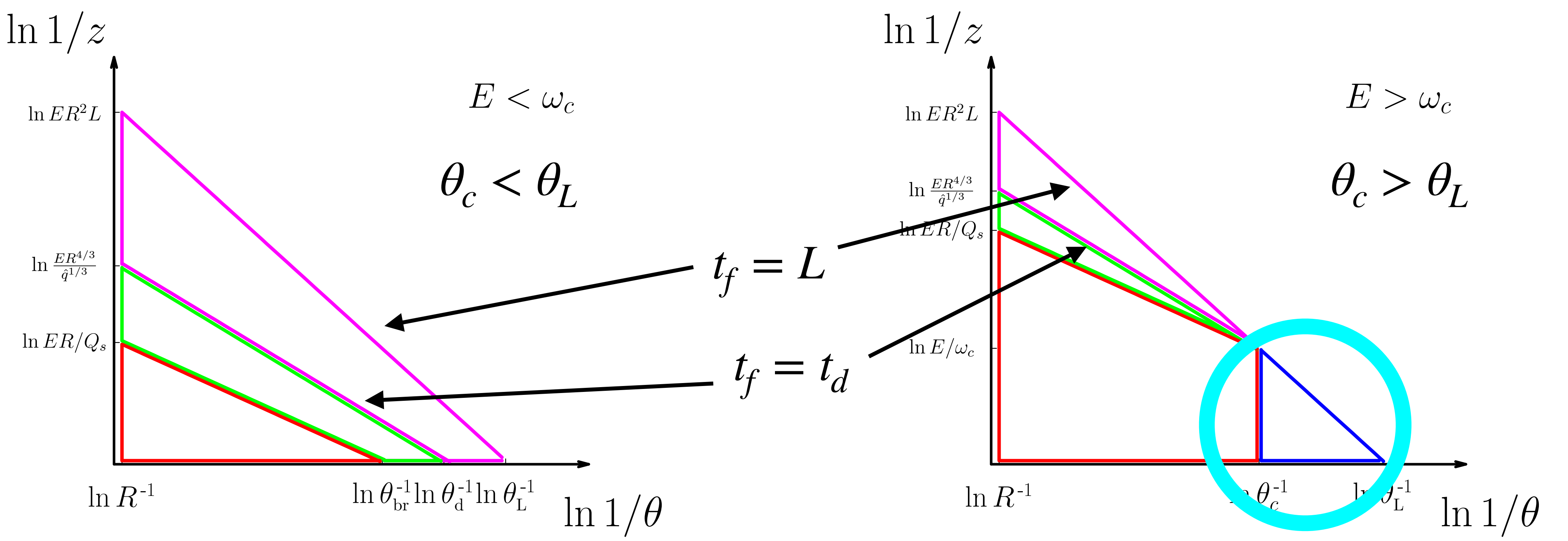

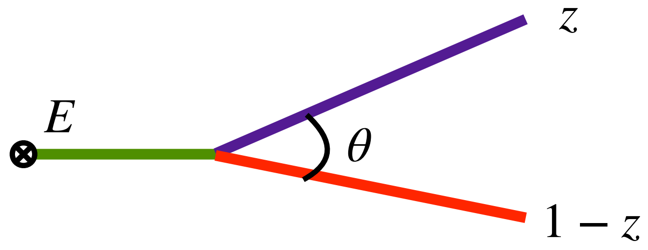

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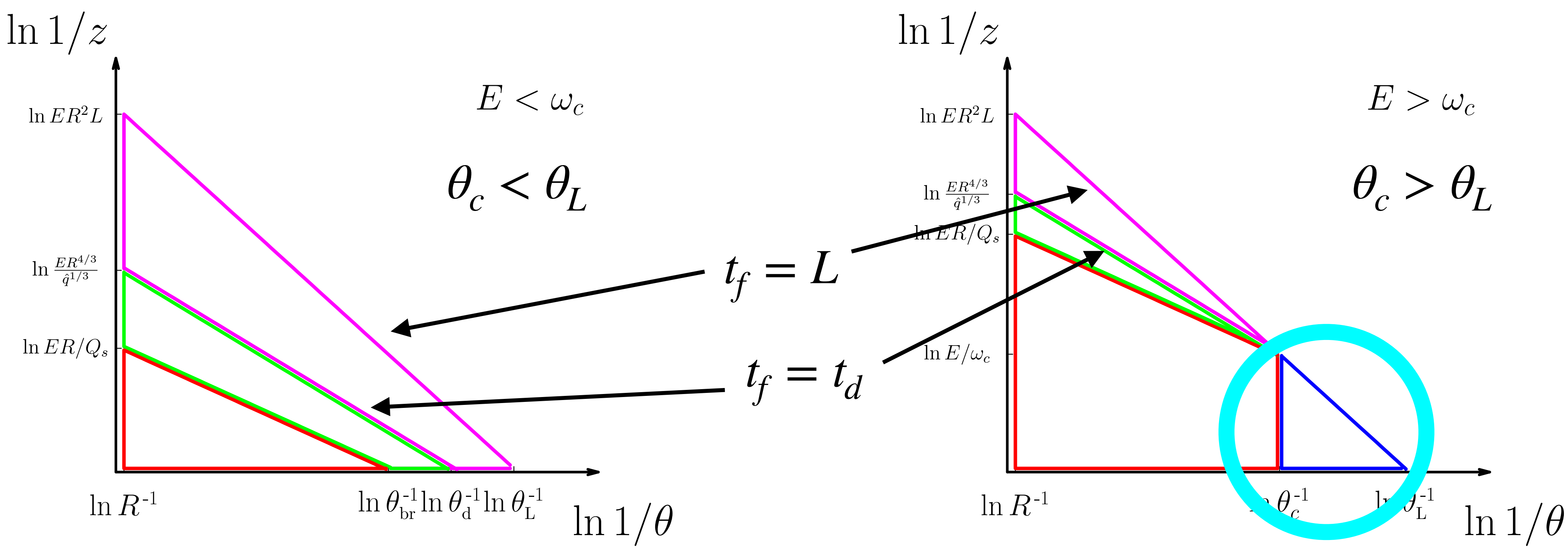
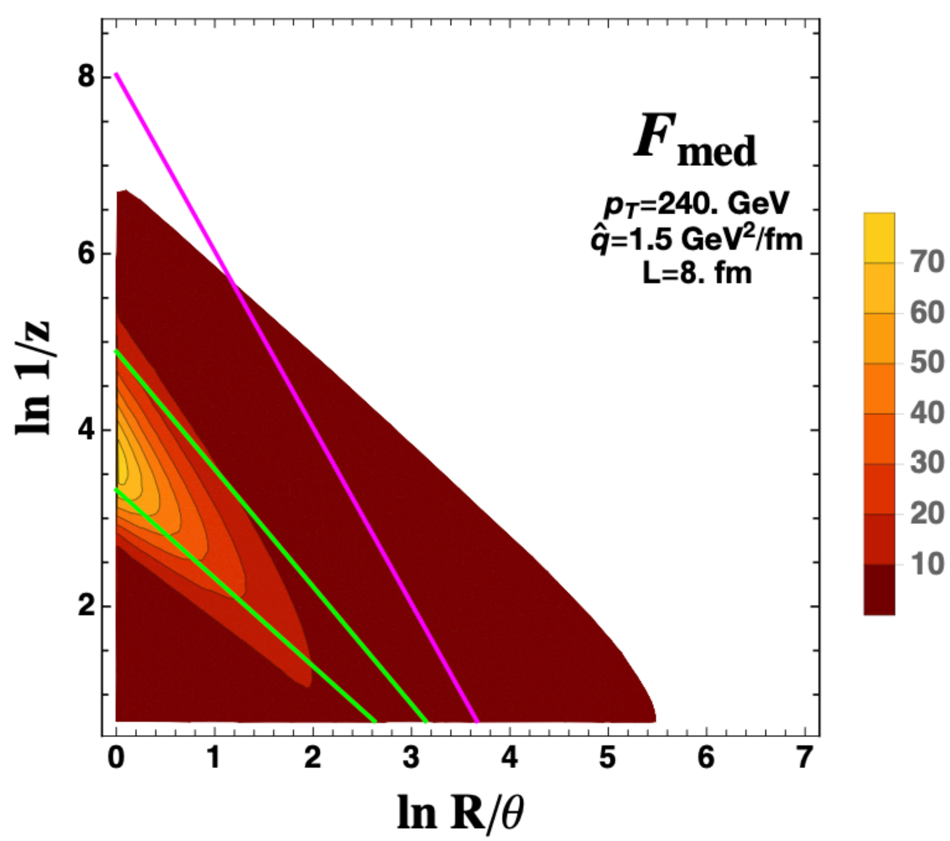
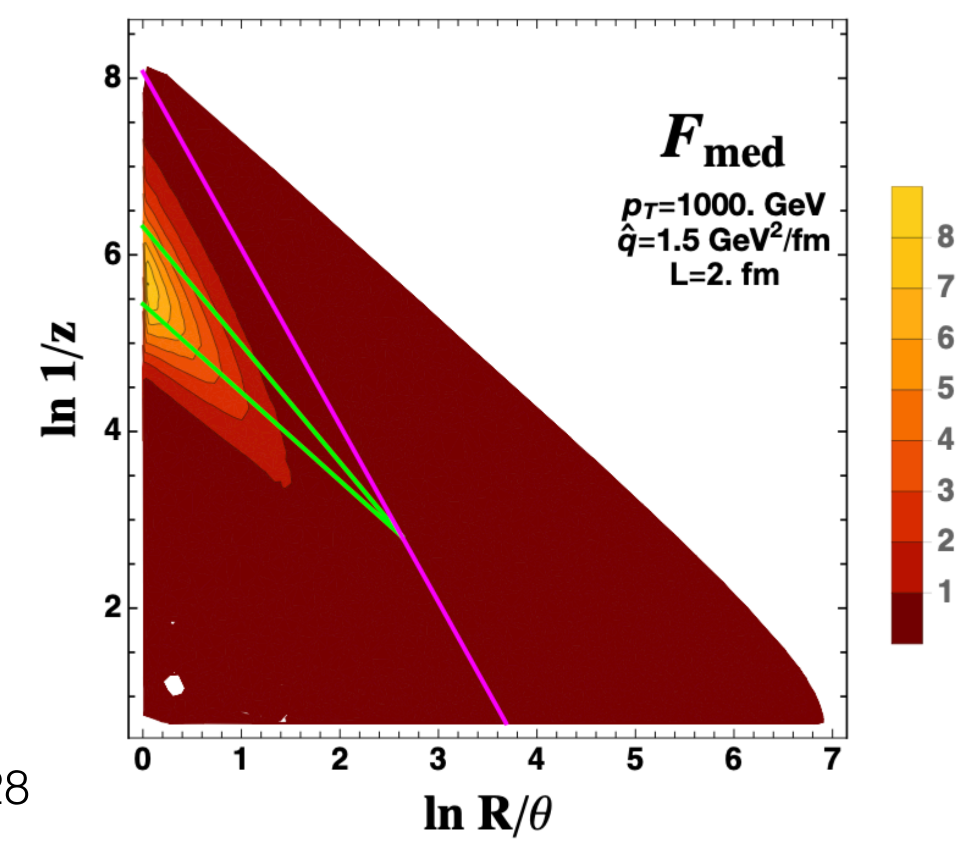


$\omega_c \sim \hat{q} L^2$

$$\frac{d\sigma_{qg}}{d\theta dz} = (1 + F_{\rm med}(z,\theta))\, \frac{d\sigma_{qg}^{\rm vac}}{d\theta dz}$$





$\gamma \to q\bar{q}$

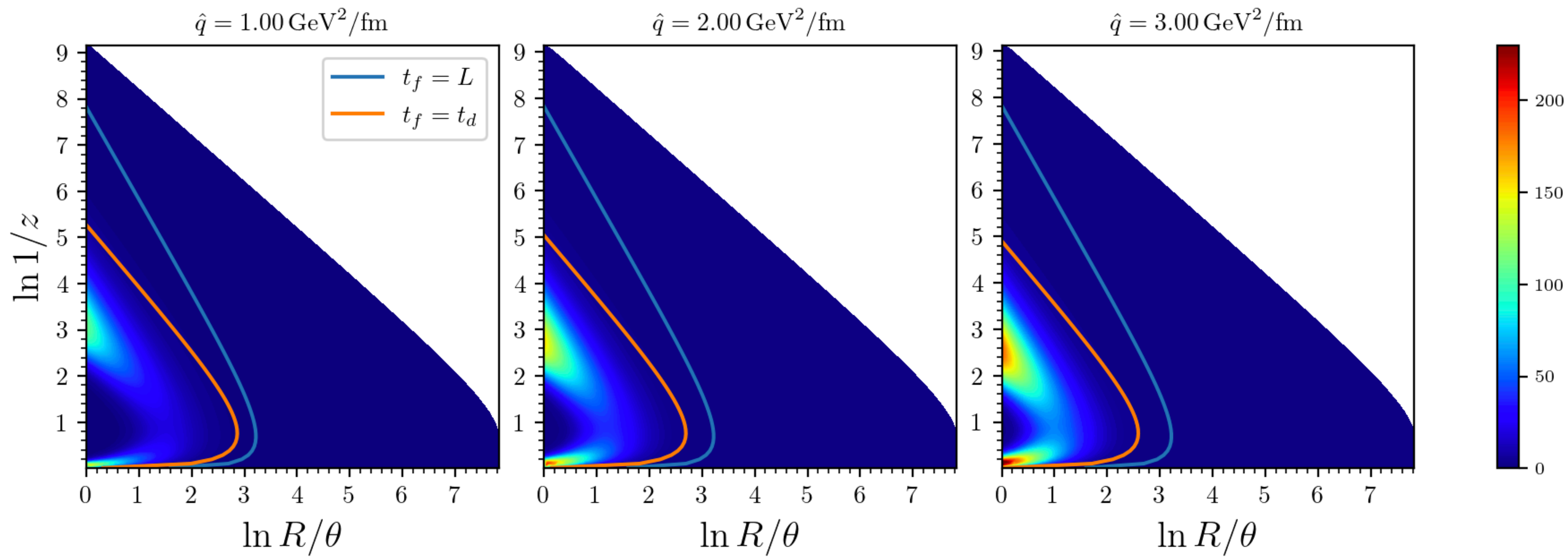
Lund planes $q \to qg$

$$\frac{d\sigma_{qg}}{d\theta dz} = (1 + F_{\text{med}}(z,\theta))\ \frac{d\sigma^{\text{vac}}_{qg}}{d\theta dz}$$

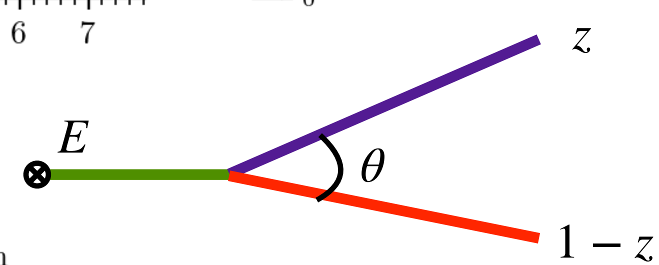
Isaksen, Tywoniuk 2107.02542

$E = 100.0\,\text{GeV}\ \ L = 10.0\,\text{fm}$

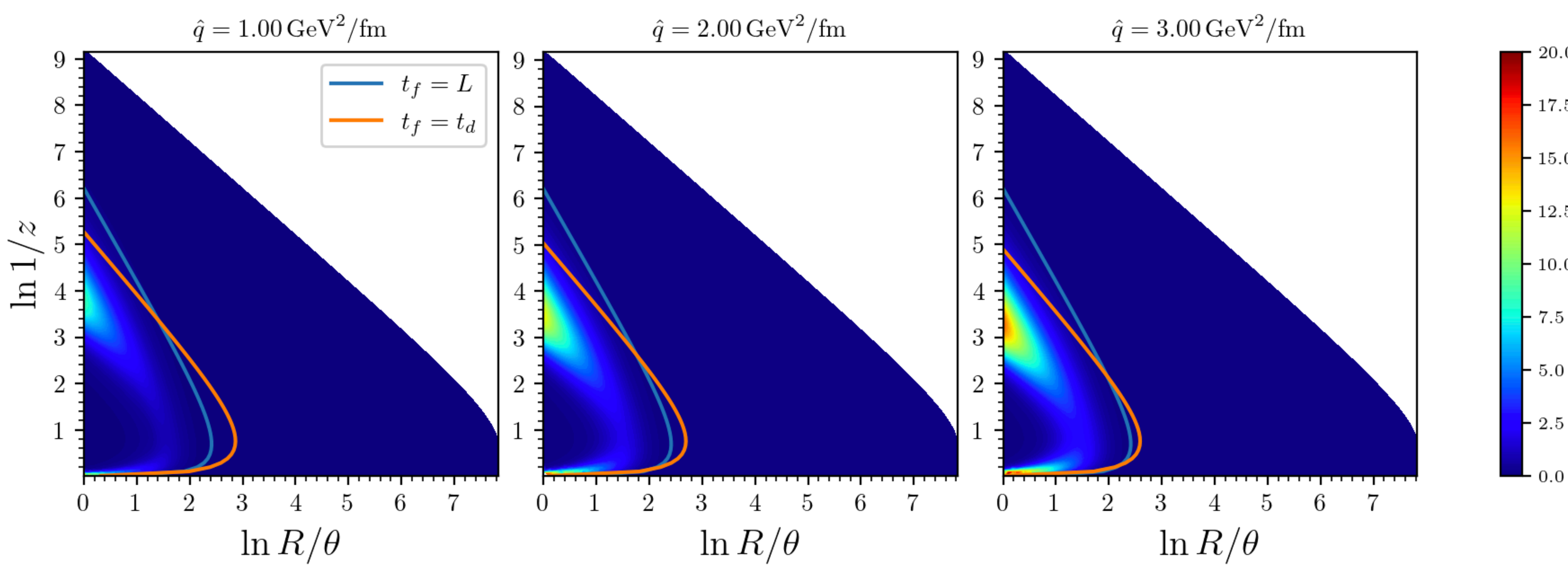
$\theta_c < \theta_L$



$E = 100.0\,\text{GeV}\ \ L = 2.0\,\text{fm}$



$\theta_c > \theta_L$



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Isaksen, Tywoniuk 2107.02542

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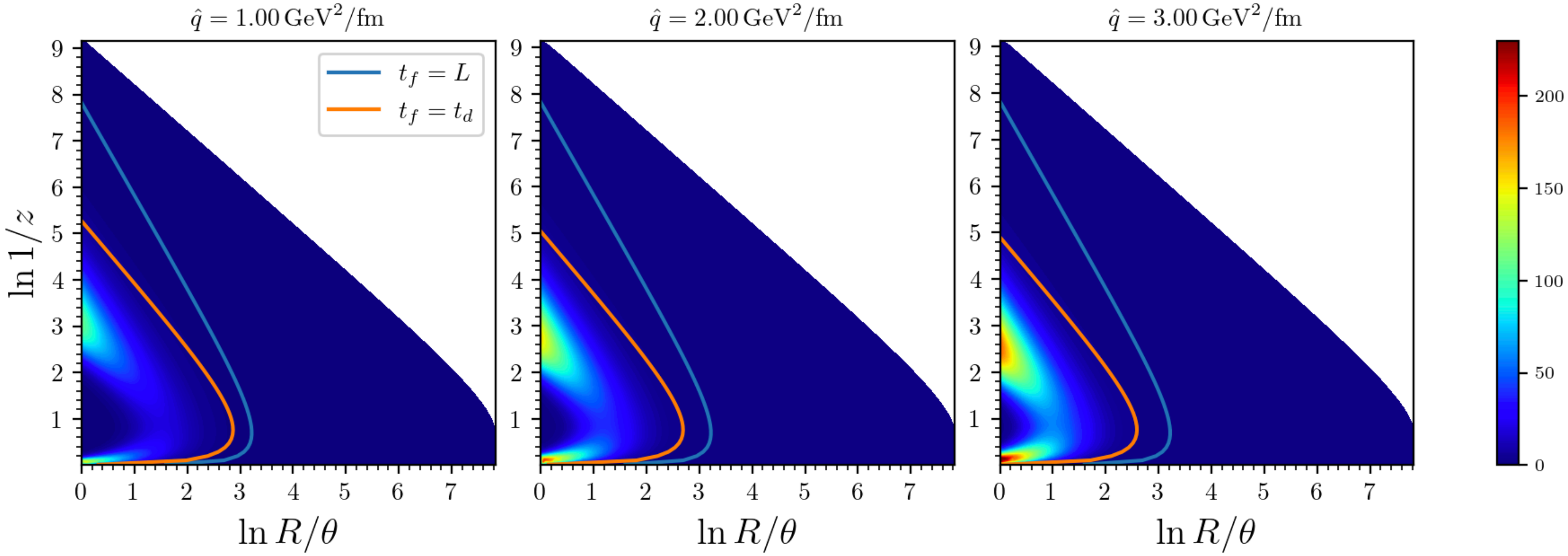
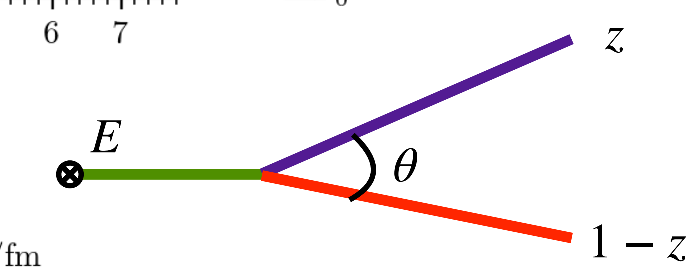
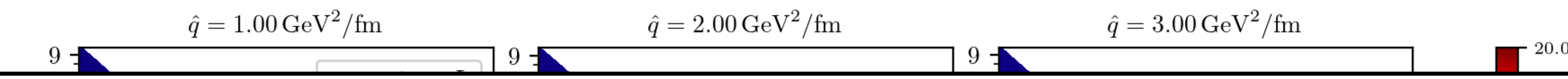


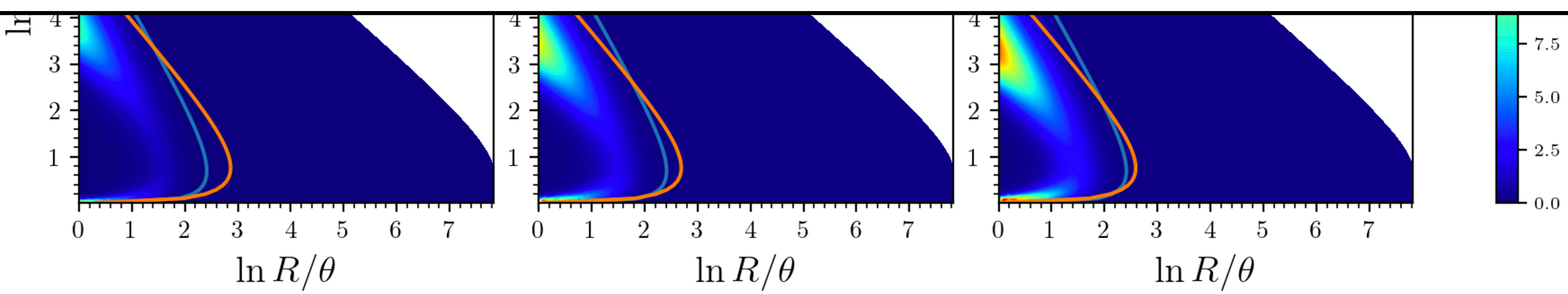


$E = 100.0\,\text{GeV}\ \ L = 2.0\,\text{fm}$



- Can we really see the different regimes?

$\theta_c > \theta_L$



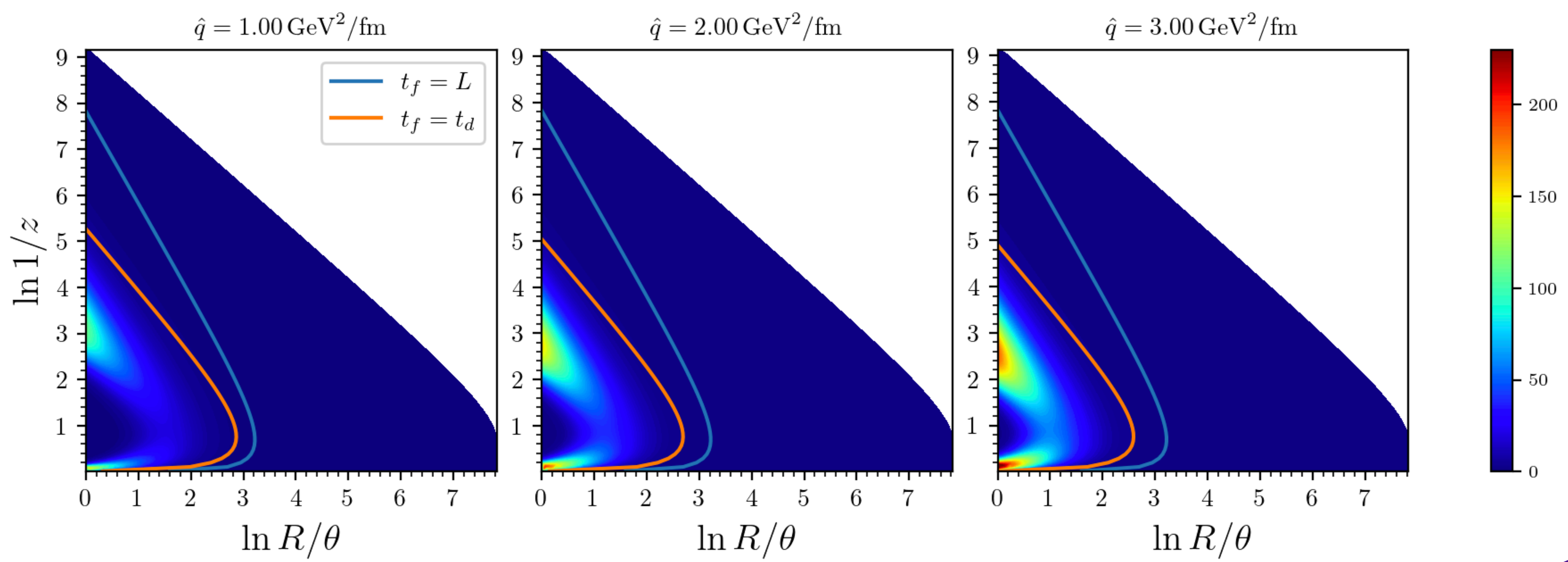
Lund planes $q \to qg$

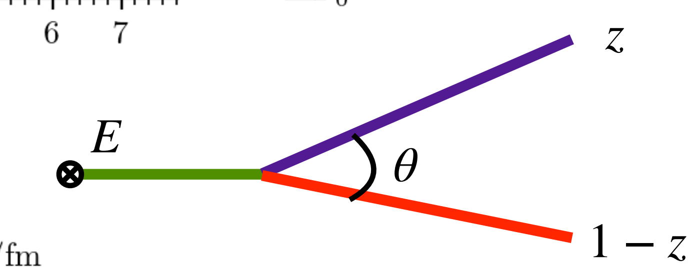
$$\frac{d\sigma_{qg}}{d\theta dz} = (1 + F_{\text{med}}(z,\theta)) \, \frac{d\sigma_{qg}^{\text{vac}}}{d\theta dz}$$

Isaksen, Tywoniuk 2107.02542

$E = 100.0\,\text{GeV} \;\; L = 10.0\,\text{fm}$

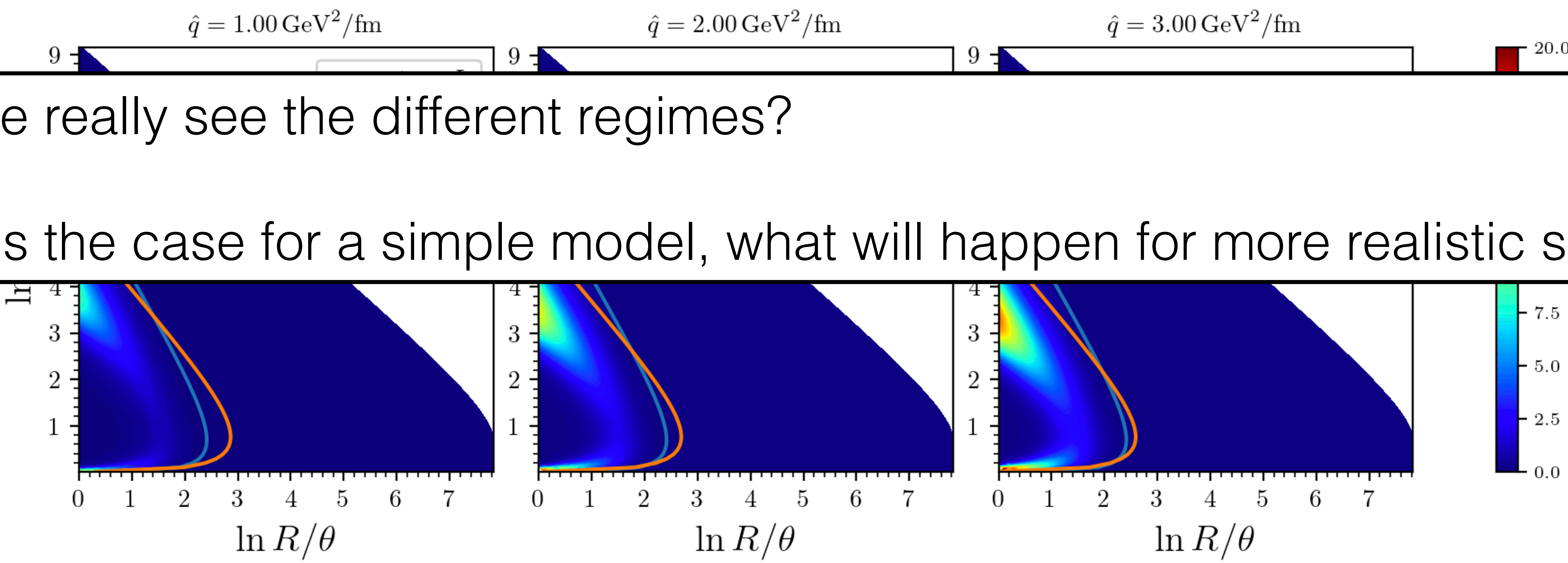
$\theta_c < \theta_L$





$E = 100.0\,\text{GeV} \;\; L = 2.0\,\text{fm}$

$\theta_c > \theta_L$



- Can we really see the different regimes?
- If this is the case for a simple model, what will happen for more realistic setups?

In-medium propagator

- Can be formally written in coordinate space in terms of a path integral

$$\mathcal{G}_R(t_2,\boldsymbol{x}_2;t_1,\boldsymbol{x}_1;\omega)=\int_{\boldsymbol{x}_1}^{\boldsymbol{x}_2}\mathcal{D}\boldsymbol{r}\,\exp\left\{\frac{i\omega}{2}\int_{t_1}^{t_2}d\xi\;\dot{\boldsymbol{r}}^2(\xi)\right\}\mathrm{P}\exp\left\{ig\int_{t_1}^{t_2}d\xi\;A_R^-(\xi,\boldsymbol{r}(\xi))\right\}$$

- Satisfies the following Schwinger-Dyson type equation

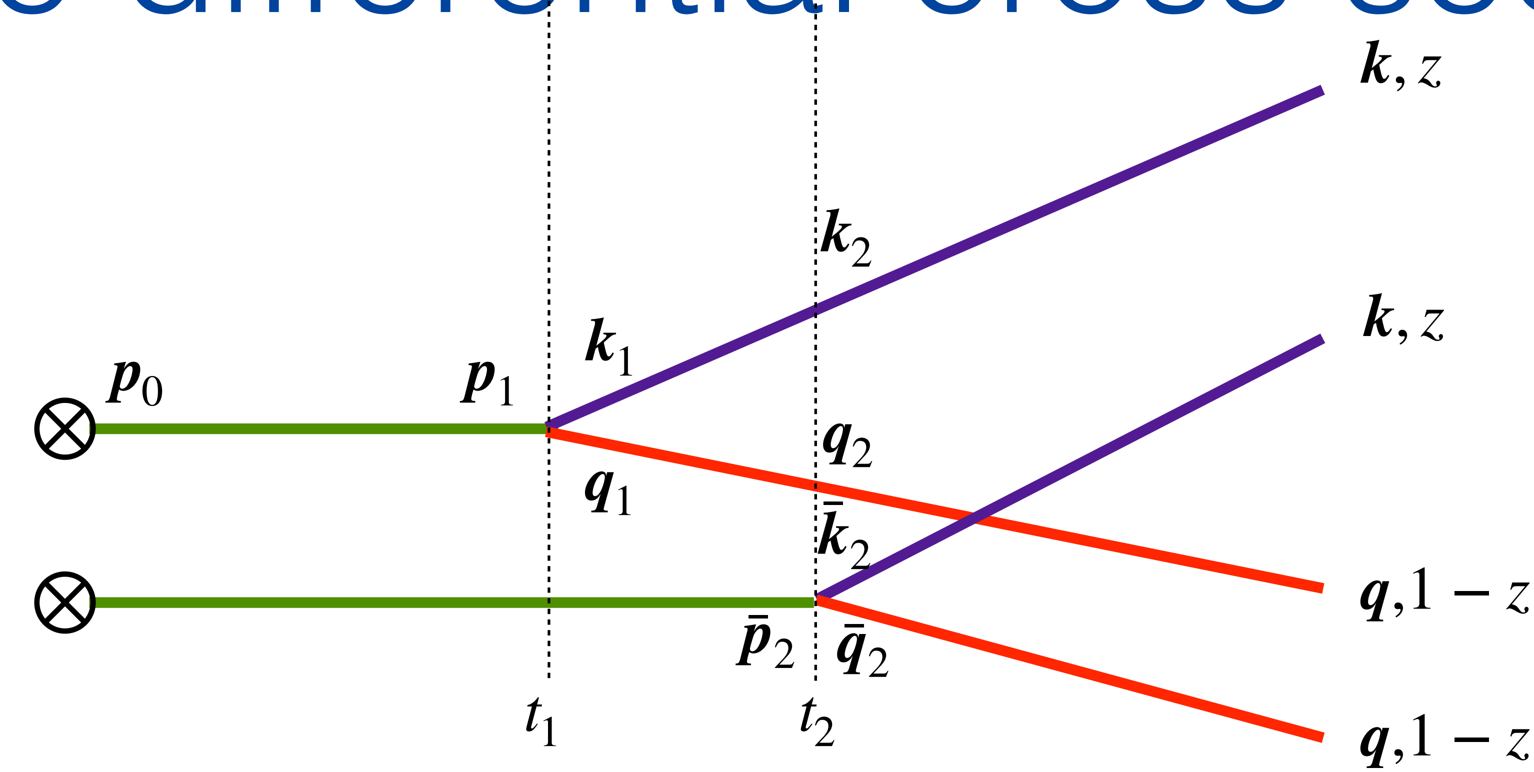
$$\begin{aligned}\mathcal{G}_R(\boldsymbol{p}_2,t_2;\boldsymbol{p}_1,t_1;\omega)&=(2\pi)^2\delta^{(2)}(\boldsymbol{p}_2-\boldsymbol{p}_1)e^{-i\frac{p_2^2}{2\omega}(t_2-t_1)}\\&+ig\int_{t_1}^{t_2}ds\;e^{-i\frac{p_2^2}{2\omega}(t_2-s)}\int_{\boldsymbol{p}'}A_R^-(s,\boldsymbol{p}_2-\boldsymbol{p}')\mathcal{G}_R(\boldsymbol{p}',s;\boldsymbol{p}_1,t_1;\omega)\end{aligned}$$

- And convolution relations

$$\int_{\boldsymbol{p}_2}\mathcal{G}_R(\boldsymbol{p}_3,t_3;\boldsymbol{p}_2,t_2;\omega)\mathcal{G}_R(\boldsymbol{p}_2,t_2;\boldsymbol{p}_1,t_1;\omega)=\mathcal{G}_R(\boldsymbol{p}_3,t_3;\boldsymbol{p}_1,t_1;\omega)$$

$$\int_{\boldsymbol{p}_2}\mathcal{G}_R^\dagger(\bar{\boldsymbol{p}}_1,t_1;\boldsymbol{p}_2,t_2;\omega)\mathcal{G}_R(\boldsymbol{p}_2,t_2;\boldsymbol{p}_1,t_1;\omega)=(2\pi)^2\delta^{(2)}(\boldsymbol{p}_1-\bar{\boldsymbol{p}}_1)$$

Double differential cross section

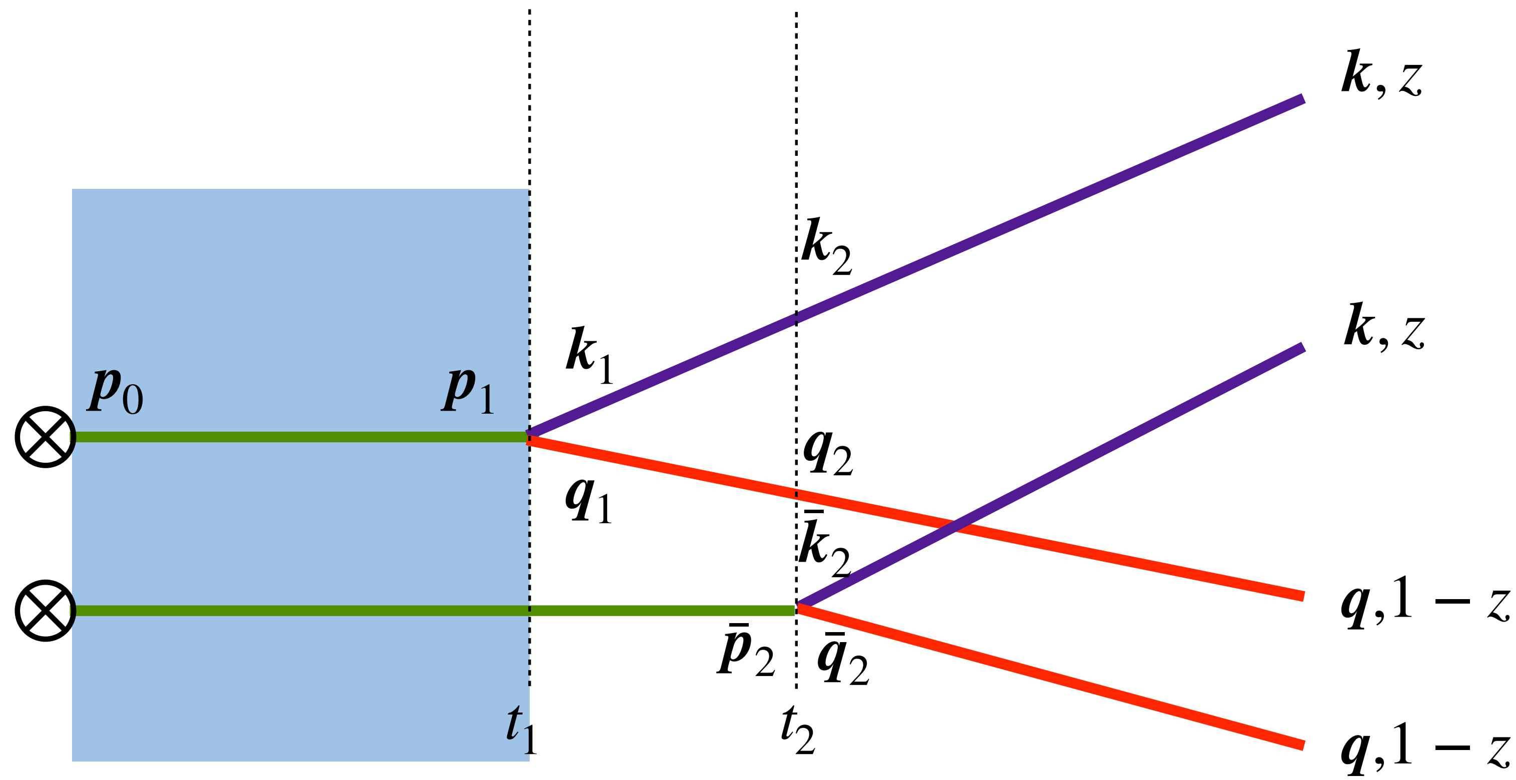


- The locality of the medium averages $\langle A^-(t)A^-(t')\rangle \propto \delta(t-t')$ implies that at any given time:
  - Averages can be factored into regions with constant number of particles
  - The sum of all momenta in the amplitude is equal to the sum of all momenta in the conjugate amplitude
  - When considering the ensemble of all particles in the amplitude and conjugate amplitude, the overall color state is always a singlet

Blaizot, Iancu, FD, Mehtar-Tani 1209.4585

Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section



Average depends on total momentum transfer only

$$= \mathcal{P}_{R_a}(\boldsymbol{p}_1 - \boldsymbol{p}_0; t_1, t_0)$$

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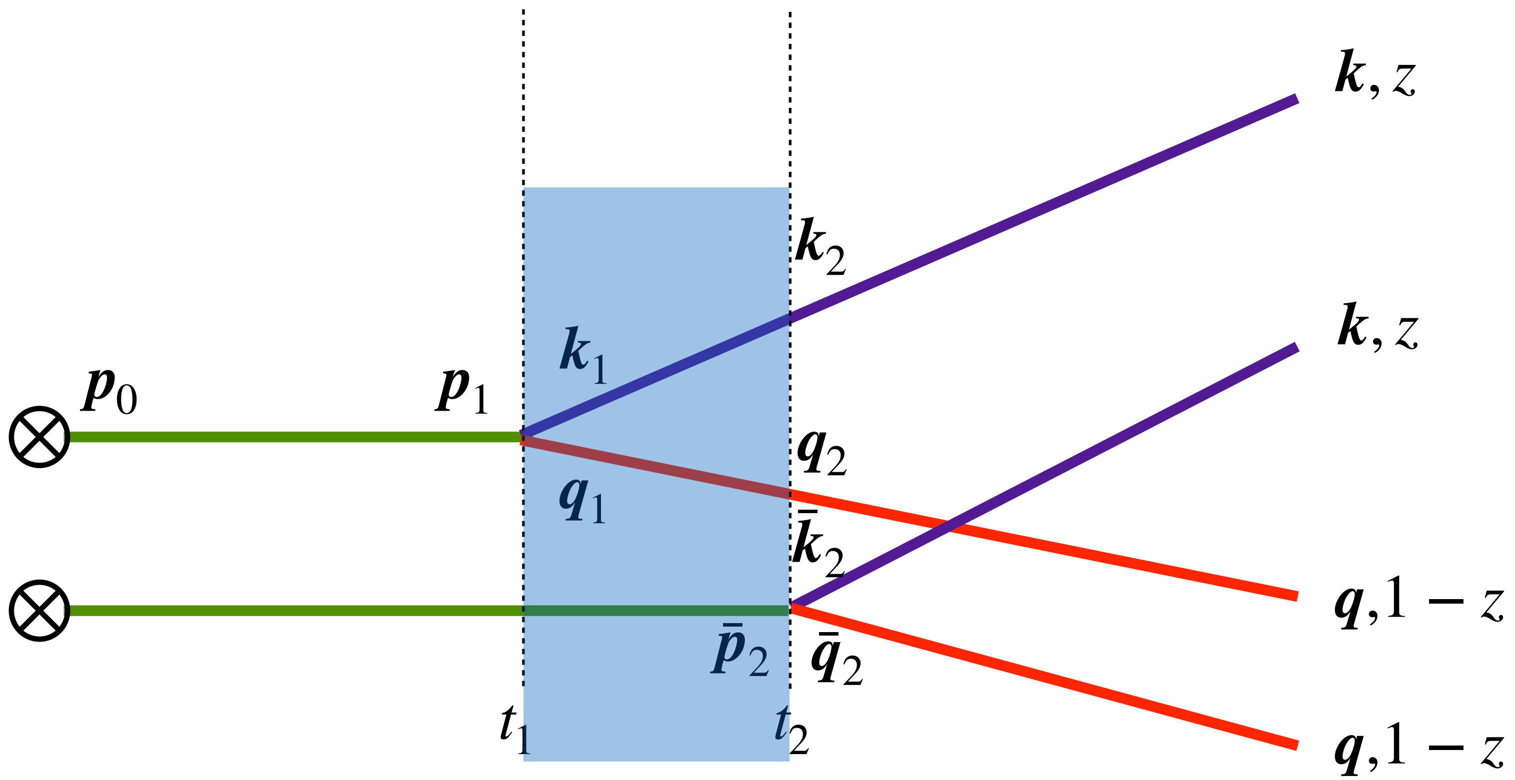

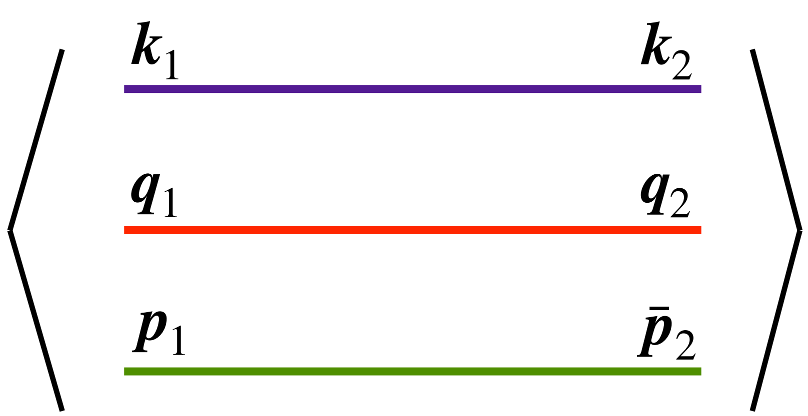

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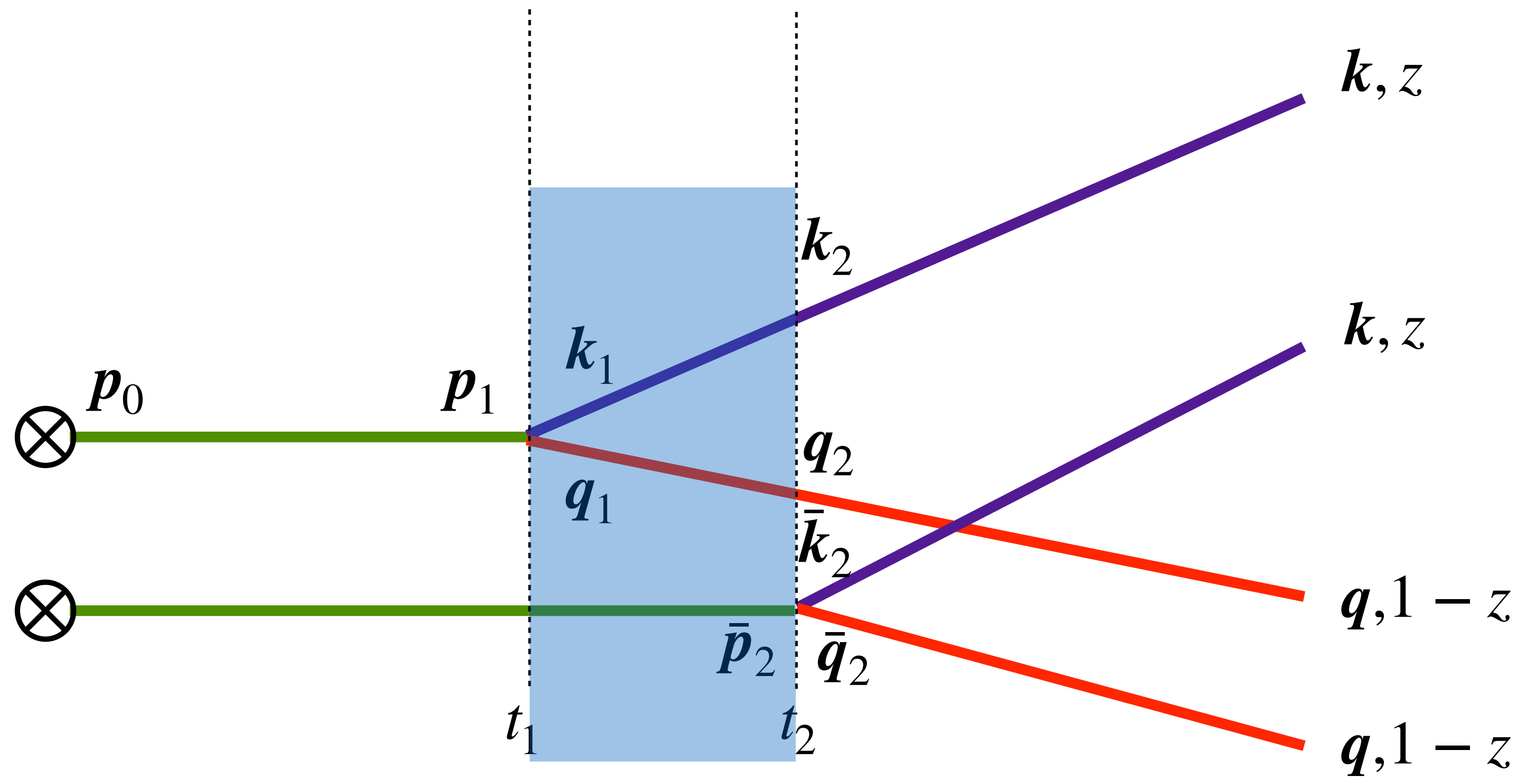
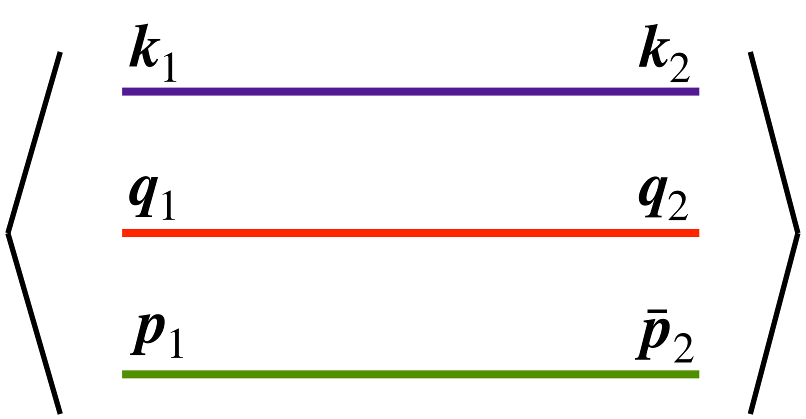


$$\boldsymbol{k}_1 + \boldsymbol{q}_1 = \boldsymbol{p}_1 \qquad \boldsymbol{k}_2 + \boldsymbol{q}_2 = \bar{\boldsymbol{p}}_2$$



Blaizot, Iancu, FD, Mehtar-Tani 1209.4585

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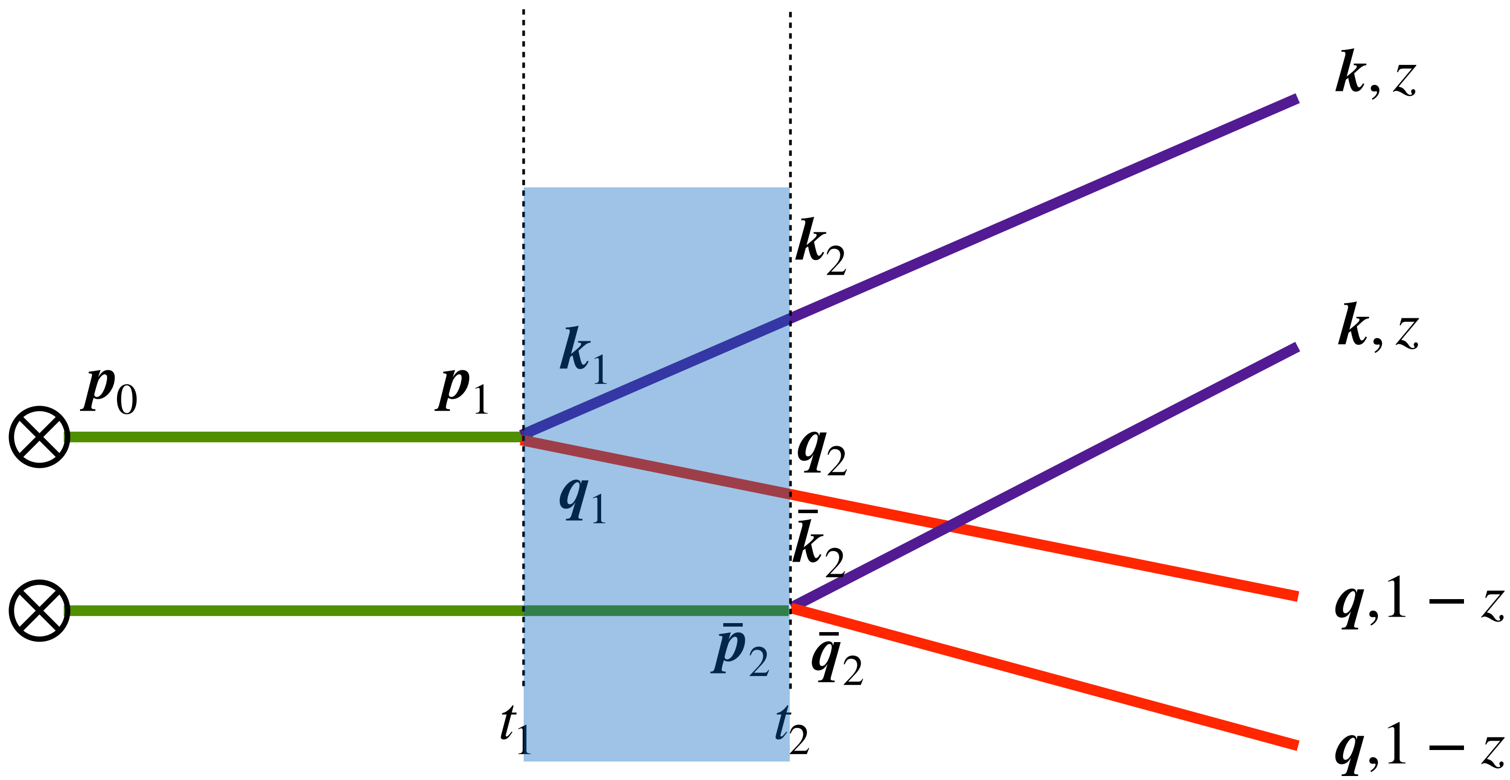
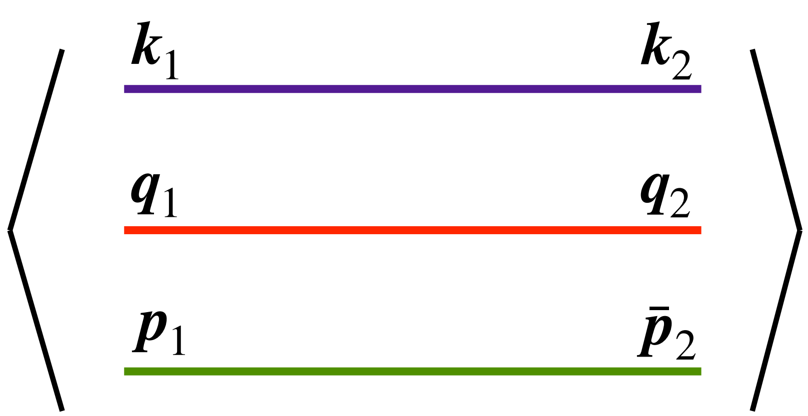
Double differential cross section



Same as vertex factors

$$\boldsymbol{k}_1 + \boldsymbol{q}_1 = \boldsymbol{p}_1 \qquad \boldsymbol{k}_2 + \boldsymbol{q}_2 = \bar{\boldsymbol{p}}_2 \qquad \boldsymbol{l}_1 = (1-z)\boldsymbol{k}_1 - z\boldsymbol{q}_1 \qquad \boldsymbol{l}_2 = (1-z)\boldsymbol{k}_2 - z\boldsymbol{q}_2$$



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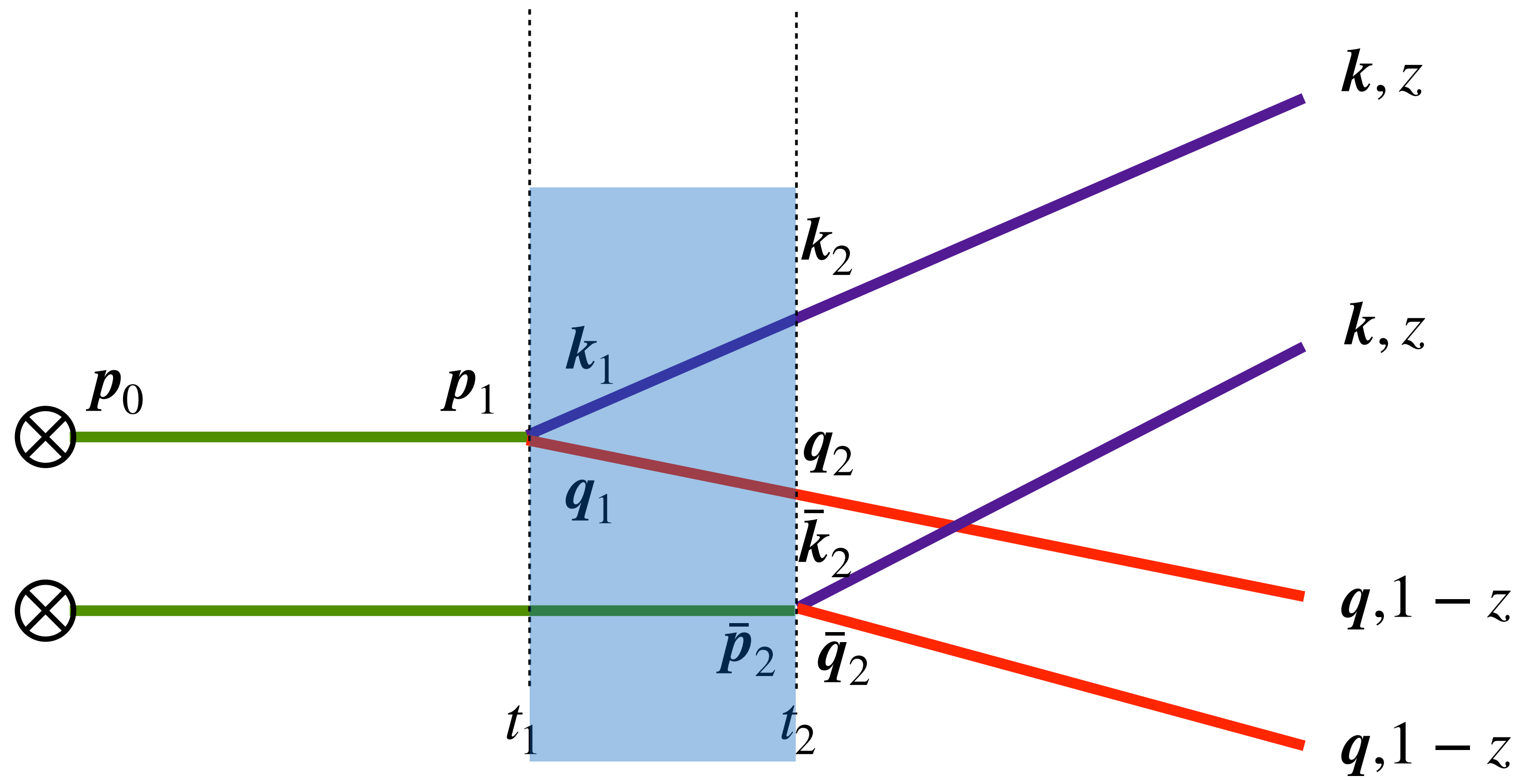

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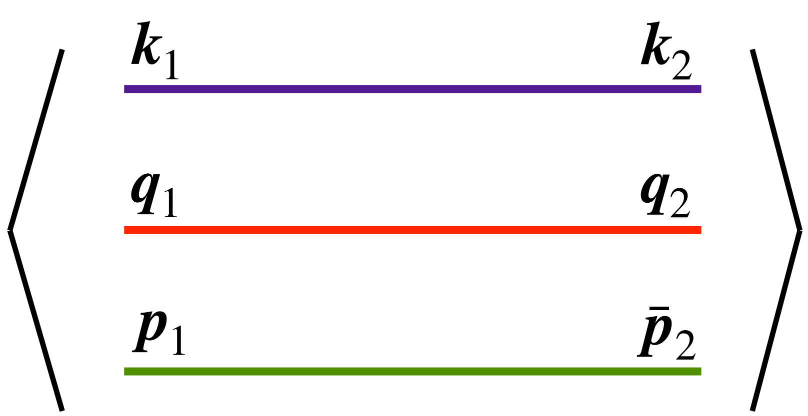

Average depends on $\boldsymbol{l}_1, \boldsymbol{l}_2, \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1$

$$\mathcal{K}^{(3)}(\boldsymbol{l}_2, t_2; \boldsymbol{l}_1, t_1; \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1, z)$$

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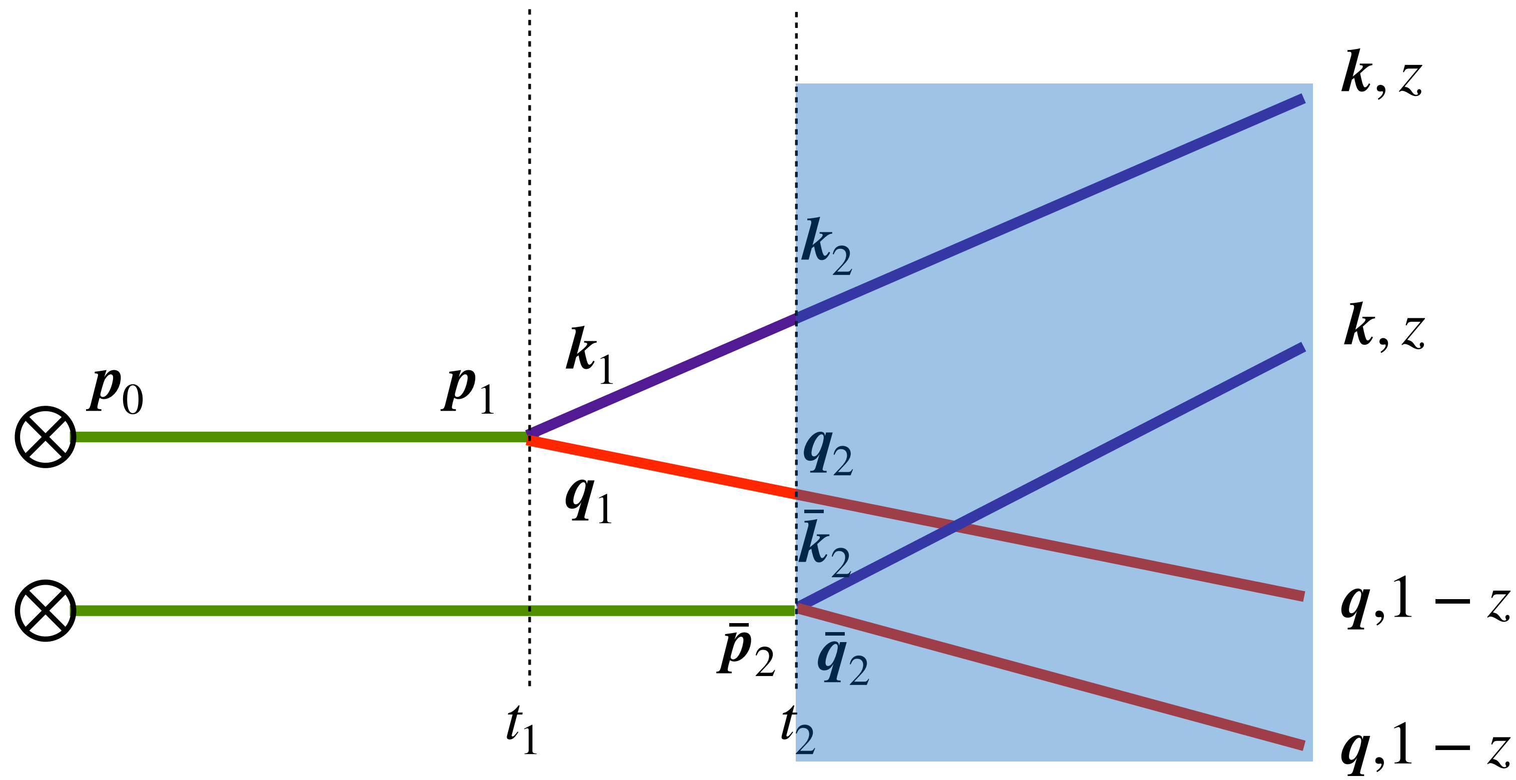
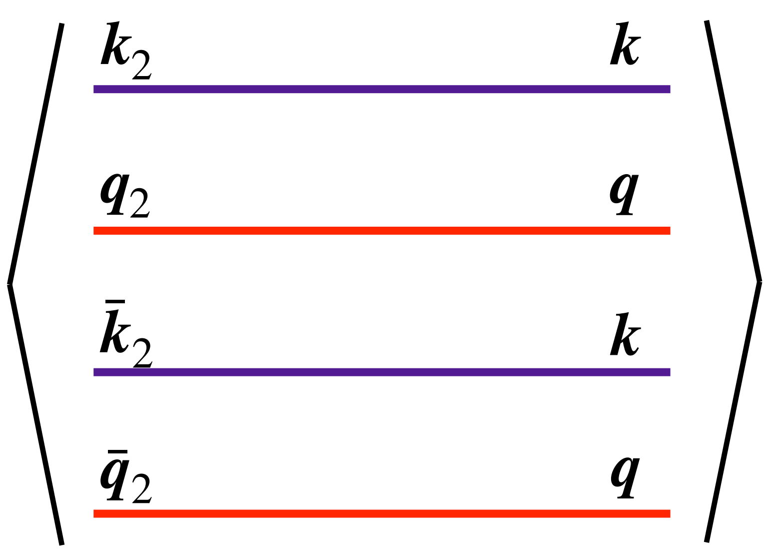
Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section





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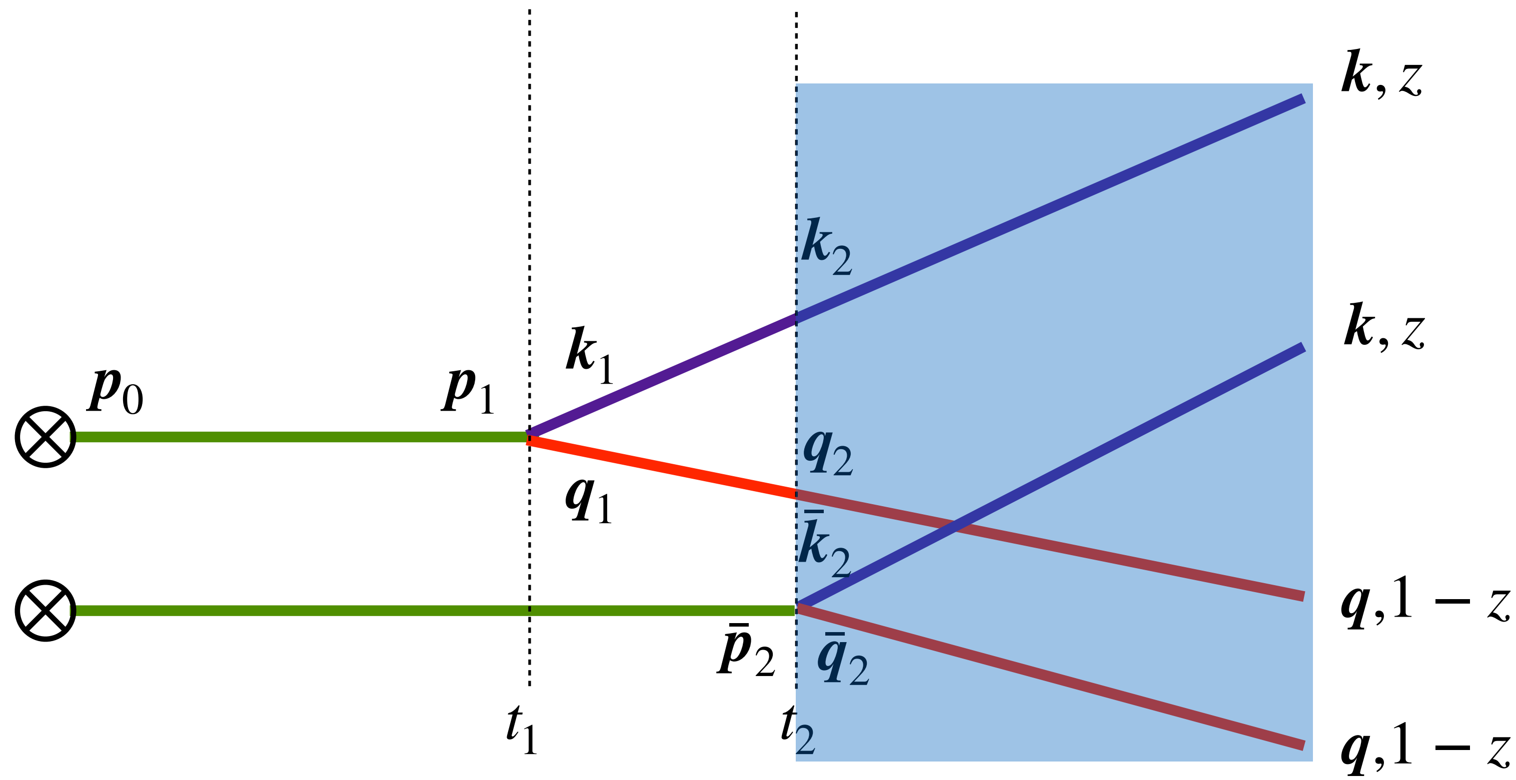
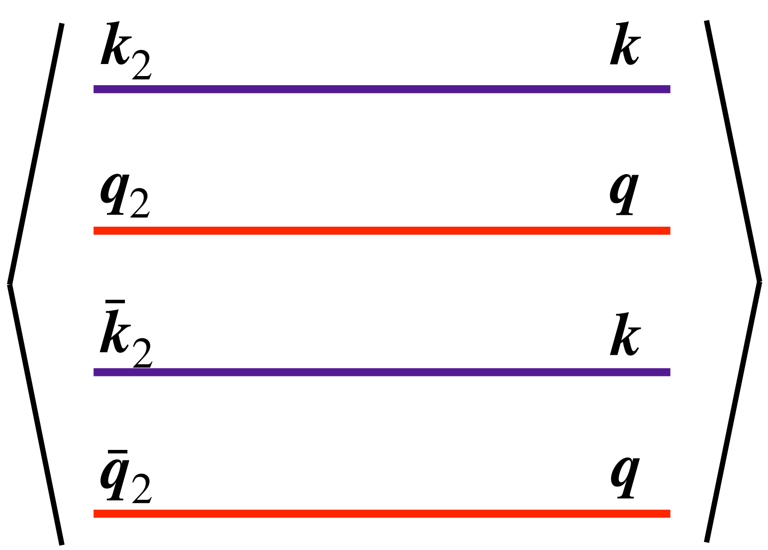


$$k_2 + q_2 = \bar{k}_2 + \bar{q}_2$$



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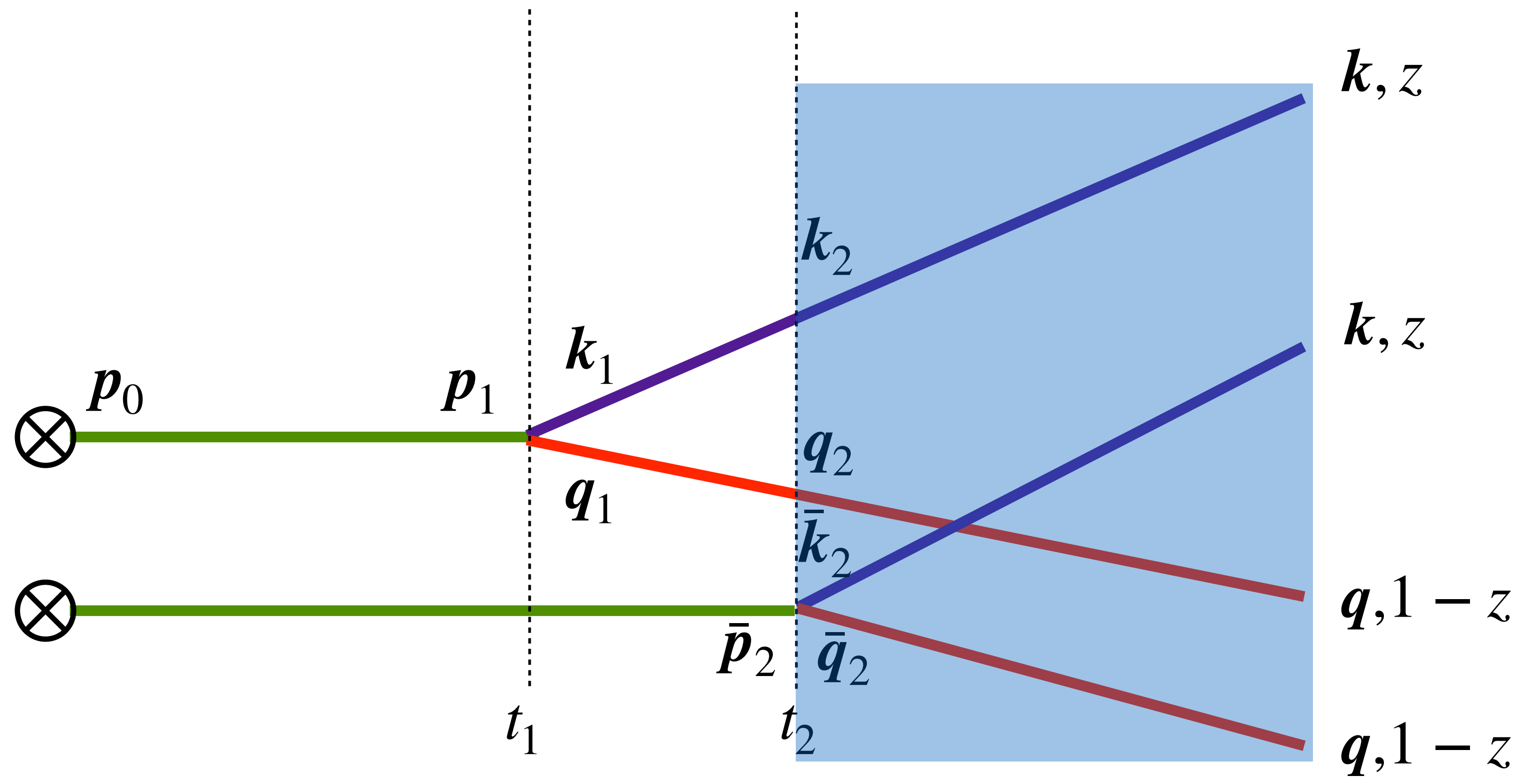
Double differential cross section

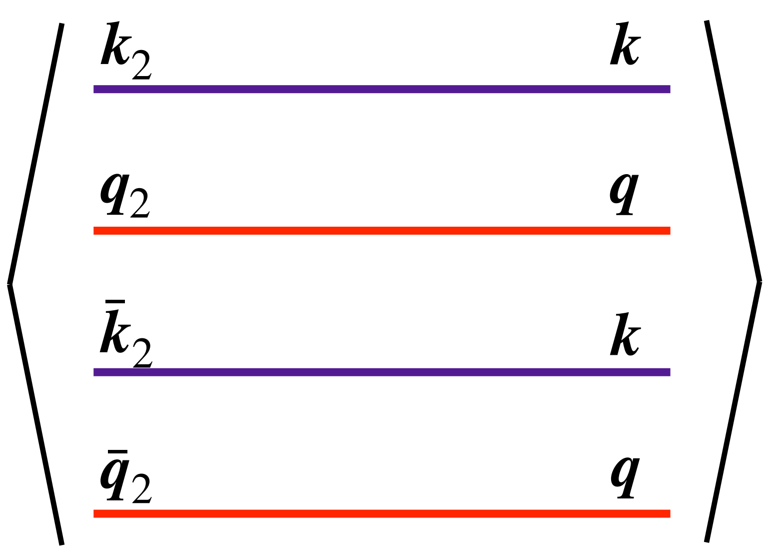


$$\boldsymbol{k}_2 + \boldsymbol{q}_2 = \bar{\boldsymbol{k}}_2 + \bar{\boldsymbol{q}}_2$$

$$\boldsymbol{l}_2 = (1-z)\boldsymbol{k}_2 - z\boldsymbol{q}_2$$

$$\bar{\boldsymbol{l}}_2 = (1-z)\bar{\boldsymbol{k}}_2 - z\bar{\boldsymbol{q}}_2$$

$$\boldsymbol{l} = (1-z)\boldsymbol{k} - z\boldsymbol{q}$$



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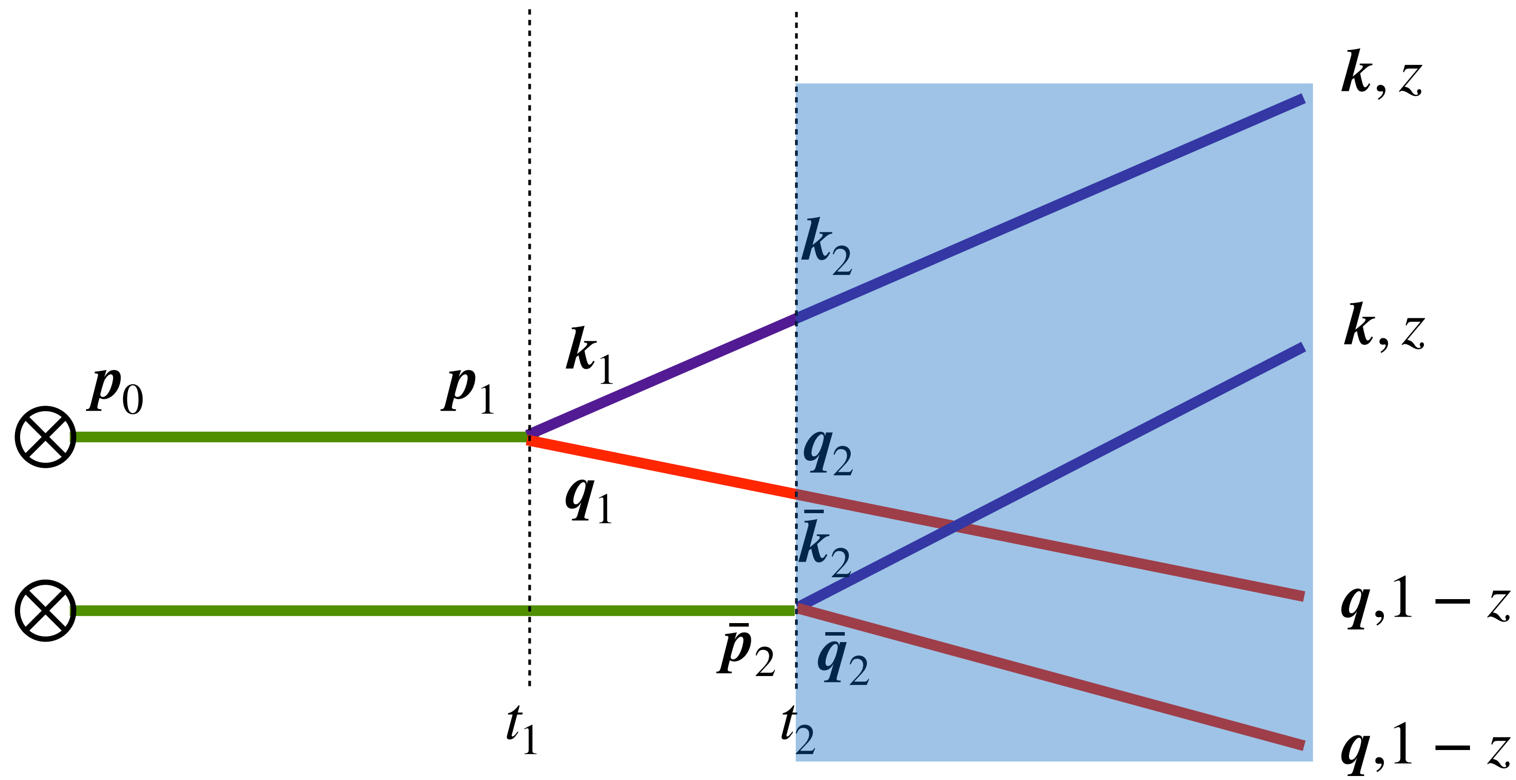
Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section

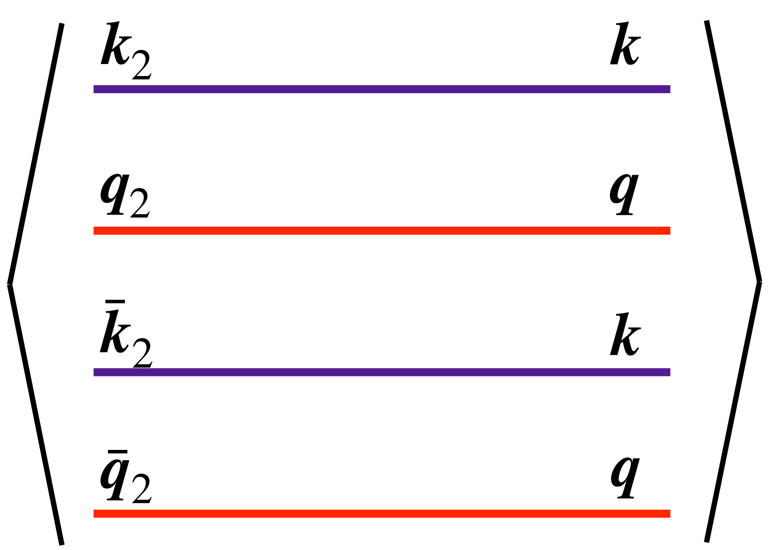


$$\boldsymbol{k}_2 + \boldsymbol{q}_2 = \bar{\boldsymbol{k}}_2 + \bar{\boldsymbol{q}}_2$$

$$\boldsymbol{l}_2 = (1-z)\boldsymbol{k}_2 - z\boldsymbol{q}_2$$

$$\bar{\boldsymbol{l}}_2 = (1-z)\bar{\boldsymbol{k}}_2 - z\bar{\boldsymbol{q}}_2$$

$$\boldsymbol{l} = (1-z)\boldsymbol{k} - z\boldsymbol{q}$$



Average depends on $\boldsymbol{l}, \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2$, and $\boldsymbol{k} + \boldsymbol{q} - \boldsymbol{k}_2 - \boldsymbol{q}_2$

$$\mathcal{S}^{(4)}(\boldsymbol{l}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k} + \boldsymbol{q} - \boldsymbol{k}_2 - \boldsymbol{q}_2, z)$$

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