Reflectance and Transmittance Beyond Snell and Fresnel

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Introduction

- To describe the light reflected or transmitted by common materials;
- Understanding the light collection is important in scintillation detectors in order to improve their sensitivity. The light collection depends on the way the light is reflected in the internal surfaces.
- We aim to have a physically based model, that can be adapted to a Monte-Carlo model and be computationally inexpensive.
- We should be able to measure the parameters of the model and adapted it to different situations
  - For example: measuring the reflectance in the air and extrapolate it to a liquid interface.
- Additionally, we plan could adapt these models to image rendering in computer graphics.

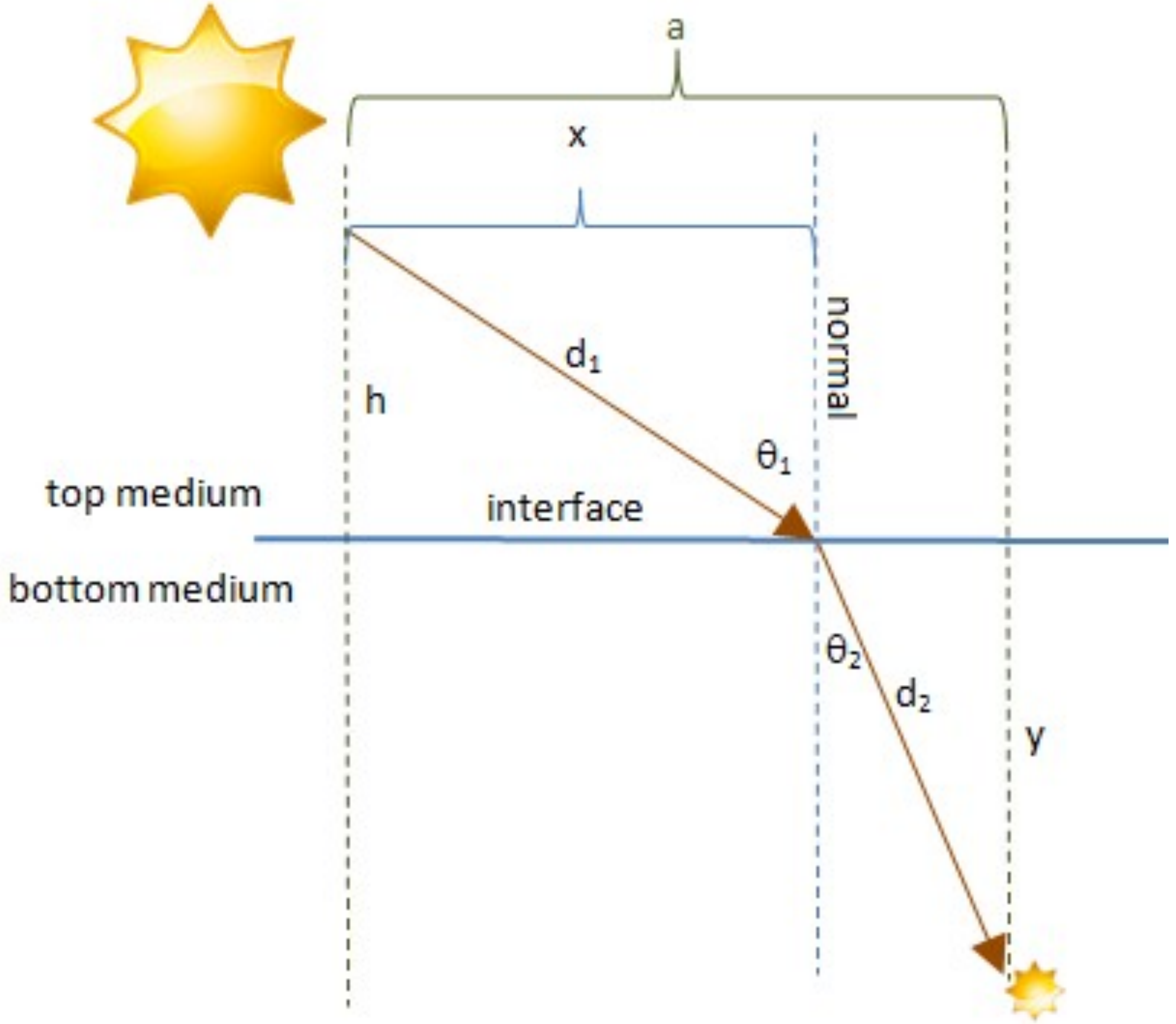
Outline

- Basics
  - Snell Law and Fresnel Formulæ
- Principles of Radiometry (science of measurement of optical radiation at any wavelength, based simply on physical measurements);
- The specular reflection:
  - The Beckmann spizzichino model
  - Shadowing-masking
- The diffuse reflection:
  - Radiative Transfer
  - Diffuse Reflectance
  - Diffuse Transmission
  - The Effect of the Interface

Laws of Reflection and Refraction

- An electromagnetic plane wave that reaches a boundary between two homogeneous media with different optical properties is split into two parts, a reflected wave and a transmitted wave, towards directions given by the laws of reflection and refraction:

$$\theta_1 = \theta_2 \quad \text{(reflection)}$$
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{(refraction)}$$

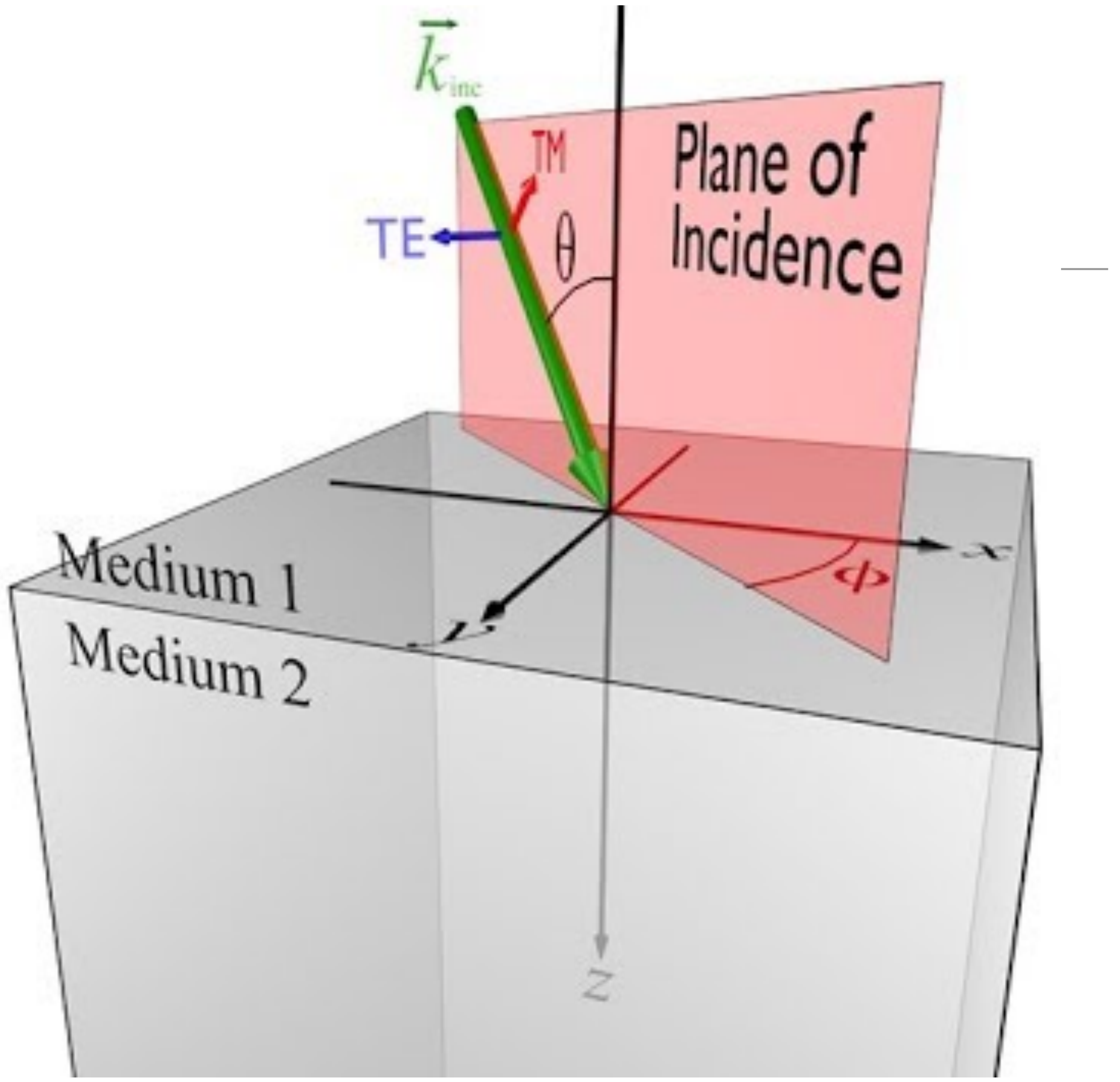


- This is the well known Snell-Descartes laws. The derivation of this law is based on the Principle of Least Time.
- Ibn Sahl proposed first this law in 984.

Claudio Ptolomeu, fl. 150 CE

Ibn Sahl, fl. 980 CE

Fresnel Formulæ - Dielectrics



- Discovered by Augustin-Jean Fresnel in 1823
- The amplitude of the reflected/refracted field is given by the Fresnel equations for the parallel (p) and perpendicular (s) to the plane of incidence.
- They can be obtained using the fact that the tangencial components of E and B has to be continuous across the surface

$$F_p = \left(\frac{n\cos\theta_i - \sqrt{1 - \frac{1}{n^2}\sin^2\theta_i}}{n\cos\theta_i + \sqrt{1 - \frac{1}{n^2}\sin^2\theta_i}} \right)^2$$

$$F_s = \left(\frac{\cos\theta_i - n\sqrt{1 - \frac{1}{n^2}\sin^2\theta_i}}{\cos\theta_i + n\sqrt{1 - \frac{1}{n^2}\sin^2\theta_i}} \right)^2$$

$$T_p = \frac{\sin 2\theta_i \sin 2\theta_t}{\sin^2(\theta_i + \theta_t)\cos^2(\theta_i - \theta_t)}$$

$$T_s = \frac{\sin 2\theta_t \cos 2\theta_i}{\sin^2(\theta_i + \theta_t)}$$

$$\theta_C > \arcsin\left(\frac{n_2}{n_1}\right)$$

Fresnel Formulæ - Metals

- In metals, the extinction coefficient **κ** has to be taken also into account:

$$\mathcal{F}_s(\theta; n, \kappa) = \frac{(n_\theta - \cos\theta)^2 + \kappa_\theta^2}{(n_\theta + \cos\theta)^2 + \kappa_\theta^2}, \qquad \textbf{(14a)}$$

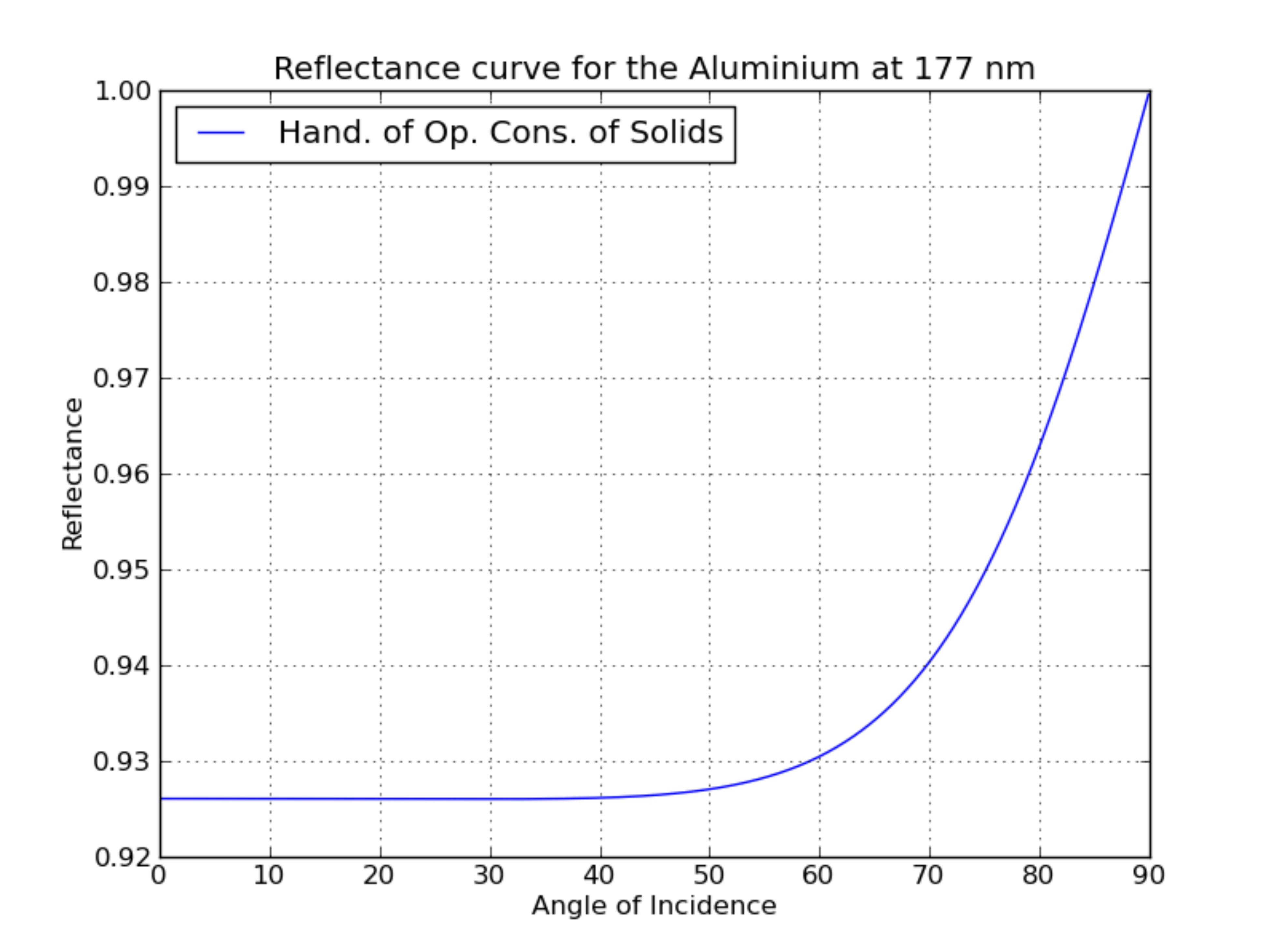
$$\mathcal{F}_p(\theta; n, \kappa) = \mathcal{F}_s \left[\frac{(n_\theta - \sin\theta\tan\theta)^2 + \kappa_\theta^2}{(n_\theta + \sin\theta\tan\theta)^2 + \kappa_\theta^2} \right], \qquad \textbf{(14b)}$$

where n_θ and κ_θ are given by:

$$n_\theta^2 = \frac{1}{2}\left[\sqrt{(n^2 - \kappa^2 - \sin^2\theta)^2 + 4n^2\kappa^2} + (n^2 - \kappa^2 - \sin^2\theta)\right]^2,$$

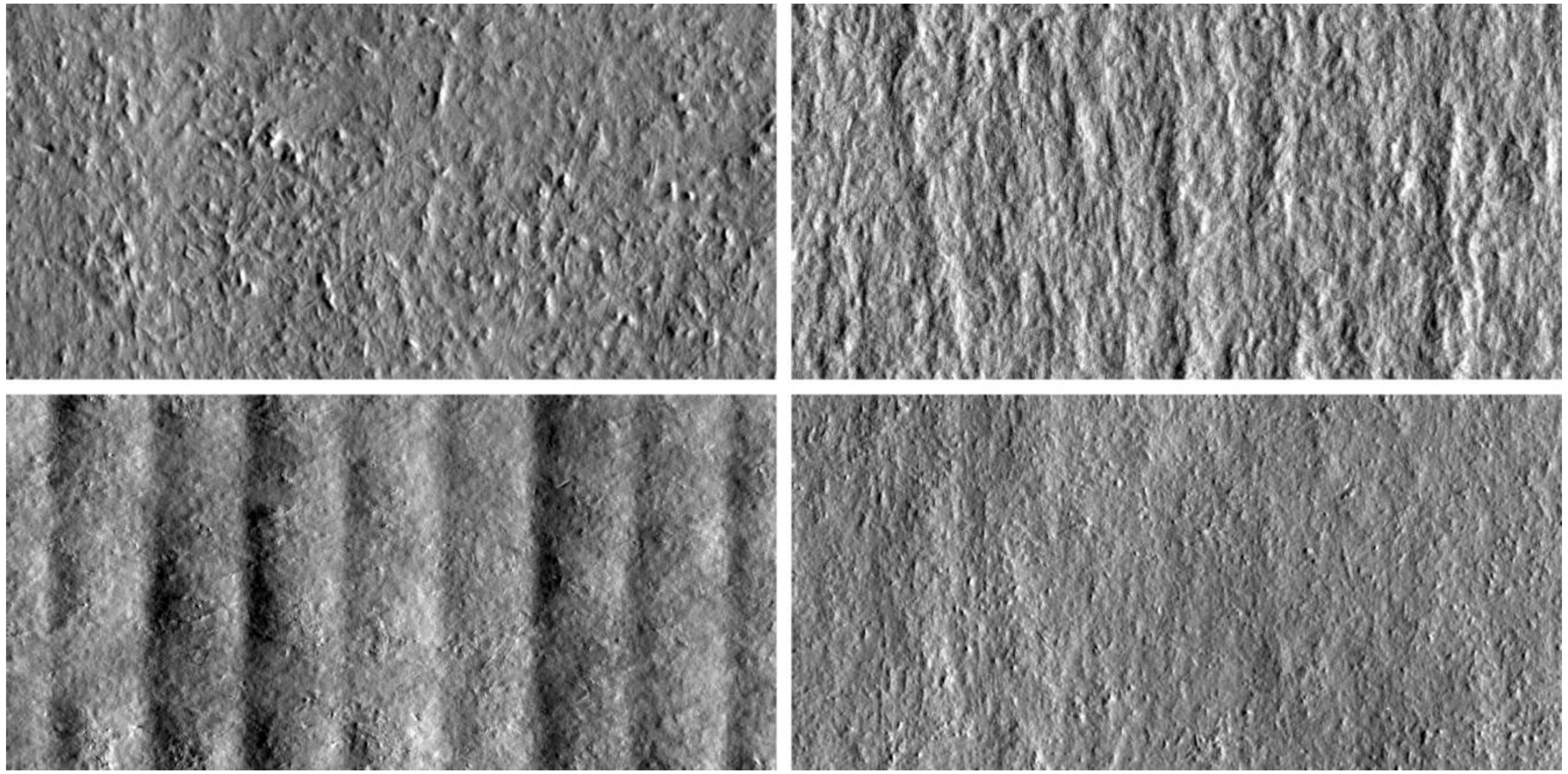
$$\kappa_\theta^2 = \frac{1}{2}\left[\sqrt{(n^2 - \kappa^2 - \sin^2\theta)^2 + 4n^2\kappa^2} - (n^2 - \kappa^2 + \sin^2\theta)\right]^2,$$

$$n = \frac{n_2}{n_1} \quad \text{and} \quad \kappa = \frac{\kappa_2}{n_1}; \quad \text{assumed } \kappa_1 = 0.$$

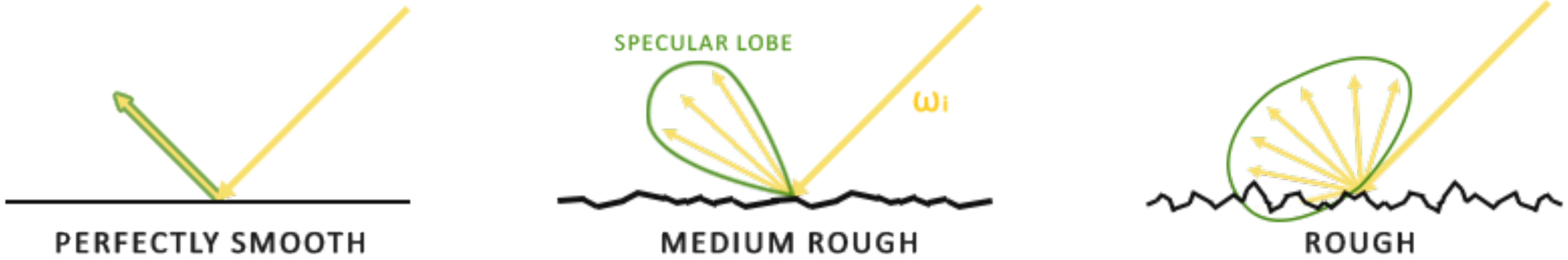


- **For smooth (and homogeneous) surfaces - that's all we need to know!**

Surface Roughness

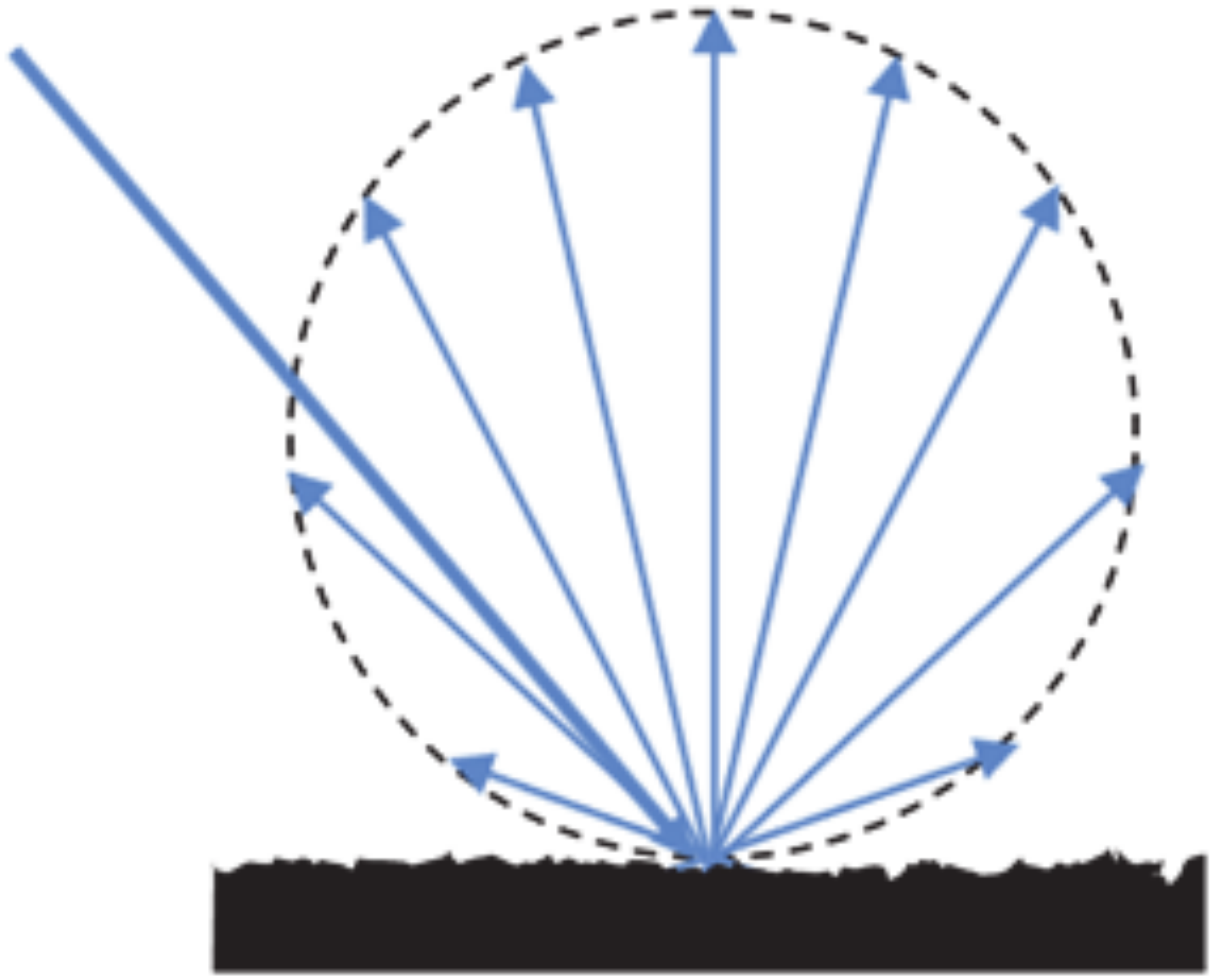


- Most of the surfaces have some kind of roughness that needs to be taken into account.
- It can be described by $z = h(x, y)$

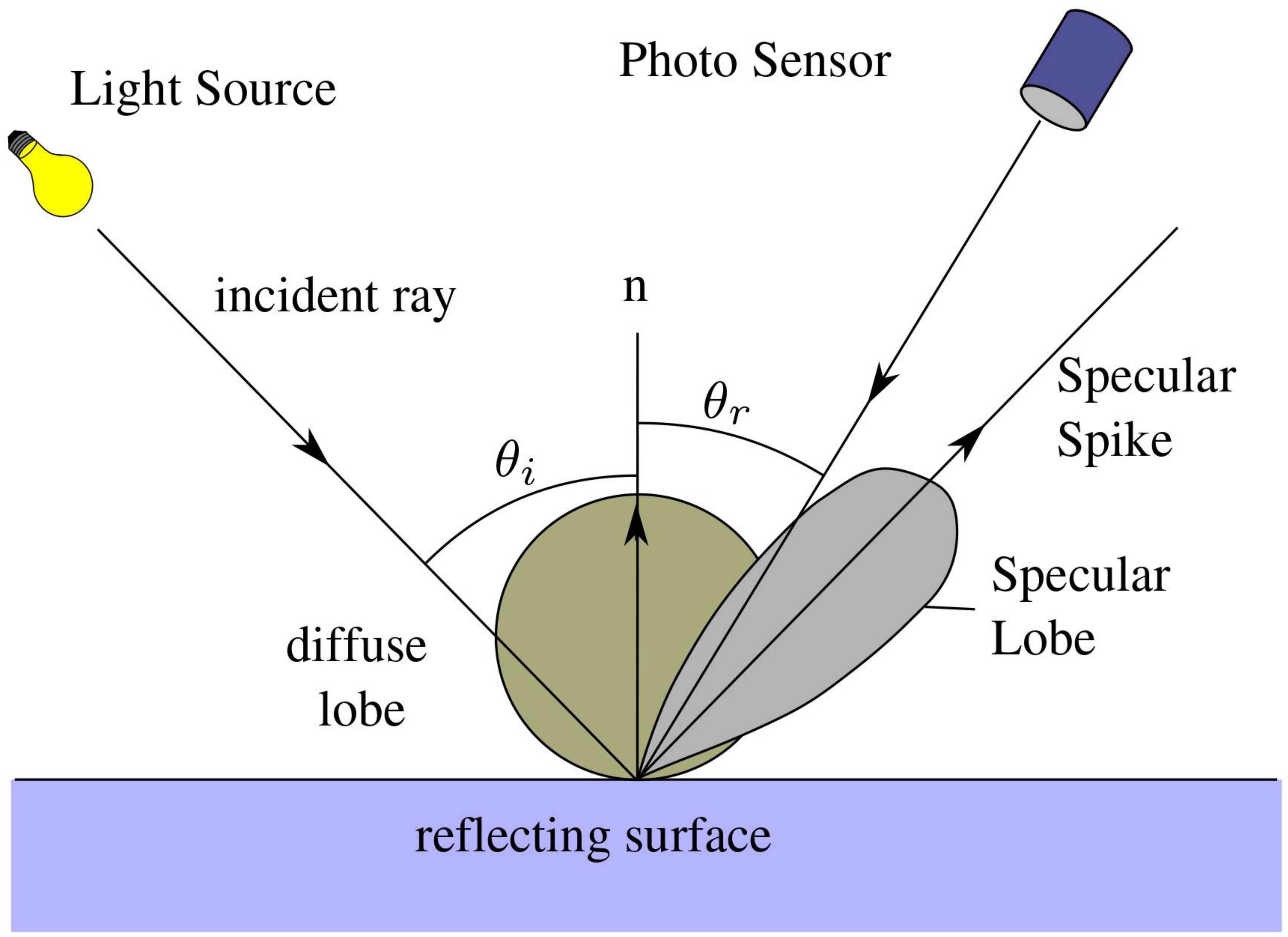


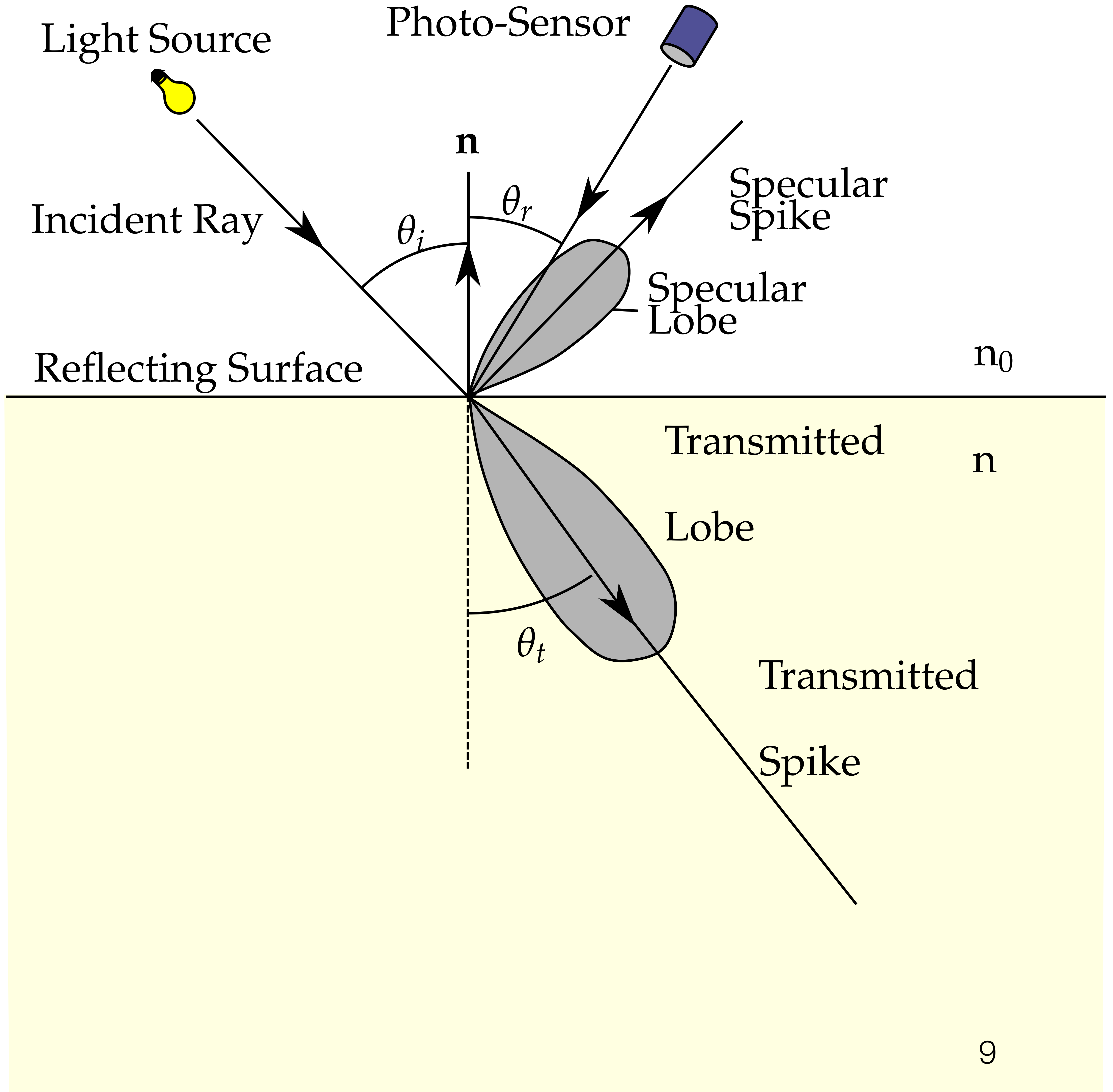
Diffuse Reflection

- Dielectrics - **mate reflection** - subsurface scattering in inhomogeneous materials, whose inhomogeneities serve as scatters centers in a otherwise uniform dielectric medium with index of refraction n.
  - Produces a reflectance pattern that is weakly dependent on the angle of incidence;
  - This process is similar to what happens in clouds
- Metals: multiple scatter that occurs in a rough surface (not that common).



Refraction and Transmission





Principles of Radiometry

Principles of Radiometry

- **Radiant Flux**: the amount of electromagnetic energy (Q) or number of photons N_{ph} received, transferred (transmitted) or emitted (reflected) by an object:

$$\Phi = \frac{\mathrm{d}Q}{\mathrm{d}t} \qquad [\mathrm{W}] \qquad\qquad \Phi = \frac{\mathrm{d}N_{\mathrm{ph}}}{\mathrm{d}t} \qquad [\mathrm{N_{ph}s^{-1}}]$$

Φ_i → Incident Flux

Φ_r → Emitted Flux

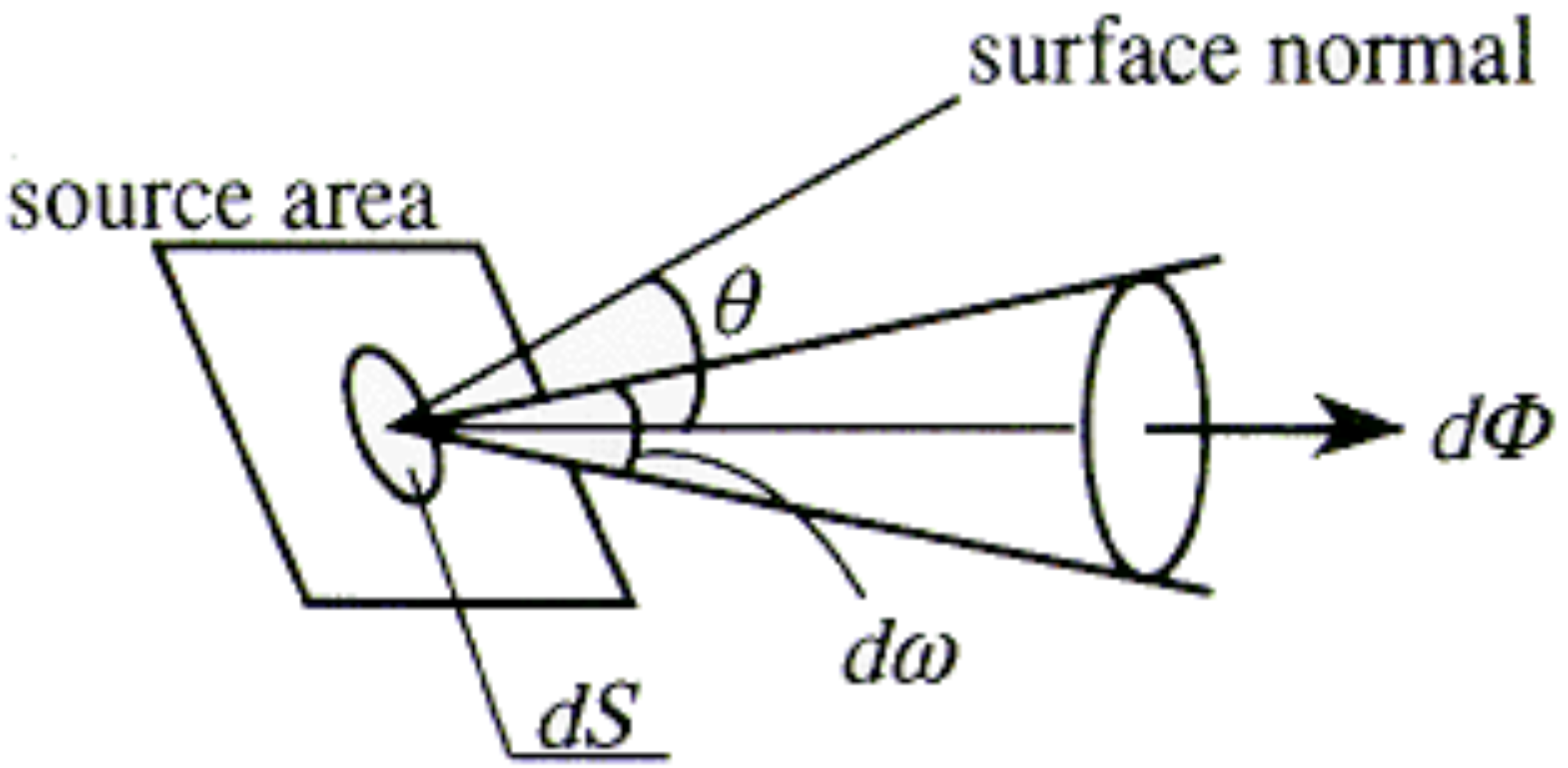
Φ_t → Transmitted Flux

- **Irradiance:**

$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$$

- **Radiance**: the flux emitted along a certain direction per unit of solid angle per unit of foreshortened area

$$L = \frac{\mathrm{d}^2\Phi}{\mathrm{d}A \cos\theta \, \mathrm{d}\Omega}$$



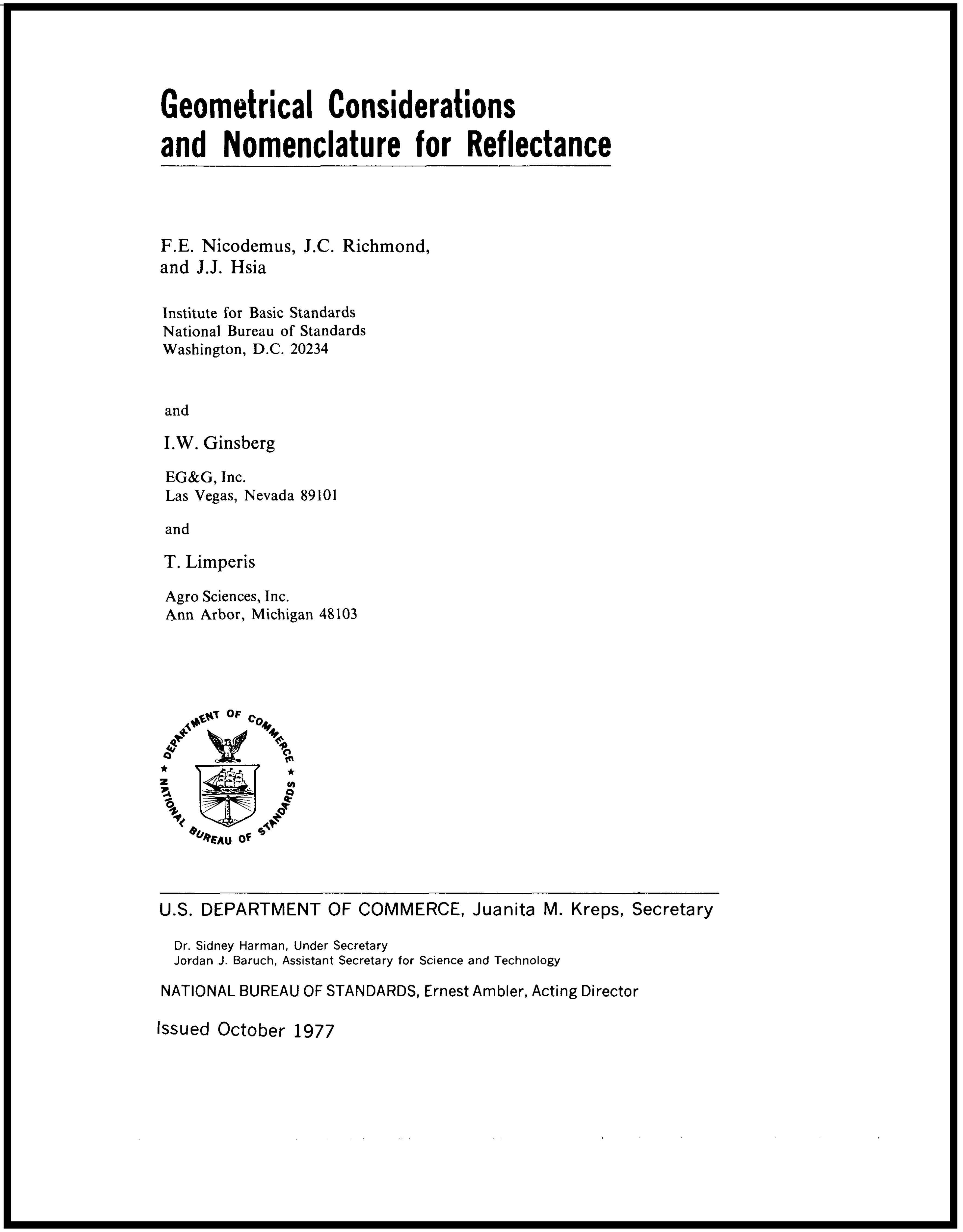
Principles of Radiometry - Reflectance

- **BSSRDF**: the most generic description of the reflectance of a surface is given by the bidirectional scattering-surface reflectance function. It is dependent of 8 variables, they are:

$$\mathrm{d}L = S\left(\theta_i, \phi_i, x_i, y_i, \theta_r, \phi_r, x_r, y_r\right) \mathrm{d}\Phi_i$$

- **BRDF** (Bidirectional reflected distribution function): it is defined as the ratio between the differential radiance and the irradiance of the surface:

$$\varrho_r\left(\theta_i, \phi_i, \theta_r, \phi_r\right) = \frac{\mathrm{d}L_r\left(\theta_i, \phi_i, \theta_r, \phi_r\right)}{L_i\left(\theta_i, \phi_i\right) \cos\theta_i \mathrm{d}\Omega_i}$$


Geometrical Considerations
and Nomenclature for Reflectance

F.E. Nicodemus, J.C. Richmond,
and J.J. Hsia

Institute for Basic Standards
National Bureau of Standards
Washington, D.C. 20234

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Agro Sciences, Inc.
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U.S. DEPARTMENT OF COMMERCE, Juanita M. Kreps, Secretary

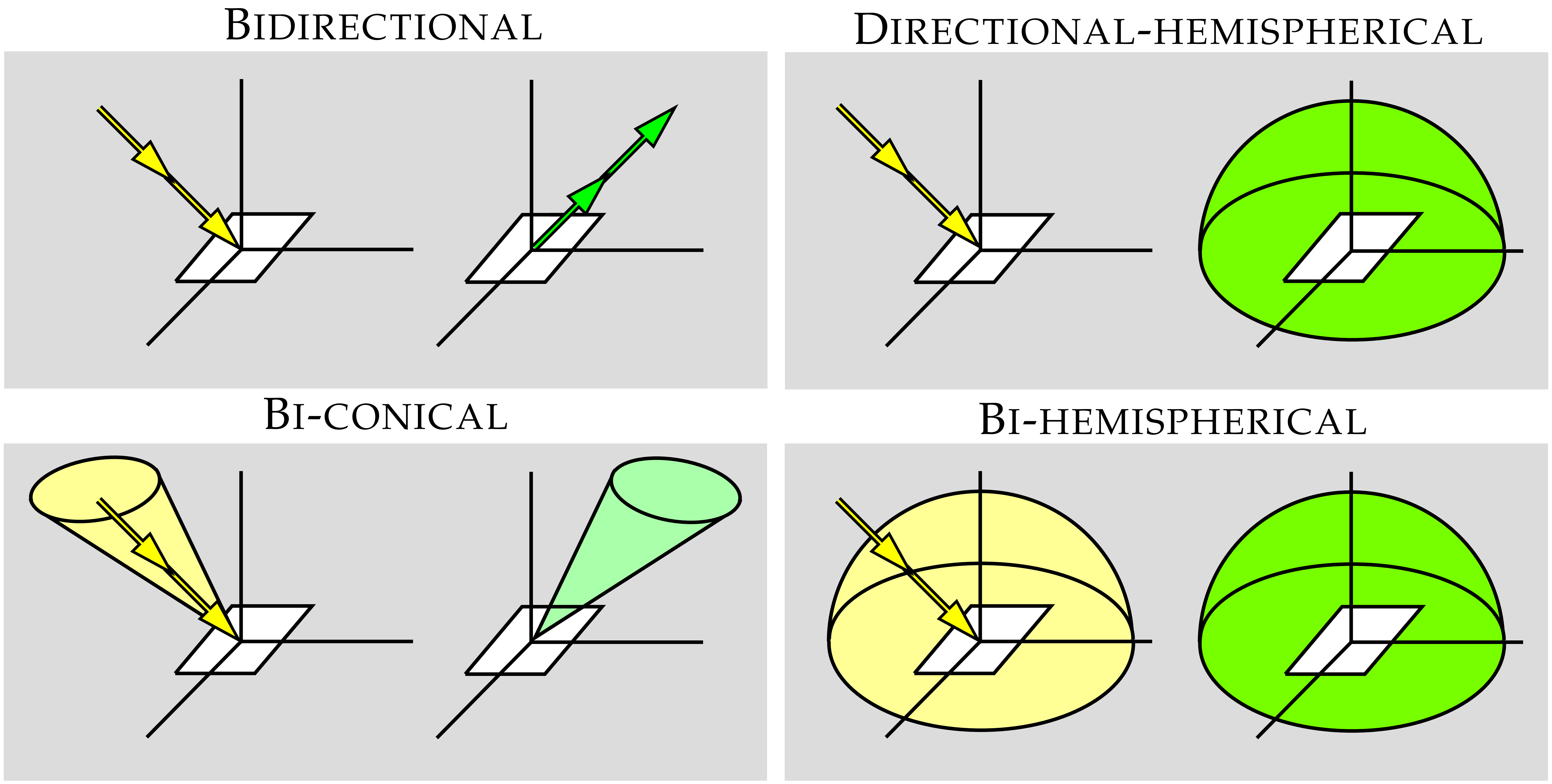
Dr. Sidney Harman, Under Secretary
Jordan J. Baruch, Assistant Secretary for Science and Technology

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Acting Director

Issued October 1977

Reflection Geometries

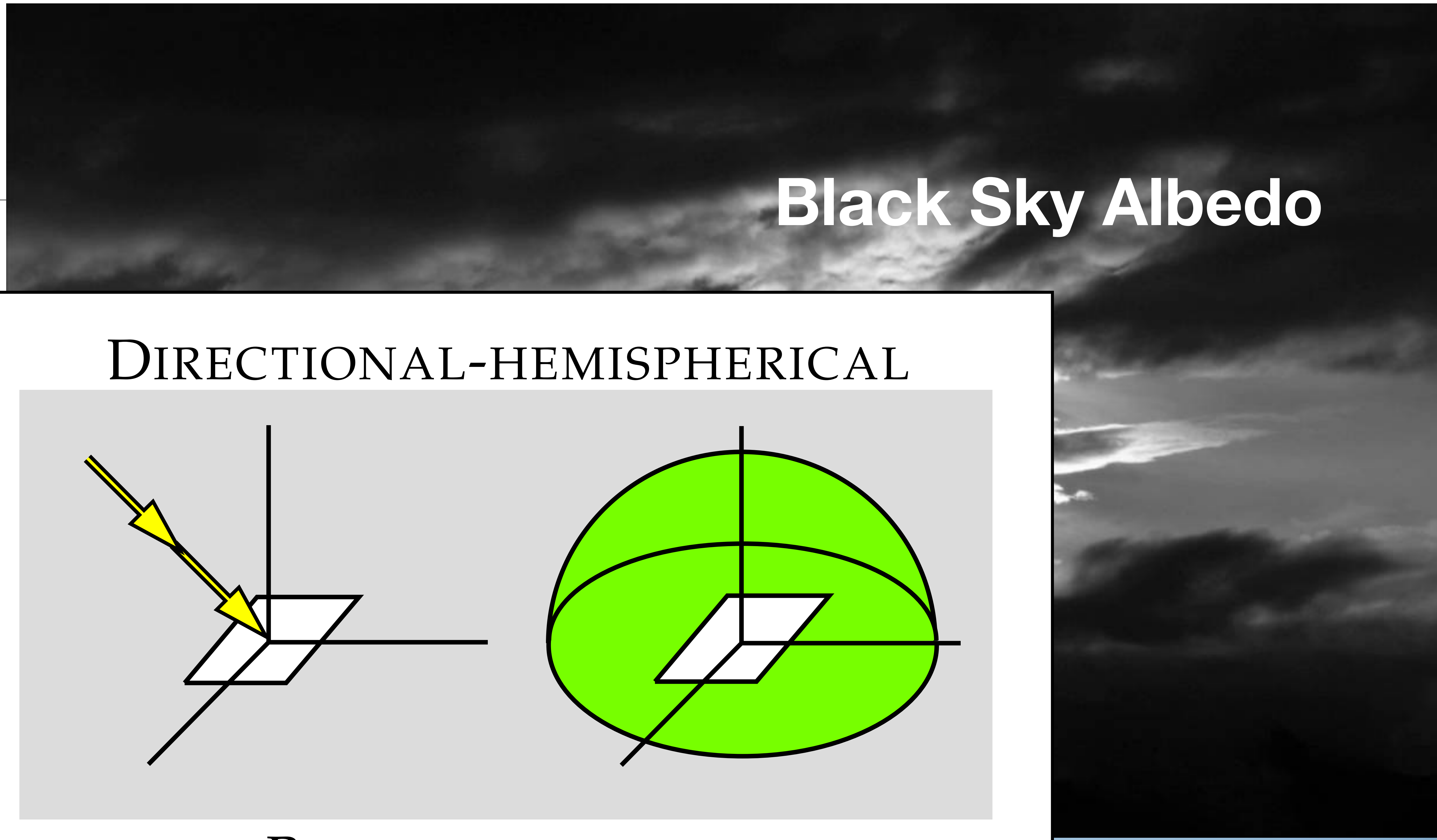
$$R^{DH}(\theta_i, \phi_i; 2\pi) = \int_{2\pi} \varrho(\theta_i, \phi_i, \theta_r, \phi_r)\, \mathrm{d}\Omega_r$$

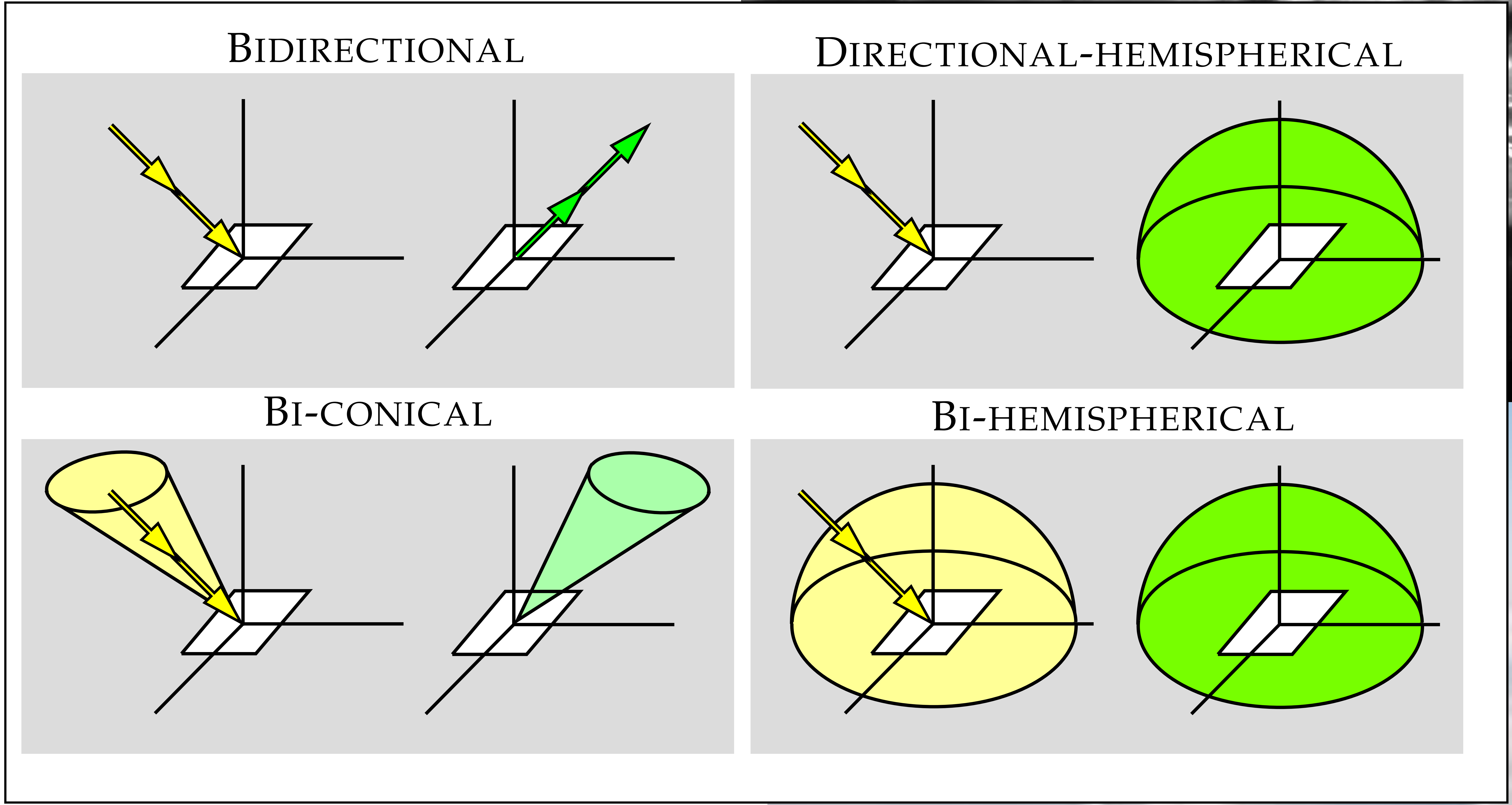


$$R^{DH}(\theta_i, \phi_i; 2\pi) = \int_{2\pi} \varrho(\theta_i, \phi_i, \theta_r, \phi_r)\, \mathrm{d}\Omega_r$$

$$R^{BH}(2\pi; 2\pi) = \frac{1}{\pi} \int_{2\pi} \int_{2\pi} \varrho(\theta_i, \phi_i, \theta_r, \phi_r)\, \mathrm{d}\Omega_r \cos\theta_i \mathrm{d}\Omega_i$$

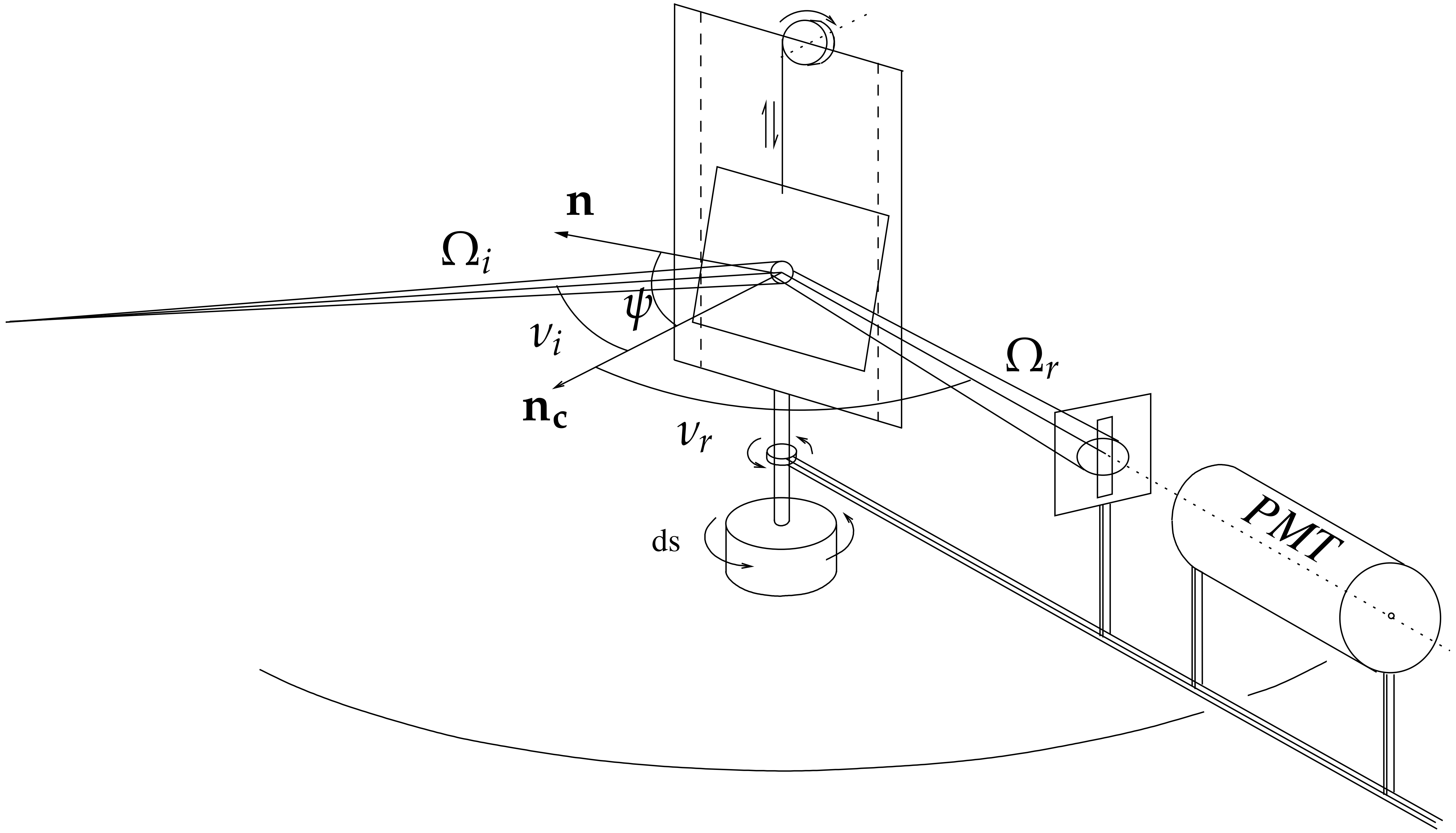
What we mean by Reflection





How can we measure it?

- Goniometer - Directional/Directional (more conical/ conical)

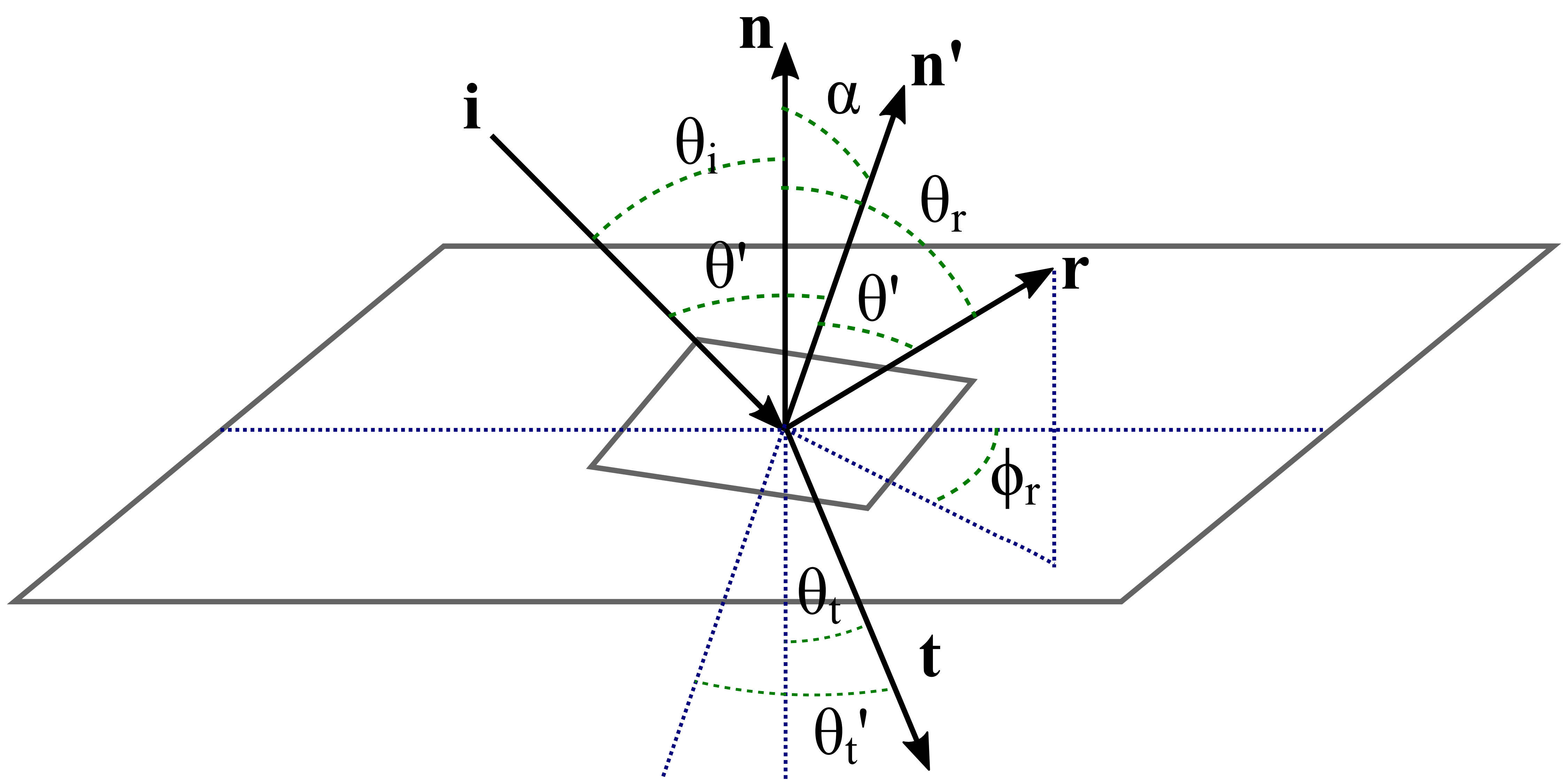


- Total Integrating Sphere - Directional (more conical)/hemispherical reflectance

The Specular Reflection

Local Variables

- In rough surfaces, it is useful to define local variables:



Global Variables

$$\mathbf{n}$$

$$\theta_i \quad \theta_r \quad \theta_t$$

Local Variables

$$\mathbf{n}'$$

$$\theta' = \theta_i' = \theta_r'$$

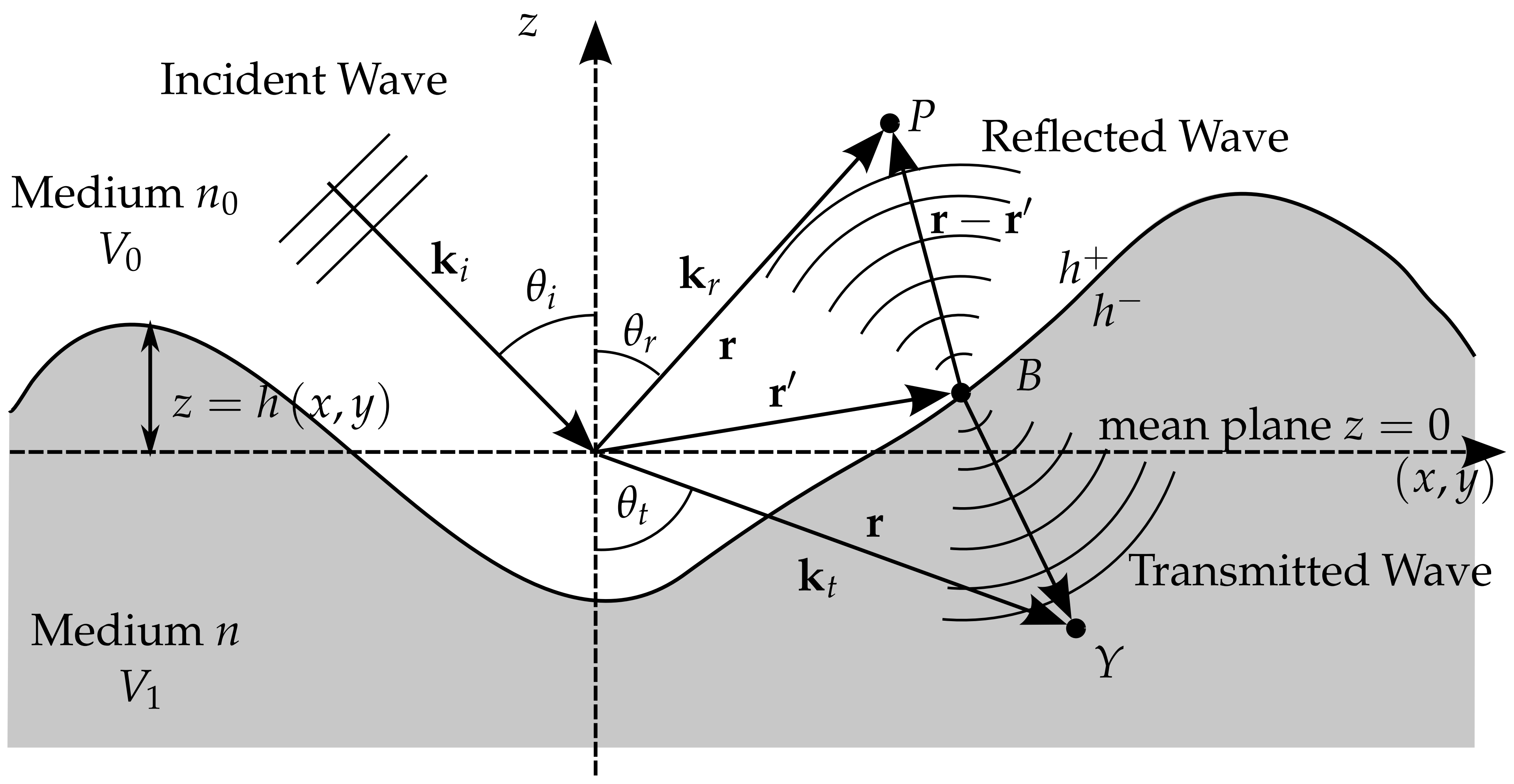
$$\cos\alpha = \mathbf{n} \cdot \mathbf{n}'$$

Light Reflection in a Rough Surface - The Physical Model

Let's assume that an incident plane wave arrives to a rough surface. Each point of the rough surface will originate a spherical plane wave that will interfere between each other. To get the scattered field, we use the following approximations:

- Fraunhofer diffraction limit - the light is observed at great distance from the surface.
- Kirchhoff approximation (tangent plane approximation) each point of the surface has the same optical properties of its tangent plan defined by the local normal **n’**
- Small slopes approximation - the local normal **n’** is not far from the global normal.

With this, the scattering field is given by:



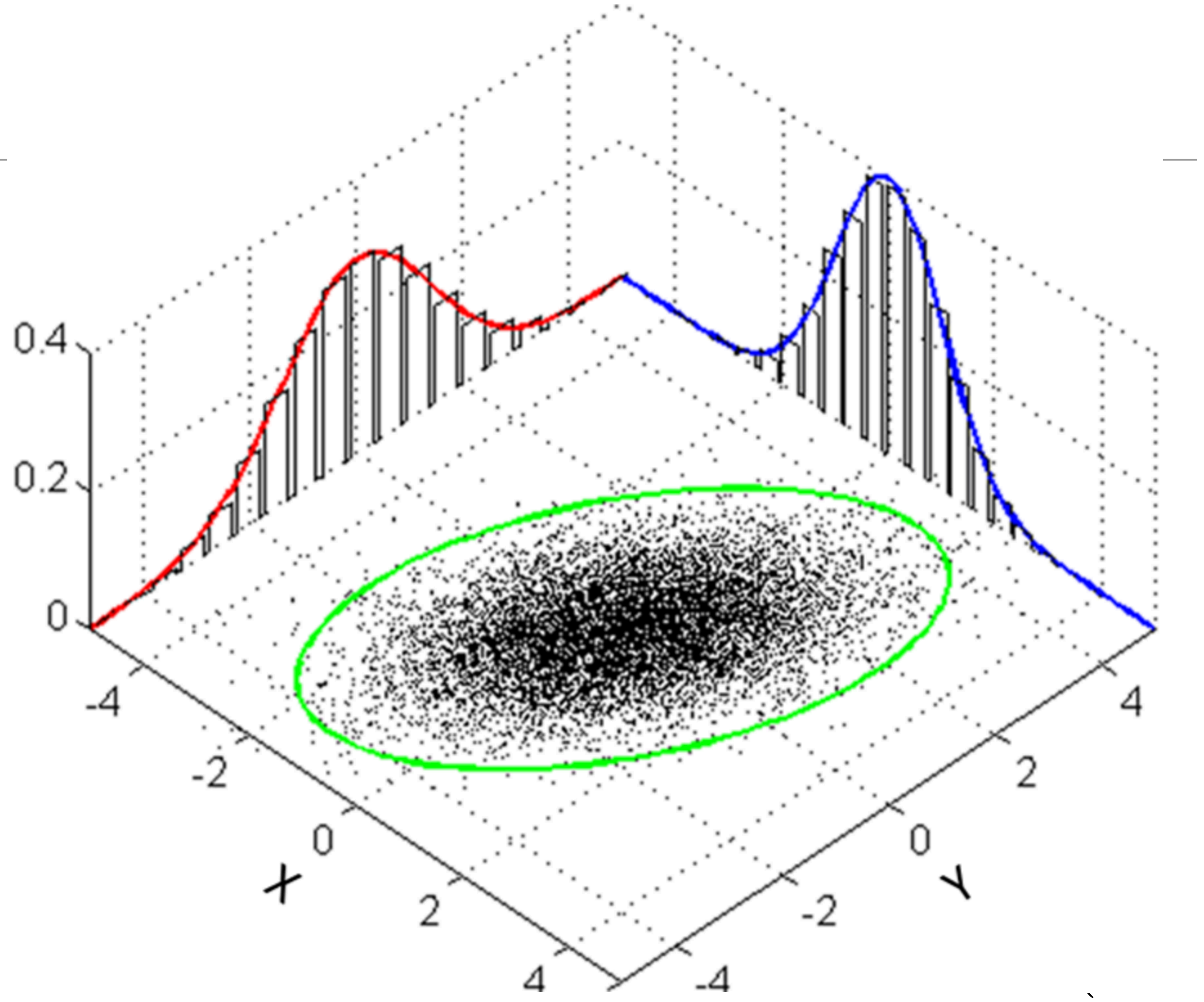
$$\psi(\mathbf{r}) = \frac{\mathrm{i}\psi_0 k_0 \exp(\mathrm{i}k_0 r)}{2\pi r} R\cos\theta' \int_A \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{u})\,\mathrm{d}A$$

With $[(1-R)\,\mathbf{i} + (1+R)\,\mathbf{r}]\cdot\mathbf{n}' = 2R\cos\theta'$

Vector wave change

Vector position in the surface

Light Reflection in a Rough Surface



- We assume that the irregularities of the surface are distributed randomly, causing both the electric field and intensity to fluctuate - average intensity of the field

$$\langle \psi\psi^* \rangle = \frac{\psi_0^2 k_0^2 \cos^2\theta'}{4\pi^2 r^2} \int_A \int_{A'} \exp\{\mathrm{i}[k_x (x - x') + k_y (y - y')]\} \cdot \langle \exp\{\mathrm{i}k_z [h(x,y) - h(x',y')]\}\rangle \,\mathrm{d}A\mathrm{d}A'$$

- We can define a joint characteristic function that depends on the joint distribution of the roughness of the surface p_2

$$\chi_2 = \langle \exp\{\mathrm{i}k_z [h(x,y) - h(x',y')]\}\rangle = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p_2(z,z') \exp[\mathrm{i}k_z (z - z')] \,\mathrm{d}z\mathrm{d}z'$$

- Gaussian joint probability function written as function of a correlation function:

$$P_2(z,z') = \frac{1}{\sqrt{2\pi[1 - C(\tau,T)]}\sigma_h} \times \exp\left[-\frac{z^2 + z'^2 - 2C(\tau,T)zz'}{2\sigma_h^2[1 - C(\tau,T)]}\right]$$

$$C(\tau) = \frac{1}{\sigma_h^2}\langle h(x_1,y_1)\cdot h(x_2,y_2)\rangle$$

- For the gaussian joint probability function, the characteristic function is given by:

$$\chi_2 = \exp\left[-k_z^2\sigma_h^2 (1 - C(\tau,T))\right]$$

- τ corresponds to the distance of the two points in the (x, y) plane

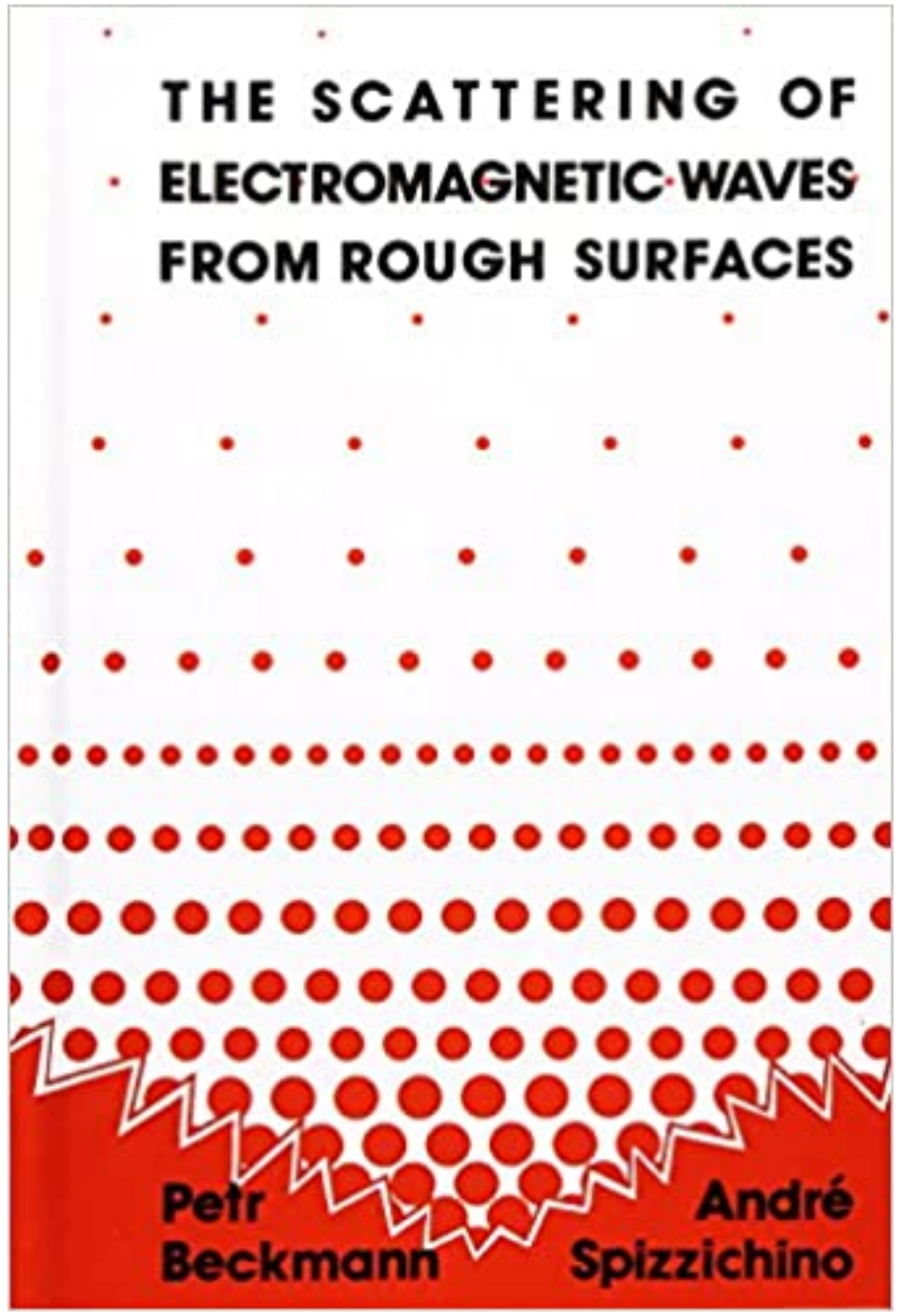
Light Reflection in a Rough Surface - The Physical Model

- Now, to get this integral we change this to cylindrical coordinates, expand it in a Taylor series to get something like (for the Gaussian bivariate function):

$$f_r = \mathcal{S}\chi_1\chi_1^*\left\{\int_0^{+\infty} J_0\left(k_z\tau\right)\tau\mathrm{d}\tau + \sum_{n=1}^{+\infty}\frac{\left(k_z\sigma_h\right)^{2n}}{n!}\int_0^{+\infty}\left[C\left(\tau,T\right)\right]^n J_0\left(k_z\tau\right)\tau\mathrm{d}\tau\right\}$$

with

$$f_r = \frac{r^2\left\langle\psi\psi^*\right\rangle}{A\psi_0^2\cos\theta_i\cos\theta_r}$$



- And, if we make the assumption that the Correlation function is Gaussian, we have:

$$f_r = e^{-g}\left[\delta\left(k_z\right) + T^2\sum_{n=1}^{\infty}\frac{g^n}{2nn!}\exp\left(-\frac{g}{2n}\frac{T^2}{2\sigma_h^2}\tan^2\alpha\right)\right]$$

- Where g corresponds to the optical roughness

$$g = \left[\sigma_h\frac{2\pi}{\lambda}\left(\cos\theta_i + \cos\theta_r\right)\right]^2$$

Light Reflection in a Rough Surface - The Physical Model

$$f_r = e^{-g}\left[\delta\left(k_z\right) + T^2 \sum_{n=1}^{\infty} \frac{g^n}{2nn!} \exp\left(-\frac{g}{2n}\frac{T^2}{2\sigma_h^2}\tan^2\alpha\right)\right]$$

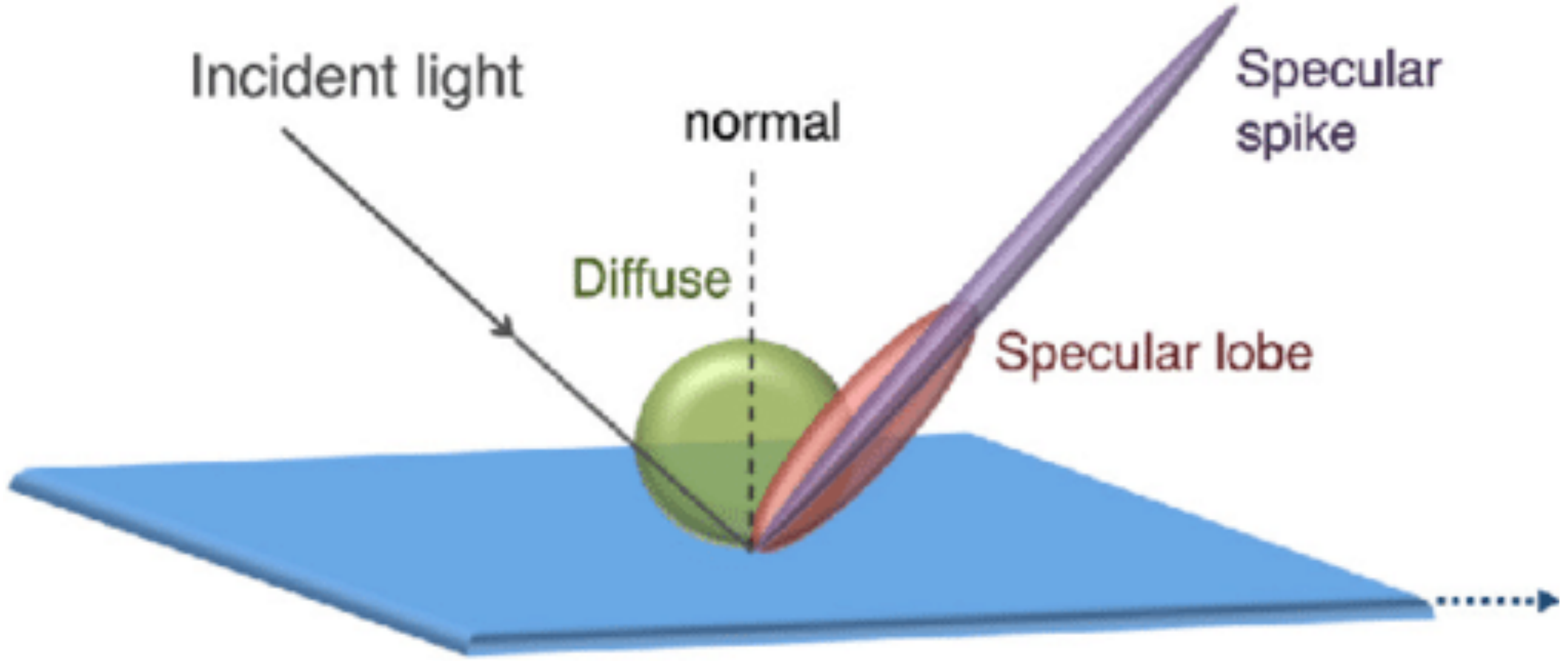
$$g = \left[\sigma_h \frac{2\pi}{\lambda}\left(\cos\theta_i + \cos\theta_r\right)\right]^2$$

Specular Spike **Specular Lobe**

- This corresponds to the well-known **Beckmann-Spizzichino model**
- The surface looks smoother for:
  - Larger angles of incidence or observation;
  - Larger photon wavelengths;
  - Larger values of roughness
- When g<<1 we have:

$$f_r = e^{-g}\left[\delta\left(k_z\right) + \frac{T^2 g}{2} \exp\left(-\frac{g}{2}\frac{T^2}{2\sigma_h^2}\tan^2\alpha\right)\right]$$

- **This only solution is only valid for Gaussian bivariate functions with a Gaussian correlation function!**



Light Reflection in a Rough Surface - The Specular Spike

- The intensity of the specular spike is given by

$$\mathcal{Q} \equiv \left|\langle \exp\left[i\mathbf{w}_z h(x,y)\right]\rangle\right|^2 \equiv \left|\int_{-\infty}^{\infty} P_z(z) \exp(i\mathbf{w}_z z) \mathrm{d}z\right|^2 \qquad k_z \equiv w_z$$

- Depends only on the characteristic function of the height distribution P_z

$$\int_{-\infty}^{+\infty} P_z(z, \sigma_z)\, \mathrm{d}z = 1$$

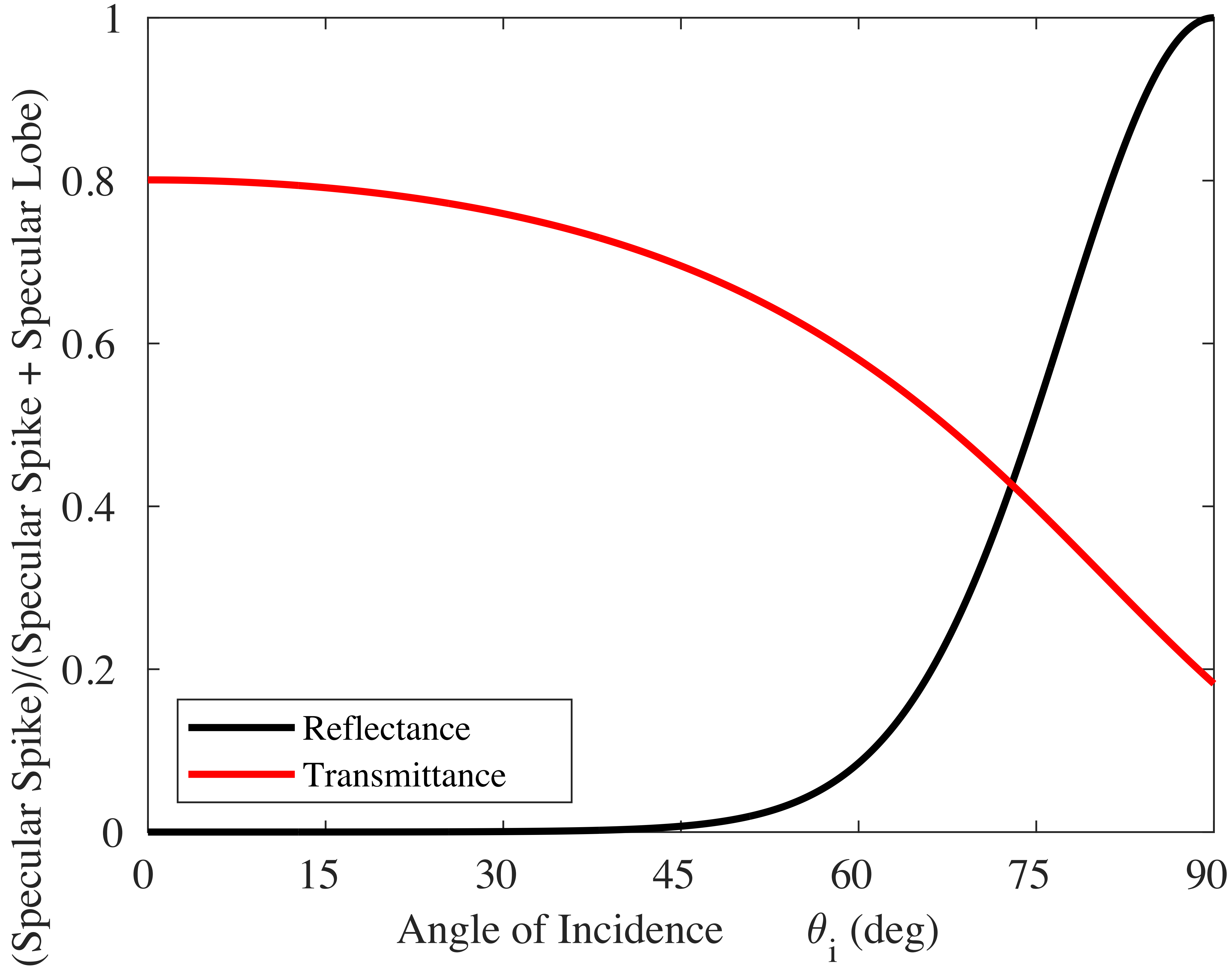
- w_z is corresponds to the z component of the vector wave change
- For the **Reflection**:

$$w_z = (\mathbf{k_r} - \mathbf{k_i}) \cdot \mathbf{n} = \frac{\mathbf{2}\pi}{\lambda}(\mathbf{n_1} \cos\theta_\mathbf{r} + \mathbf{n_1} \cos\theta_\mathbf{i}) = \frac{\mathbf{4}\pi}{\lambda} \cos\theta_\mathbf{i}$$

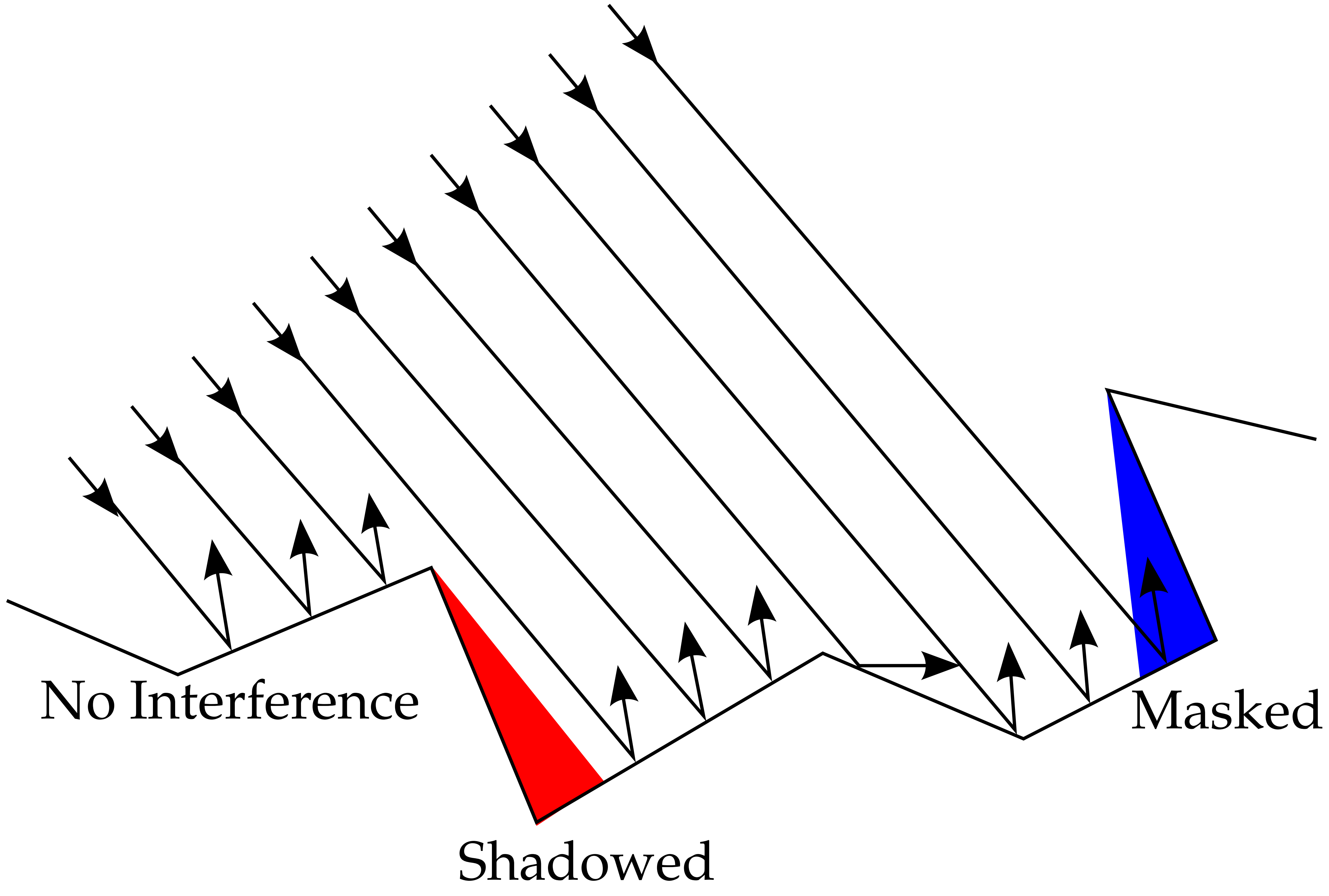
  - The surface looks rougher in the normal direction
- For the **Transmission**:

$$w_z = (\mathbf{k}_t - \mathbf{k}_i) \cdot \mathbf{n} = \frac{2\pi}{\lambda}(n_1 \cos\theta_i - n_2 \cos\theta_t)$$

  - The surface looks smoother in the normal direction



Shadowing-Masking



- **Shadowing:** the roughness creates regions in the surface that are in a shadow, i.e. they are not visible for light incoming from a specific directions.
- **Masking:** the reflected light is blocked by a prominent tip of the surface.

Diffuse Reflection

Diffuse Reflection (Bulk Reflection) - What it is?

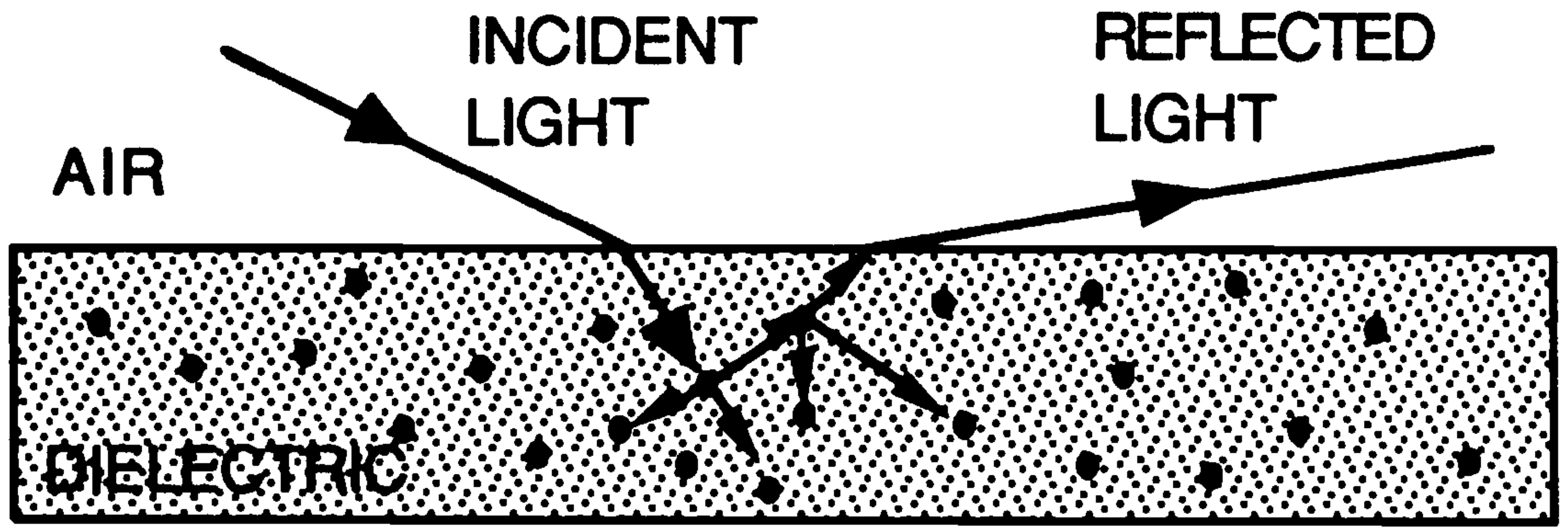
- D*iffuse Reflection:* the light is refracted to the bulk, multi-scattered in inhomogeneities of the material, and returns to the original medium.
- It is usually described by the Lambertian Model:

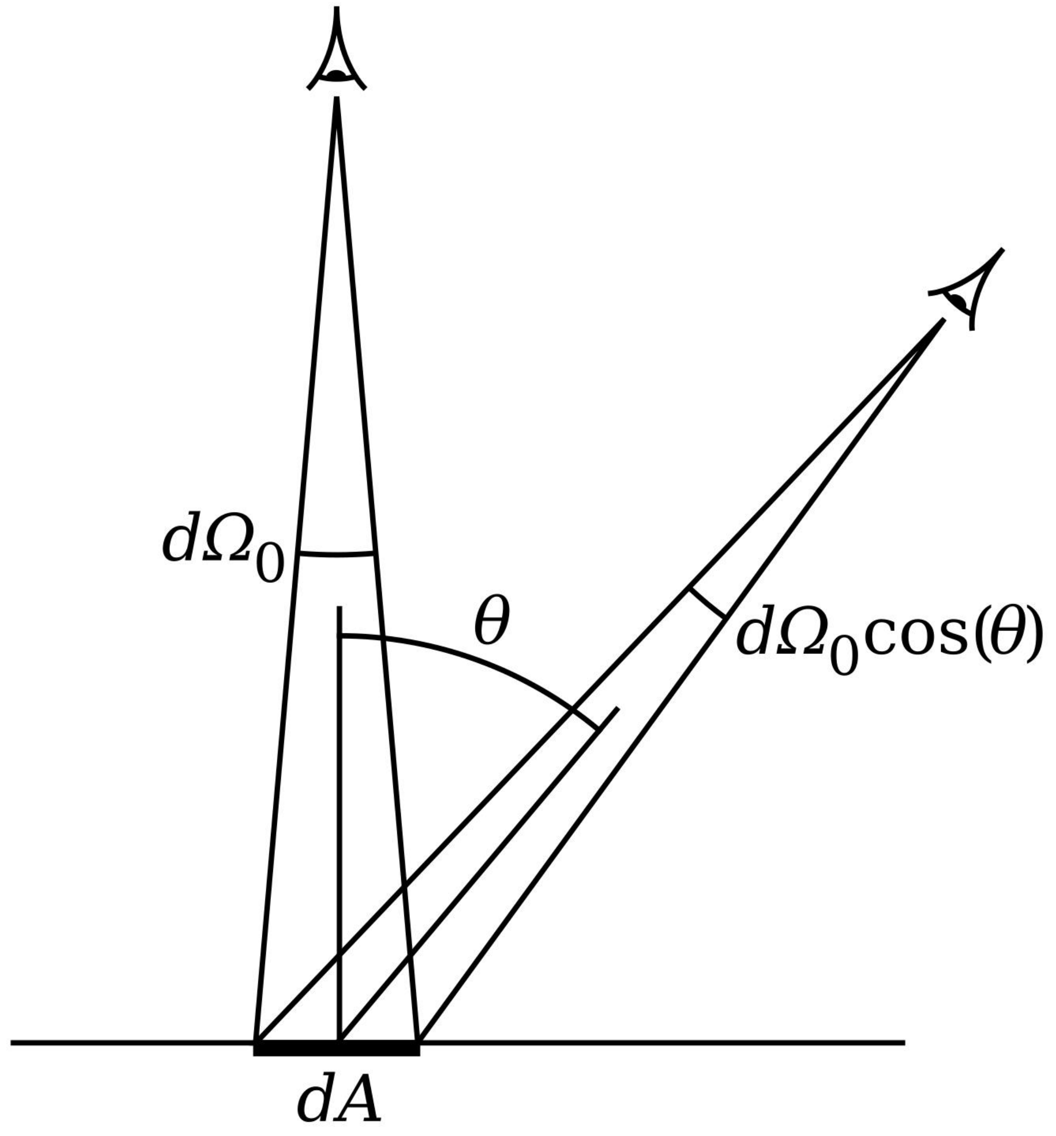
$$I = \frac{\varrho}{\pi} \cos\theta_r$$

- We will write the reflectance (and transmittances) using the reflection factor R, in which the Lambertian law takes the form:

$$R = \varrho$$

- Very rough surfaces can also produce a reflection pattern similar to the Lambertian reflection. So, there is some confusion with the term Diffuse Reflection.



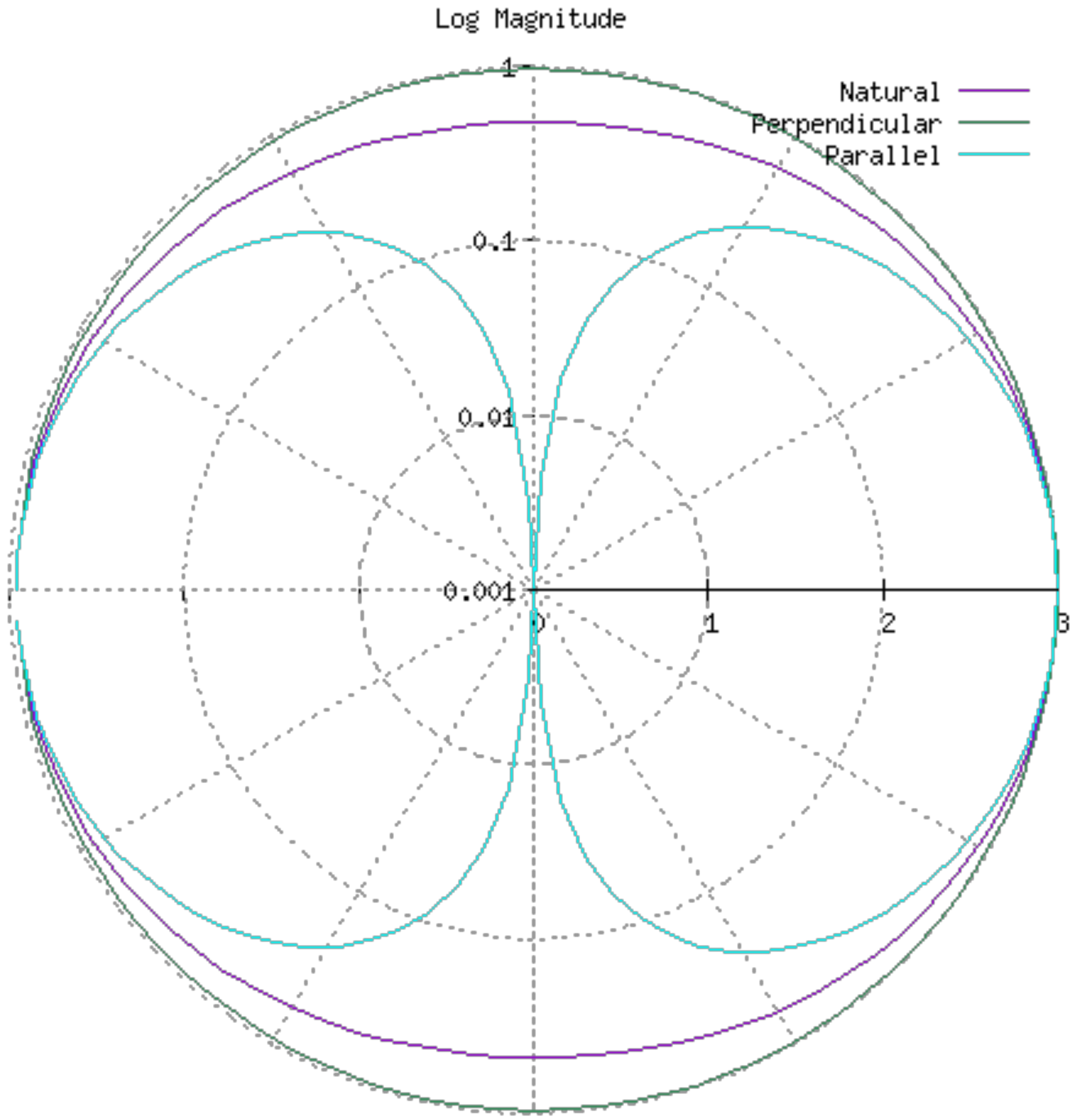


Rayleigh and Mie Scattering

- Rayleigh Scattering: scattering of the light with the molecules

$$I = I_0 \frac{8\pi^4 N\alpha^2}{\lambda^4 R^2}\left(1+\cos^2\theta\right)$$

  - It is dominant for tiny particles (up to 1/10 of the wavelength)
- Mie Scattering: scattering of a plane wave from a sphere - produces a pattern like an antenna lobe, with a sharper and more intense forward lobe for larger particles
- Geometrical Optics: Light scattering by large particles can be understood through the concepts of geometrical optics (formation of rainbow).

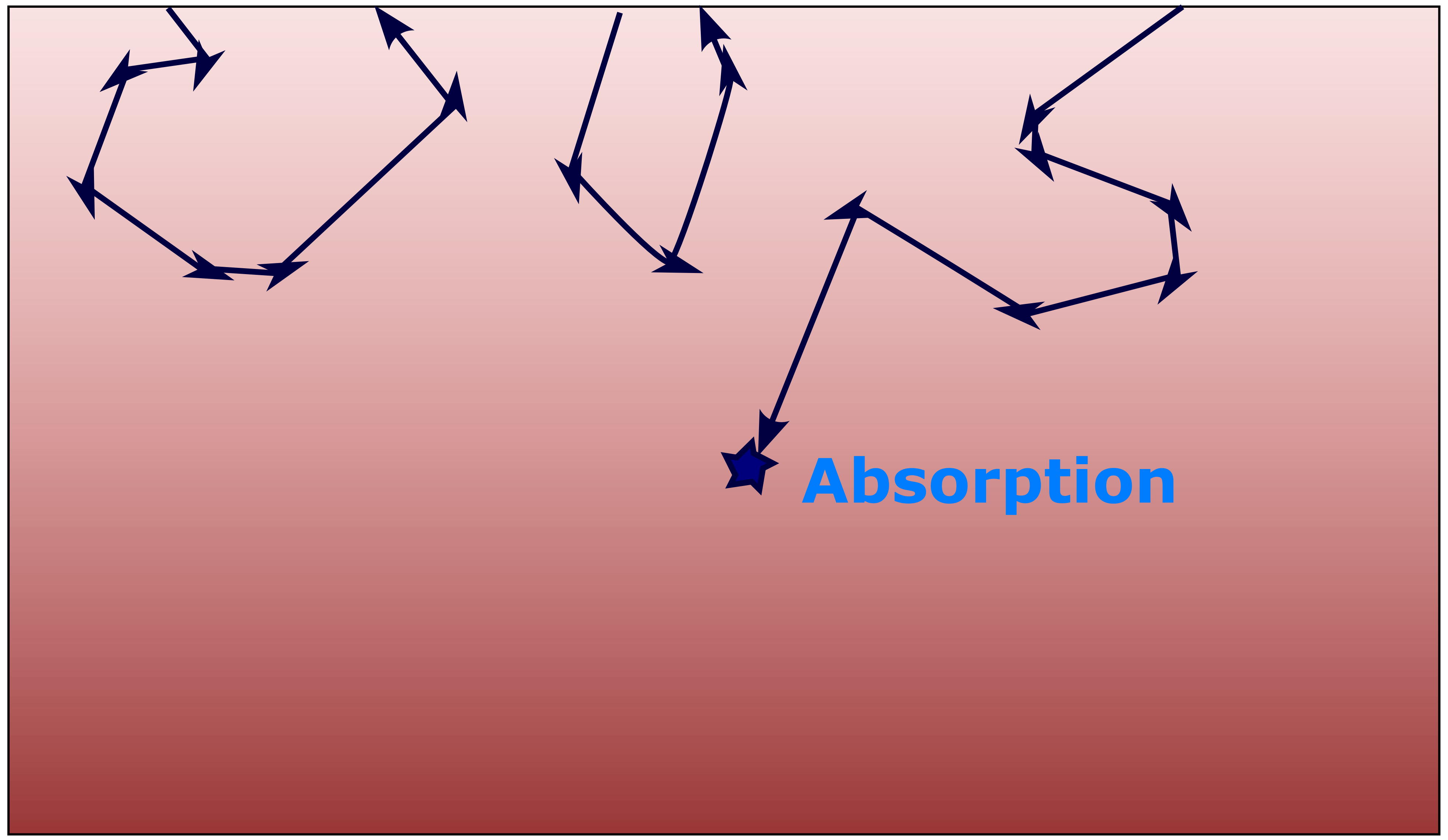


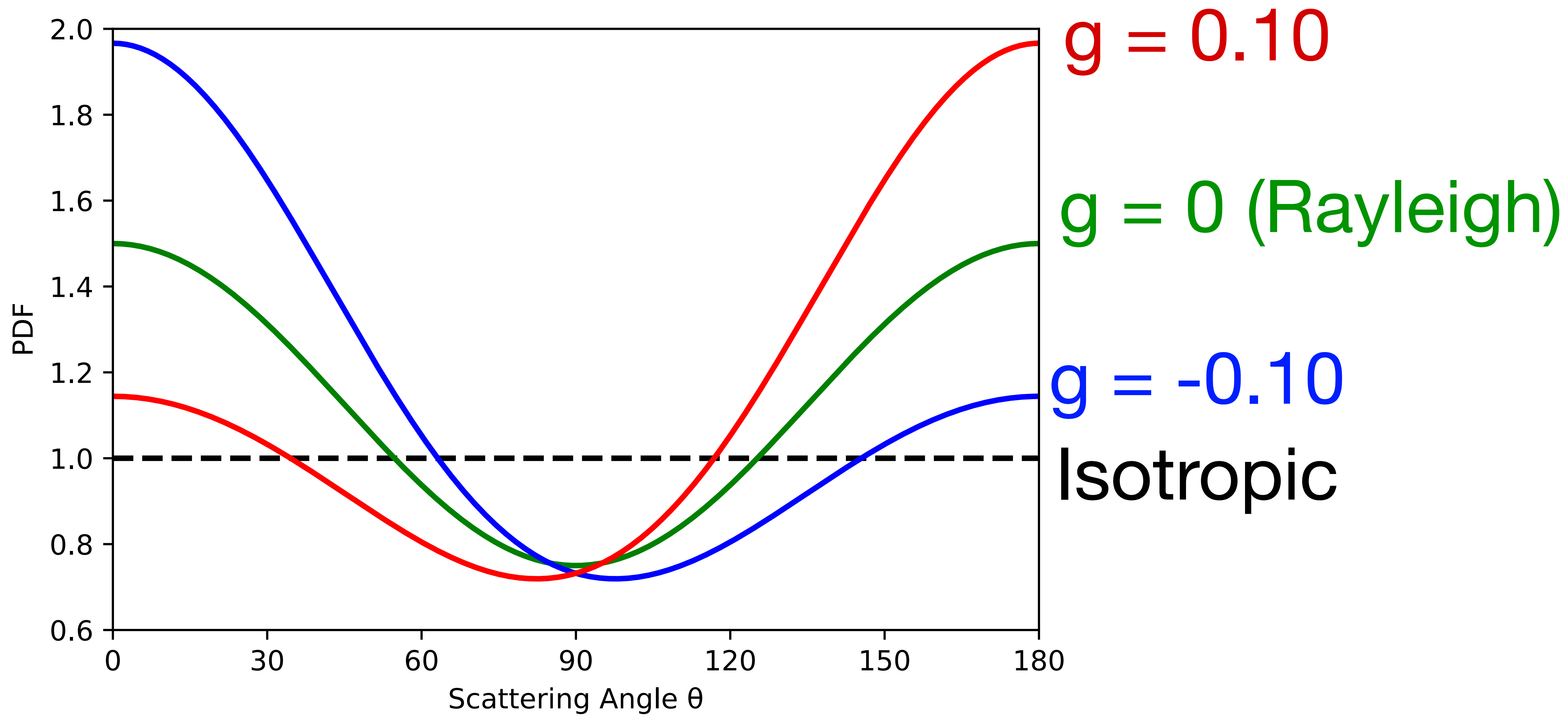
λ=500 nm and particles change from 50 nm to 5 μm

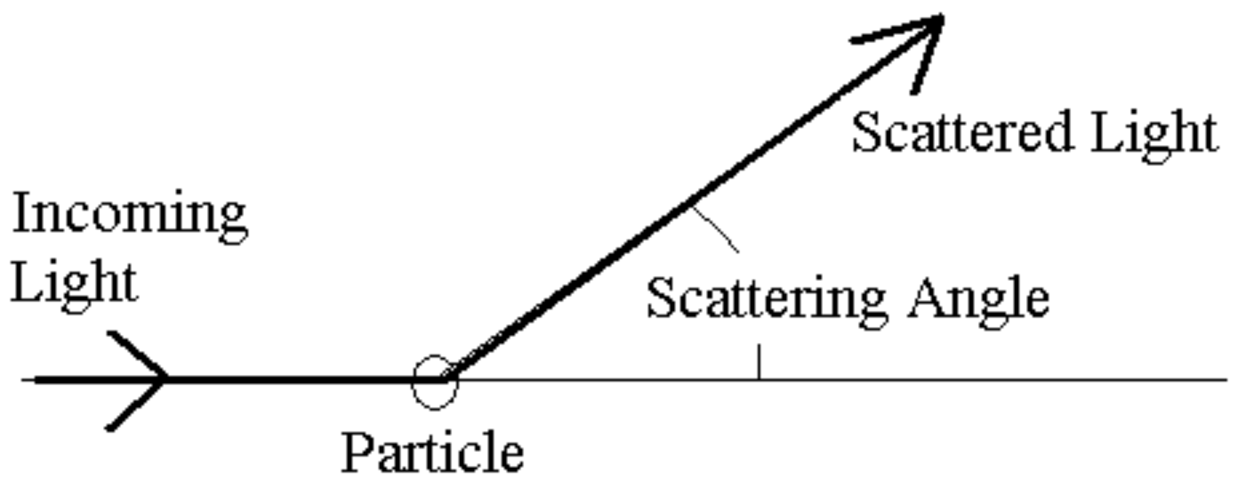
Simulation of Diffuse Reflection in a semi-infinite diffuser

- The diffuser slab has an infinite depth, and infinite x and y dimensions.
- We assume a linear scattering coefficient, α_{scat}, of 1 and a specific linear absorption coefficient, σ_{abs}.
- The simulation starts at the surface with a specific angle of immersion.
- Henyen-Greenstein equation used to generate the scattering angle:

$$P\left(\theta_s\right) = \frac{3}{2}\frac{1+\cos^2\theta}{2+g^2}\frac{1-g^2}{\left(1-2g\cos\theta_s+g^2\right)^{\frac{2}{3}}}$$

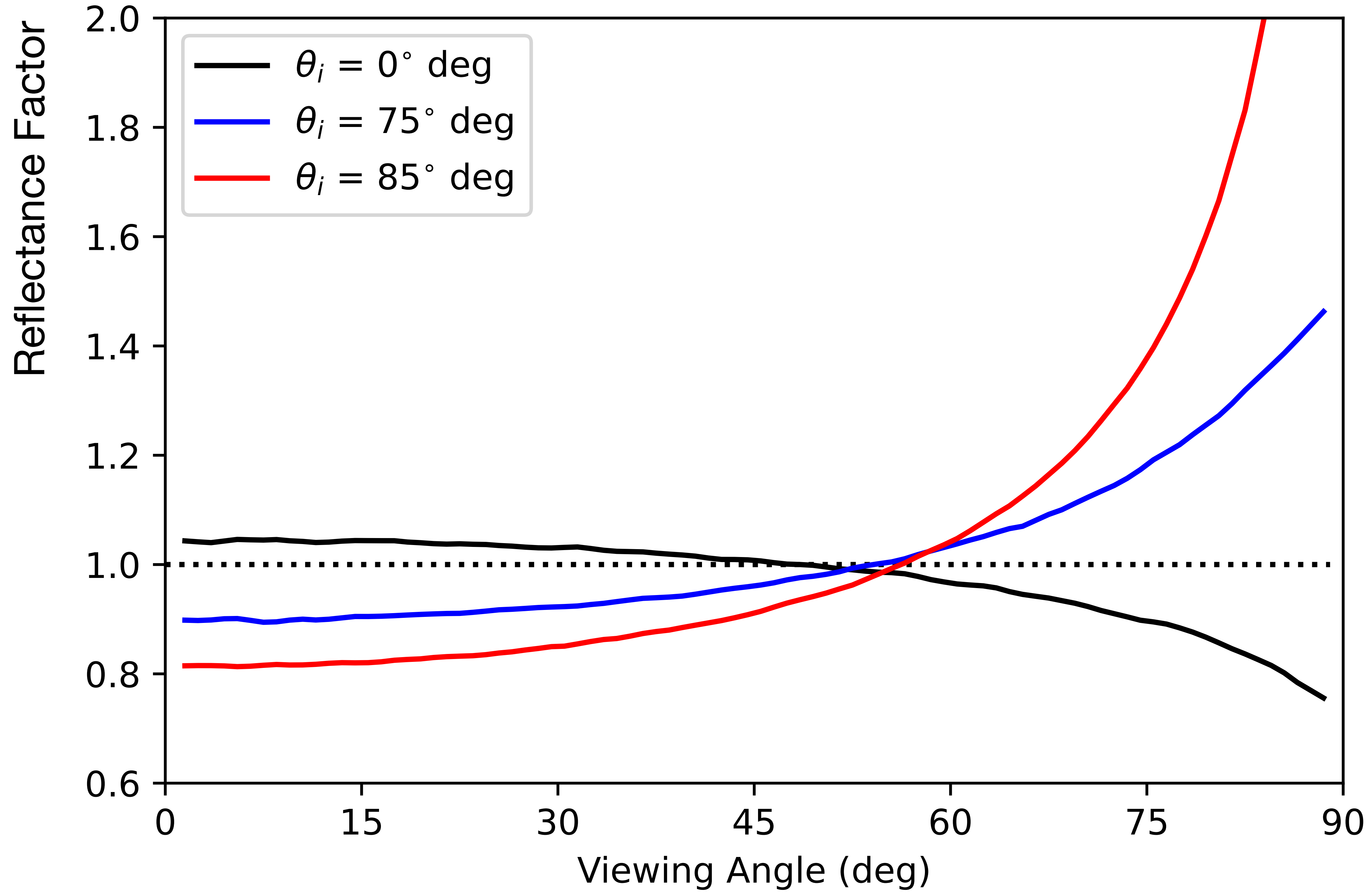






Results - Isotropic

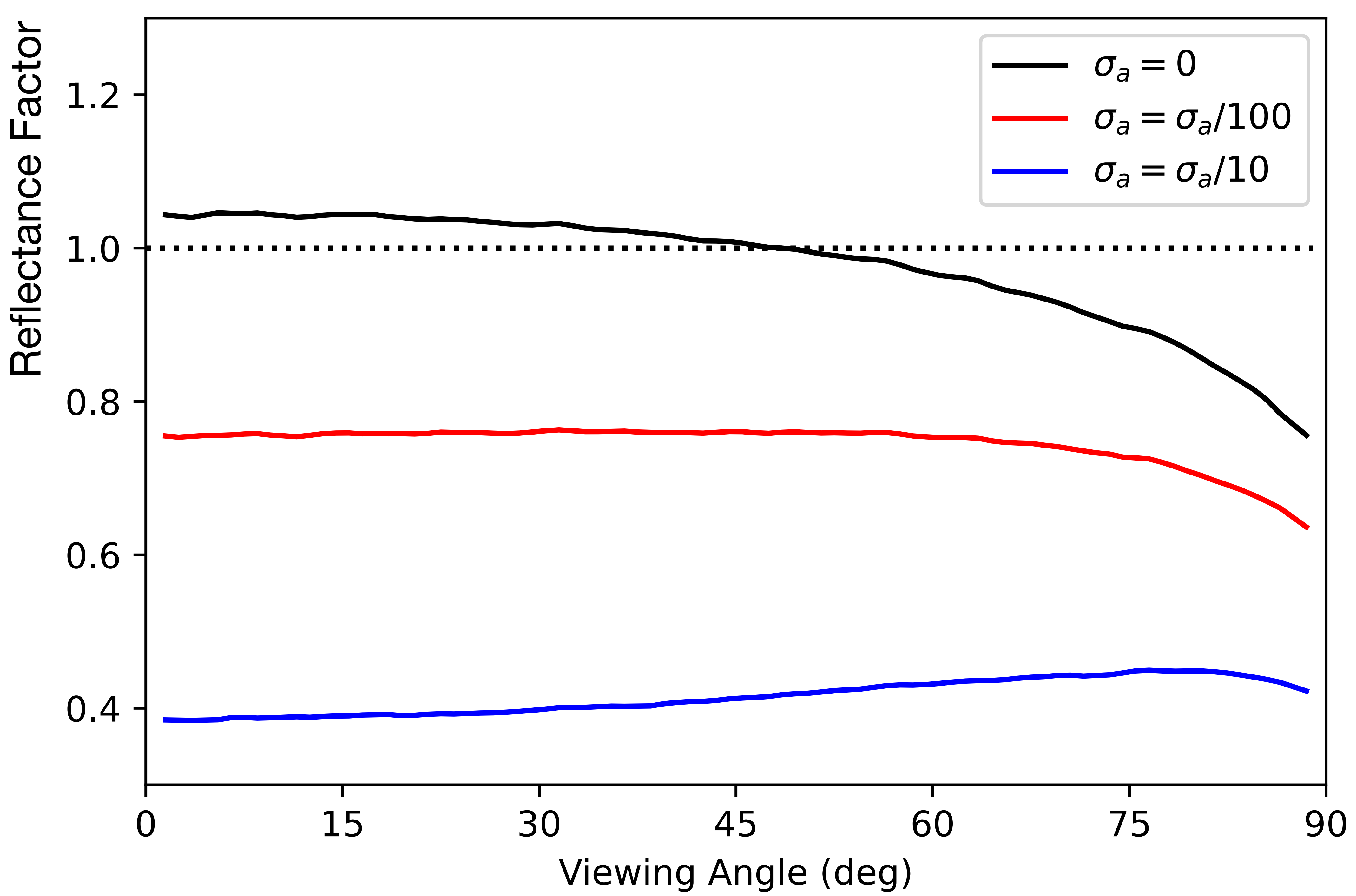
Dependence with Angle of Incidence/Immersion



Isotropic, conservative

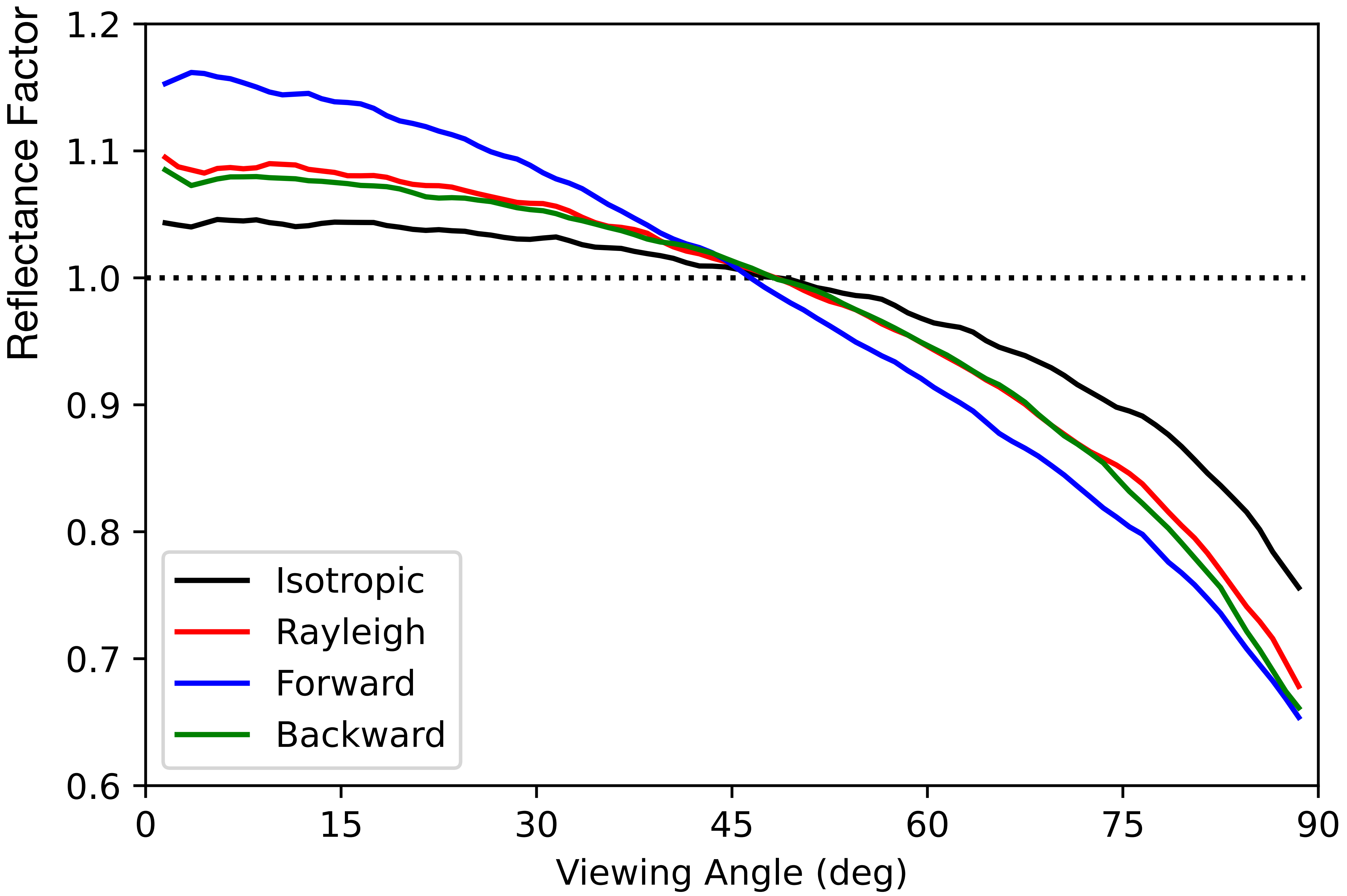
α_{abs}=0

Dependence with Absorption



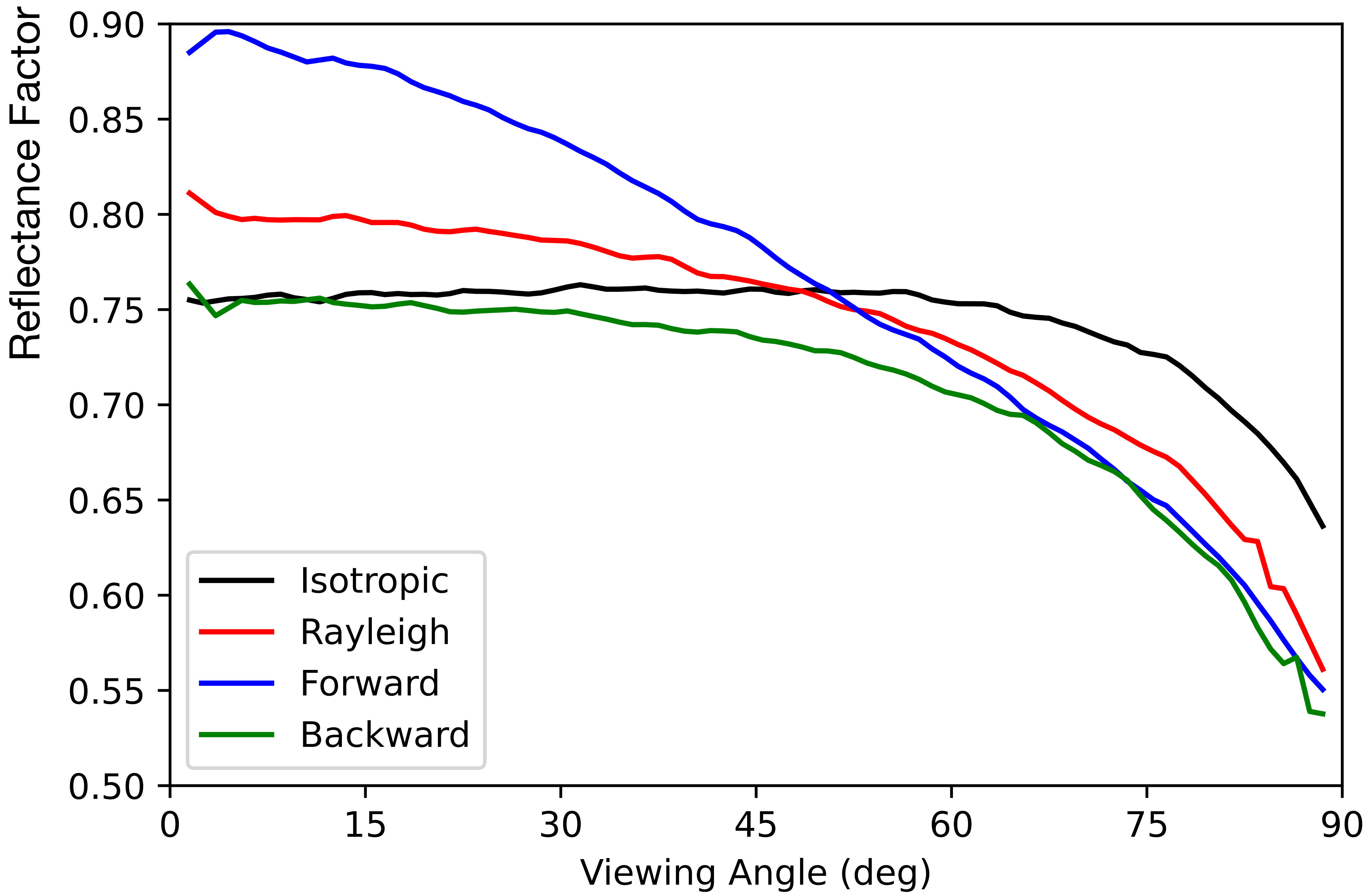
Isotropic, normal direction

Results - Dependence with the Phase Function



Conservative

θ_i = 0 deg



$\sigma\alpha_{abs} = \alpha_{scat}/100$

θ_i = 0 deg

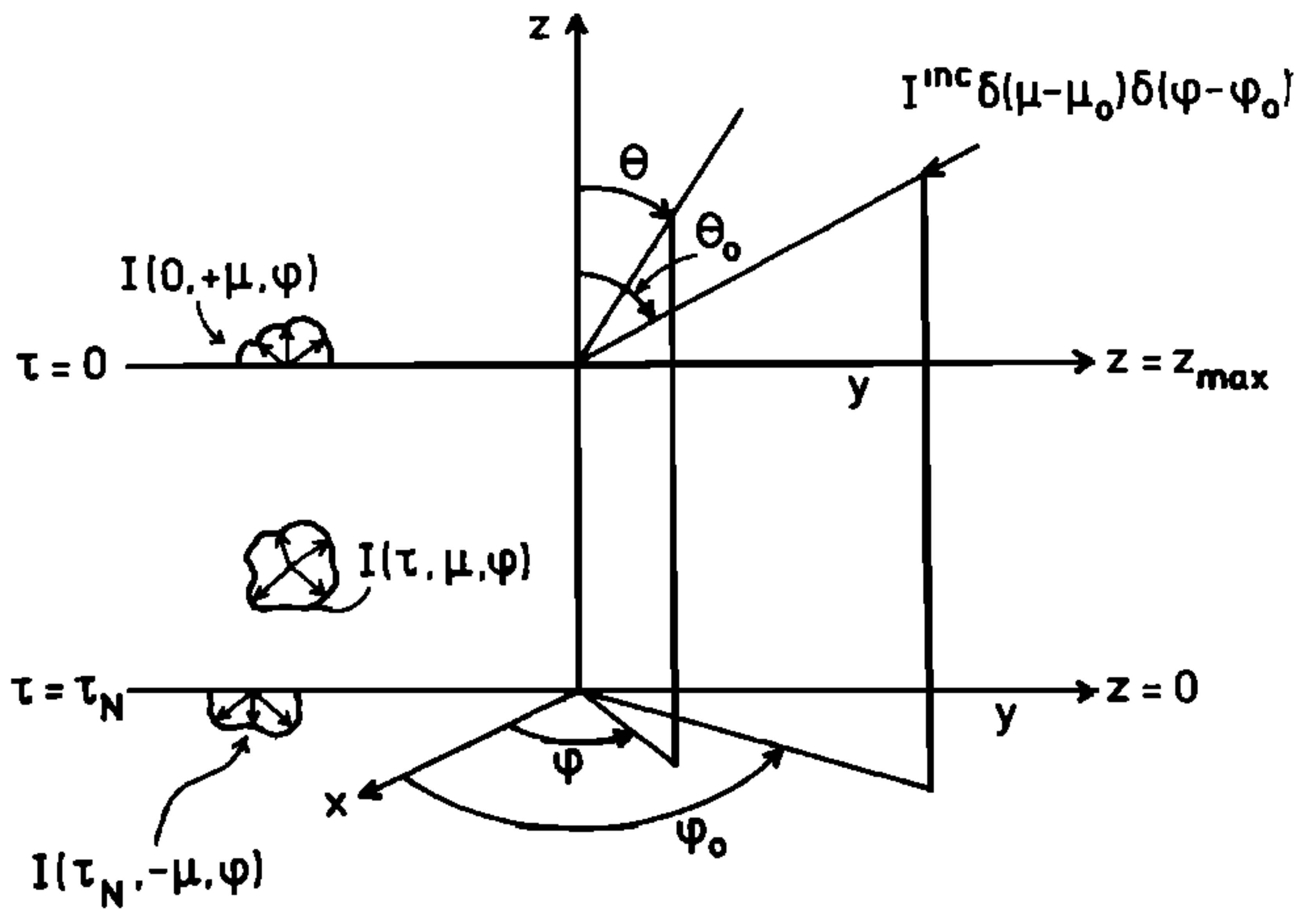
The Monte Carlo Simulations

- The Monte Carlo simulations are precise but they are also **very slow** as the light can scatter multiple times (up to 10^5 times) before returning back to the original medium or being absorbed.
- Also, the absorption cross section, the scattering cross section, the parameter g of the scattering are usually unknown (or difficult to measure) for most of the materials.
- The solution - **Radiation Transfer Theory**

Radiation Transfer Theory

- Plane -parallel: we assume that the length and width of the medium is infinite.
- Radiation Transfer equation (derived from the Boltzmann equation)

$$\mu \frac{\partial I(\tau;\mu,\phi)}{\partial \tau} = I(\tau;\mu,\phi) - \frac{1}{4\pi}\int_{-1}^{+1}\int_{0}^{2\pi} \chi(\mu,\phi;\mu',\phi')\, I(\tau;\mu,\phi)\, \mathrm{d}\mu' \mathrm{d}\phi' - \frac{1}{4} F e^{-\tau/\mu_0} \chi(\mu,\phi;-\mu_0,\phi_0)$$



- **χ** is the phase function (e.g. isotropic is 1)
- **I** corresponds to the intensity of the radiation;
- **μ = cos(θ)** - measured relative to the surface's normal. The angle ϕ is the azimuth.
- **F** is the incident flux
- **τ** corresponds the normal optical depth

Radiation Transfer Theory - The Phase Function

$$\mu \frac{\partial I(\tau;\mu,\phi)}{\partial \tau} = I(\tau;\mu,\phi) - \frac{1}{4\pi}\int_{-1}^{+1}\int_{0}^{2\pi} \chi(\mu,\phi;\mu',\phi')\, I(\tau;\mu,\phi)\,\mathrm{d}\mu'\mathrm{d}\phi' \\ -\frac{1}{4} F e^{-\tau/\mu_0} \chi(\mu,\phi;-\mu_0,\phi_0)$$

- The function χ corresponds to the phase function. It is a probability distribution function describing the angular dependence of the scattered radiation.
  - For the Isotropic case:

$$\chi(\mu) = \varpi \quad \text{where}$$

$$\varpi = \frac{\alpha_{\mathrm{scat}}}{\alpha_{\mathrm{scat}} + \alpha_{\mathrm{abs}}}$$

single scatter albedo

Linear scattering coefficient [L^{-1}]

Linear absorption coefficient [L^{-1}]

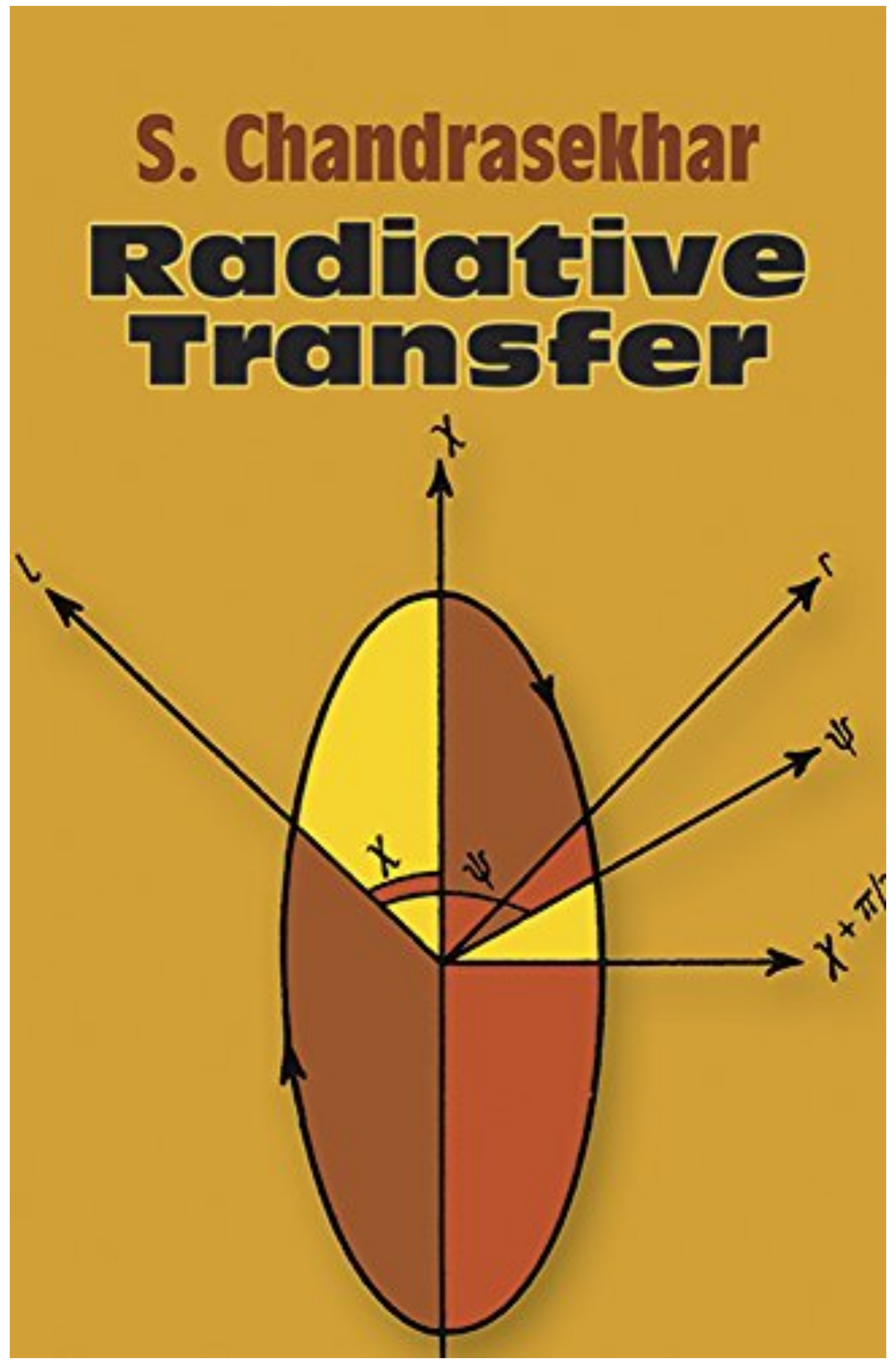
  - For the Rayleigh scattering:

$$\chi(\mu) = \frac{3}{4}\varpi\left(1 + \cos^2\theta\right)$$

  - Etc
- For non-isotropic scattering, we have a system of integral equations which are non-linear, non-homogeneous, and of high degree.

Chandrasekhar Theory

- The seminal work of Subramanyan Chandrasekhar on radiation transfer was published in 1950, the result from this work from 1944-1948.
- The first solutions to the radiation transfer equation divided the radiation field into an outward and an inward stream of intensities. Chandrasekhar divided it into 2n streams.
- He solved the radiation transfer equation for **semi-infinite** (from $\tau=0$ to $\tau=\infty$) and **finite** diffusers (in which case we have also transmittance). He also considered light polarisation.
- Here, I will present the main results for **isotropic scattering** but his book also contains the results for the Rayleigh scattering and $(1+\cos\theta)$ phase function.



Chandrasekhar Theory - Isotropic Scattering

- The diffuse-law for isotropic scattering is given by ($\mu_i=\cos(\theta i)$ and $\mu_r=\cos(\theta_r)$):

$$R\left(\mu_i;\mu_r\right)=\frac{\varpi}{4}\frac{1}{\mu_i+\mu_r}H\left(\mu_i,\omega\right)H\left(\mu_r,\omega\right)$$

- Where H are the infamous functions defined as:

$$H\left(\mu\right)=1+\varpi\mu H\left(\mu\right)\int_0^1\frac{1}{\mu+\mu'}H\left(\mu'\right)d\mu'$$

- There are analytic approximations!

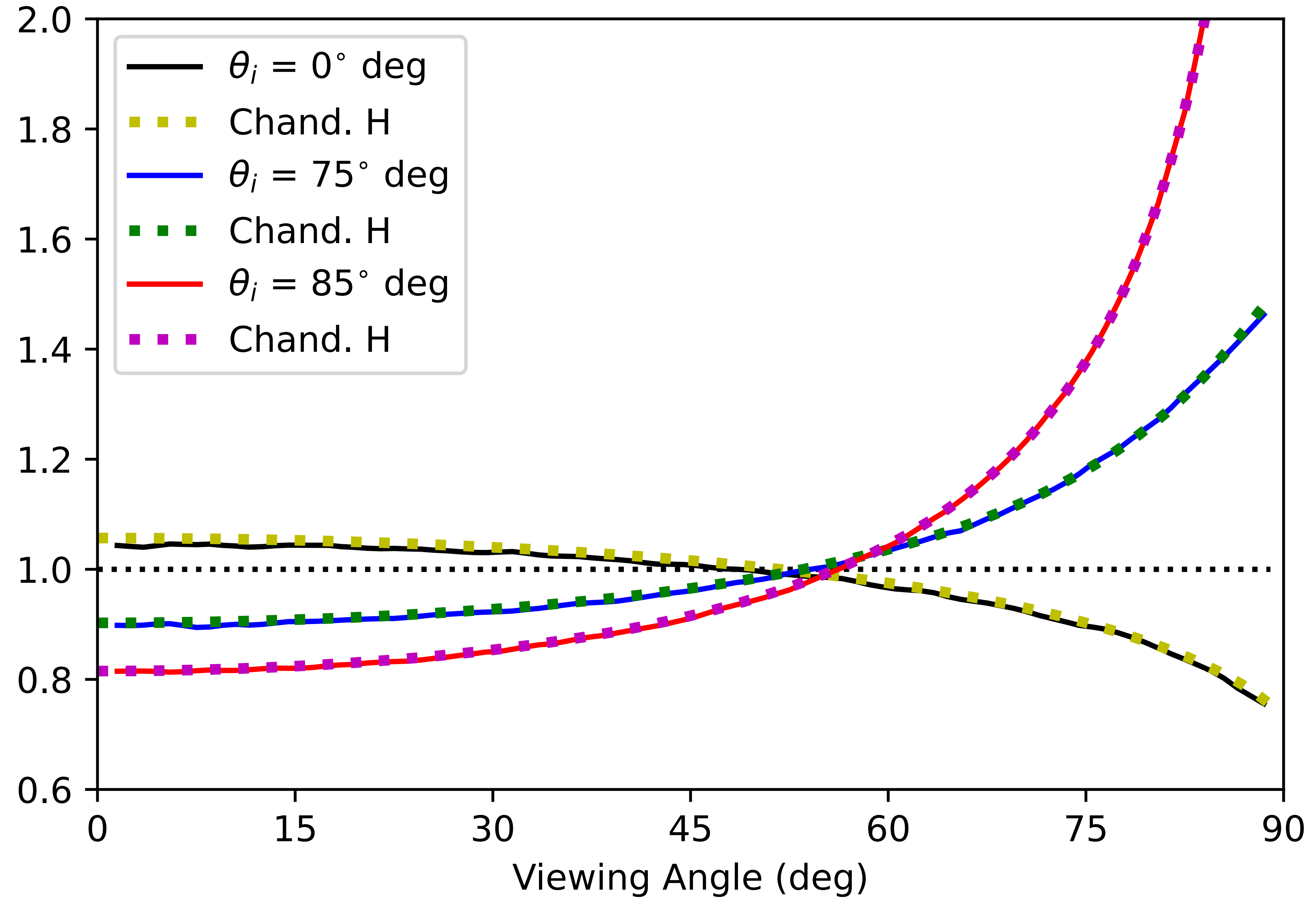
$$H\left(\mu,\varpi\right)=\frac{1+2\mu}{1-2\mu\left(\sqrt{1-\varpi}\right)}$$

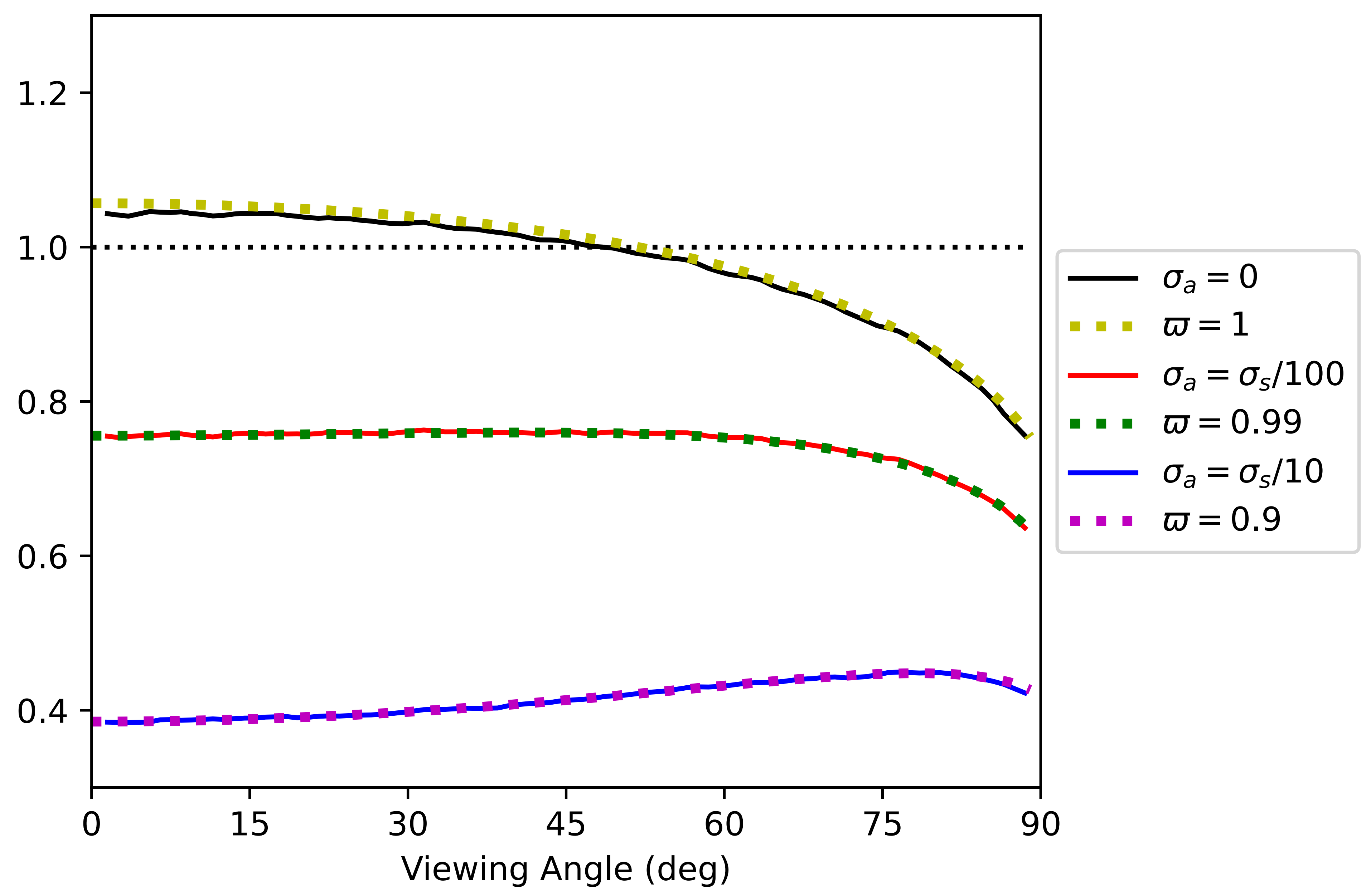
- We have the following property:

$$R\left(\mu_i;2\pi\right)=1-H(\mu_i)\sqrt{(1-\varpi)}$$

- **This is the integrated reflectance over the hemisphere - it is something that we can measure!**

Results from the Chandrasekhar Theory





- There is a small difference, especially observed in the conservative case and for normal incidence. This is actually due to the light being transmitted. The slab simulated has a finite thickness because a infinite thickness would lead to an infinite simulation.

Radiative Transfer - Transmission

- S. Chandrasekhar also included the transmission in his treatise (finite diffusers).
- If we assume isotropic scattering, transmission and reflection from a finite slab is determined from the single-scatter albedo ϖ and optical depth τ.
- The reflectance and transmittance are given by:

$$\begin{aligned} R(\mu_i, \mu_r) &= \frac{\varpi}{4} \frac{\mu_i}{\mu_i + \mu_r} \left[X(\mu_i) X(\mu_r) - Y(\mu_i) Y(\mu_r)\right] \\ T(\mu_i, \mu_t) &= \delta(\mu_i - \mu_t) e^{-\tau/\mu_i} + \frac{\varpi}{4} \frac{\mu_i}{\mu_t - \mu_i} \left[Y(\mu_t) X(\mu_i) - X(\mu_t) Y(\mu_i)\right] \end{aligned}$$

- Where the X and Y functions are given by:

$$\begin{aligned} X(\mu) &= 1 + \frac{\varpi}{2} \mu \int_0^1 \frac{1}{\mu + \mu'} \left[X(\mu) X(\mu') - Y(\mu) Y(\mu')\right] \\ Y(\mu) &= e^{-\tau/\mu} + \frac{\varpi}{2} \mu \int_0^1 \frac{1}{\mu - \mu'} \left[Y(\mu) X(\mu') - X(\mu) Y(\mu')\right] \end{aligned}$$

- As expected, when τ approaches the infinite X->H and Y->0.
- The calculation of the X and Y are an *interesting* mathematical problem. There are no known analytical approaches. Also Y has a zero for μ = μ'.
- Also, to get the hemispherical reflectances we need to integrate this over the hemisphere.

Radiative Transfer

- We also created sims for transmission for different slabs defined as:

$$s = d \cdot \alpha_{\text{scat}}$$

  - Where d corresponds to the thickness in units of length.

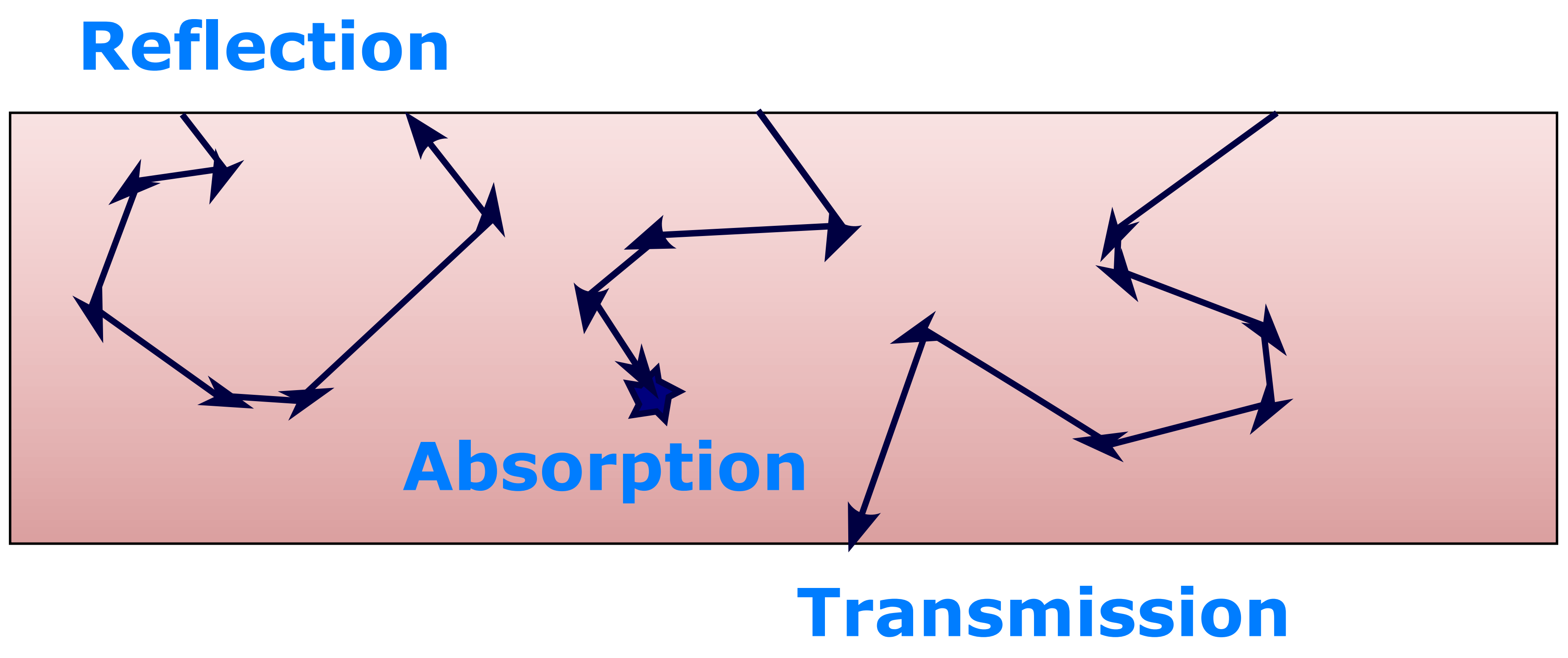
- The optical depth is given by:

$$\tau = d \cdot (\alpha_{\text{scat}} + \alpha_{\text{abs}})$$

- We calculated the reflection and transmission using the X and Y functions, for different values of ϖ and τ. This is difficult and not efficient to be used in simulations.

- For the conservative case (ϖ=1), we have the

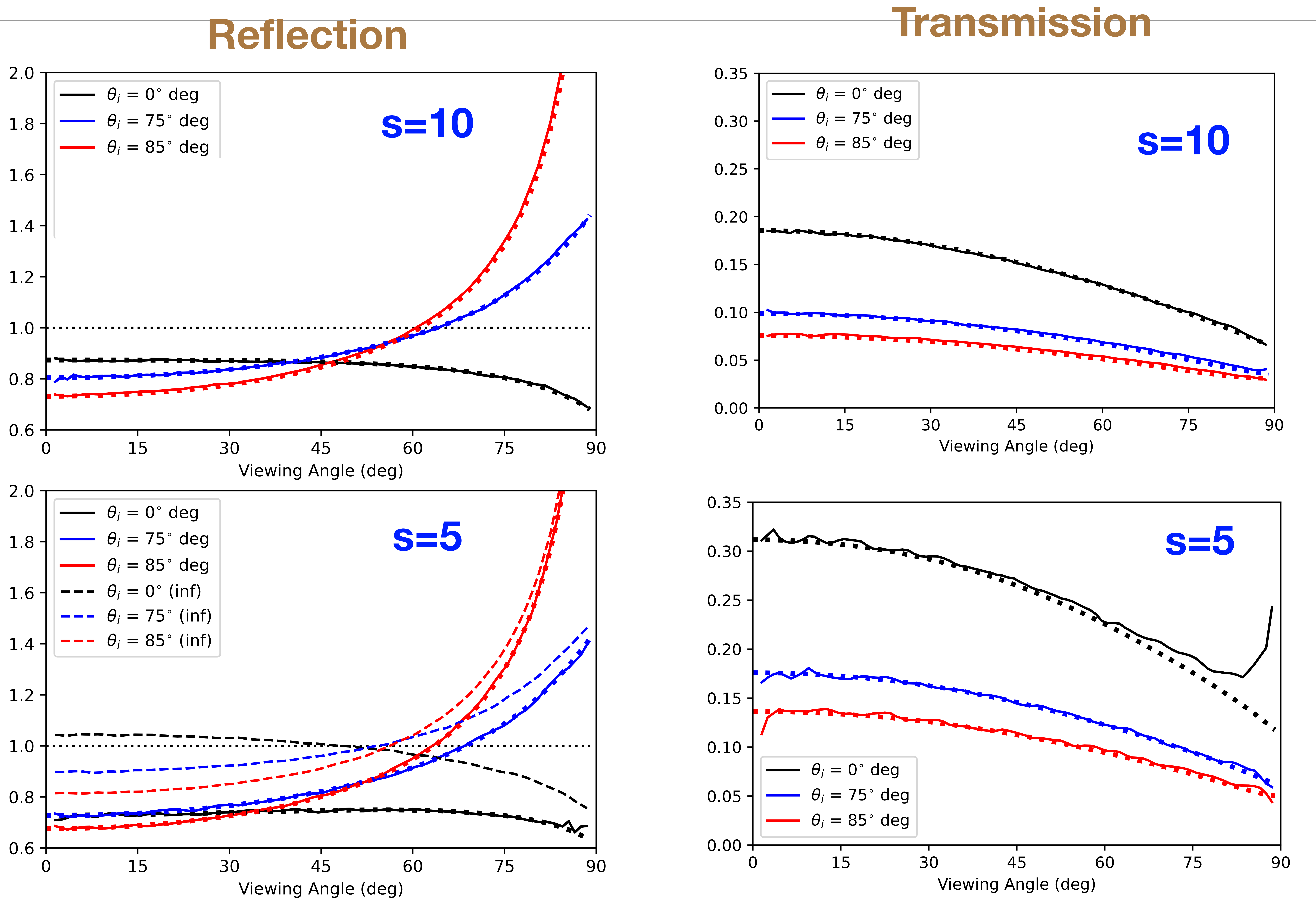
$$X \simeq \left(1 - \frac{1}{\tau + 1.42}\right) H \qquad Y \simeq \frac{1}{\tau + 1.42} H$$



Radiative Transfer - Results from Sims

Dotted lines are the predictions from the Chandrasekhar theory for a finite (X and Y functions) assuming isotropic scattering

Dashed lines are the predictions from the Chandrasekhar theory for a semi-infinite diffuser (s=∞)



Diffuse Lobe - Effect of the Interface

- The light has to be first refracted to the diffuser:

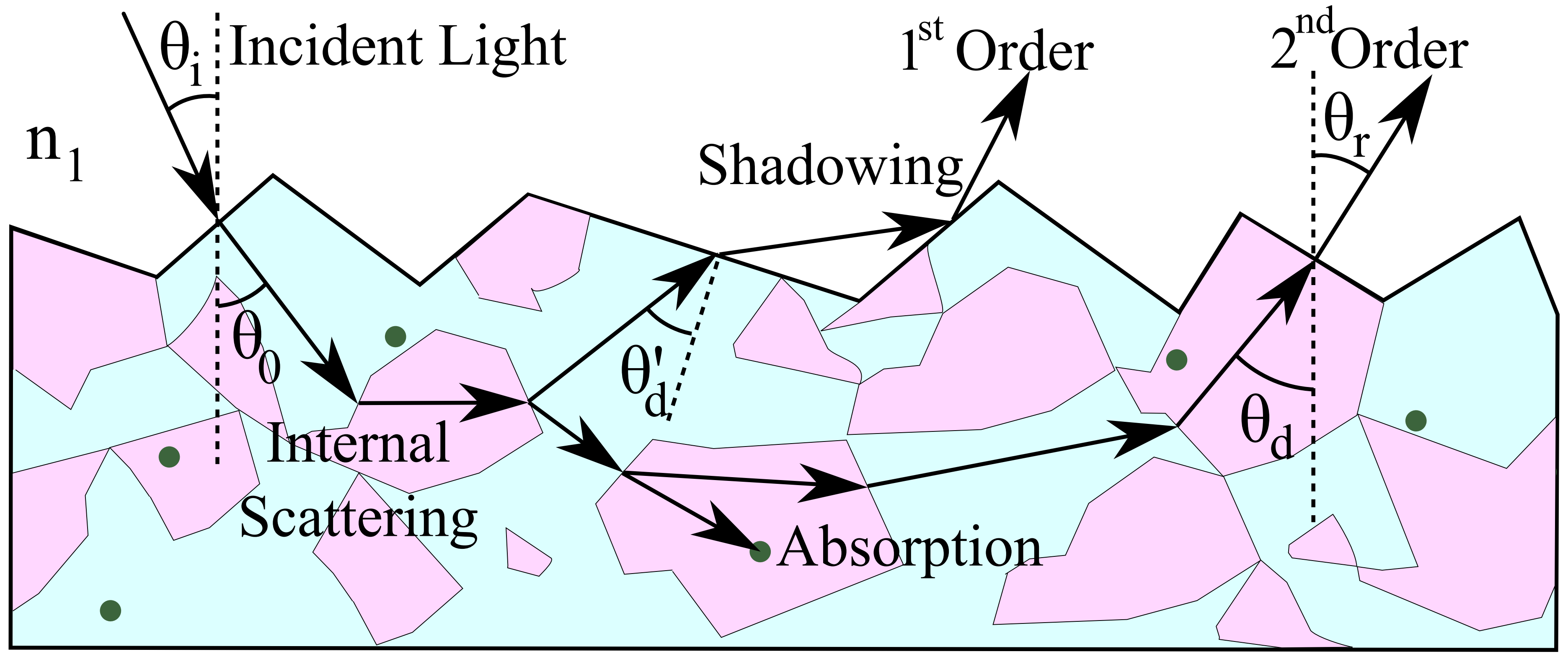
$$1 - F\left(n;\theta_i\right)$$

- And refracted from the diffuser:

$$1 - F\left(n;\theta_r\right)$$

F is the probability of reflection

Calculated with the Fresnel Equations



- The light reflected has additional additional scattering, where it can be absorbed or not. Assuming the **Lambertian model — R = ρ_m** in which $\boldsymbol{\rho_m}$ corresponds to the probability that the light *survives* in each multiple scattering process, we have:

$$R = \left[1 - F\left(n;\theta_i\right)\right] \cdot \left\{\rho_m + \rho_m\left[\rho_m F\left(n;\theta'\right)\right] + \rho_m\left[\rho_m F\left(n;\theta'\right)\right]^2 + ...\right\} \cdot \left[1 - F\left(n;\theta_r\right)\right]$$

1st order **2nd order** **3rd order**

- Which results in:

$$R = \frac{\rho_m}{1 - \bar{F}(n)\rho_m}\left[1 - F\left(n;\theta_i\right)\right] \cdot \left[1 - F\left(n;\theta_r\right)\right]$$

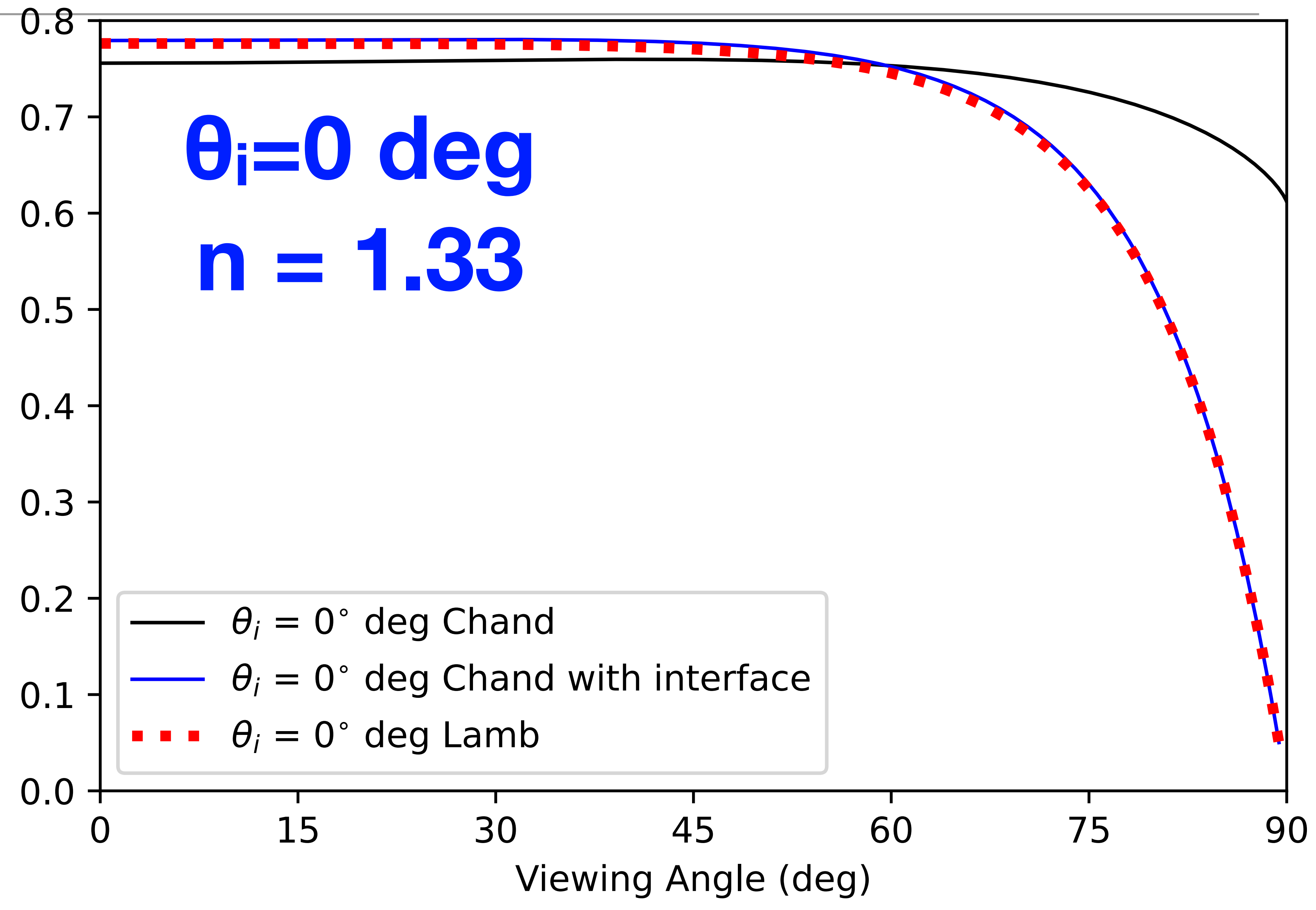
With $$\bar{F}(n) = 2\pi\int_0^{\pi/2} F\left(n,\theta'\right)\cos\theta'\sin\theta' d\theta' d\phi'$$

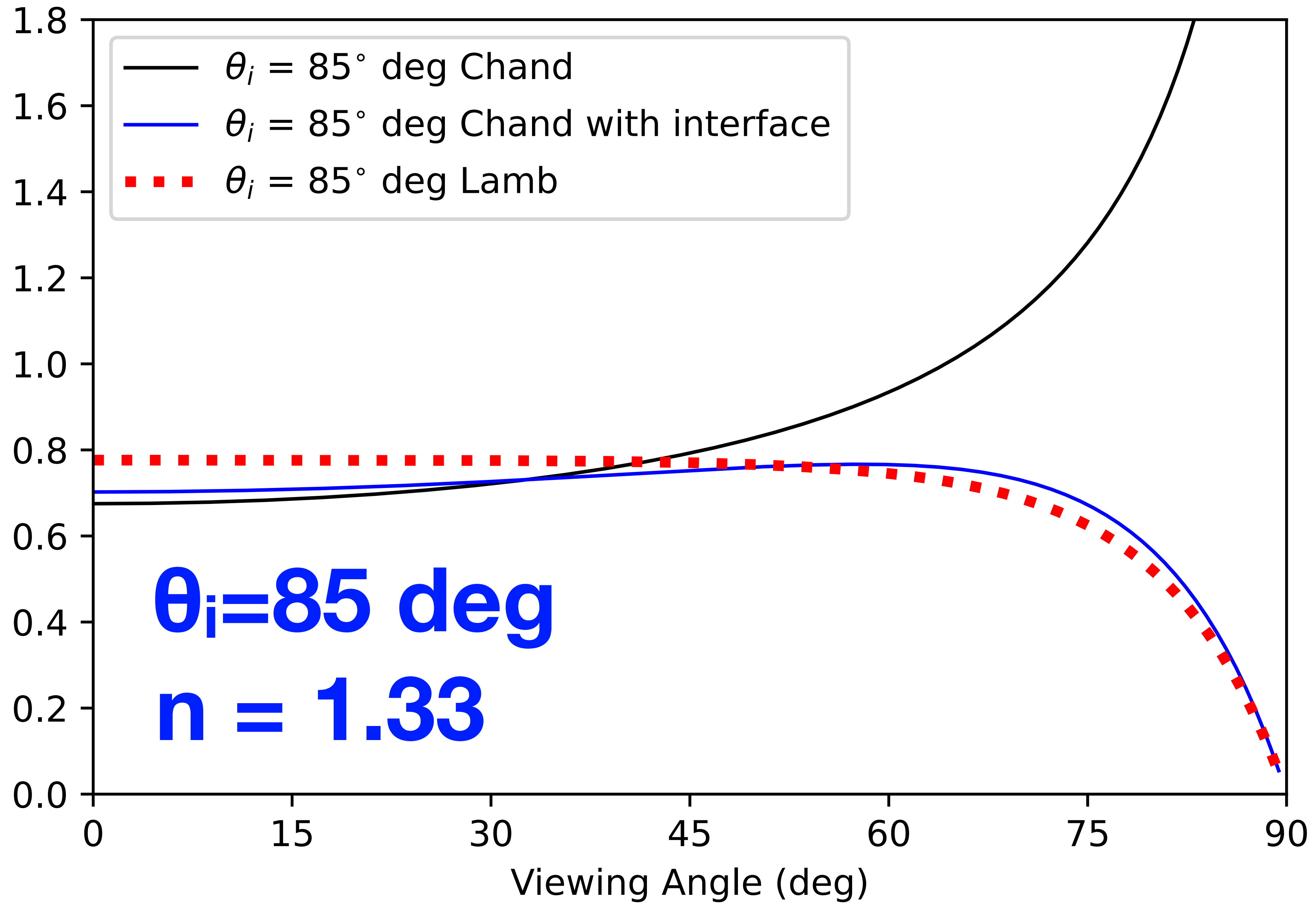
Diffuse Lobe - Effect of the Interface - What this Means?

- The effect of the interface was included in both the Chandrasekhar model and the Lambertian model.
- In many situations the Lambertian model is sufficient to describe the diffuse reflectance, when:
  - The relative refractive index is larger than 1;
  - The angle of incidence is small;
- What happens when the first medium where the photon originally travels changes?

$$R = \frac{\rho_m}{1 - \bar{F}(n)\rho_m}\left[1 - F\left(n;\theta_i\right)\right] \cdot \left[1 - F\left(n;\theta_r\right)\right]$$

  - When a diffuser is immersed in a liquid, the **reflectance should increase** because F(n) approaches zero.



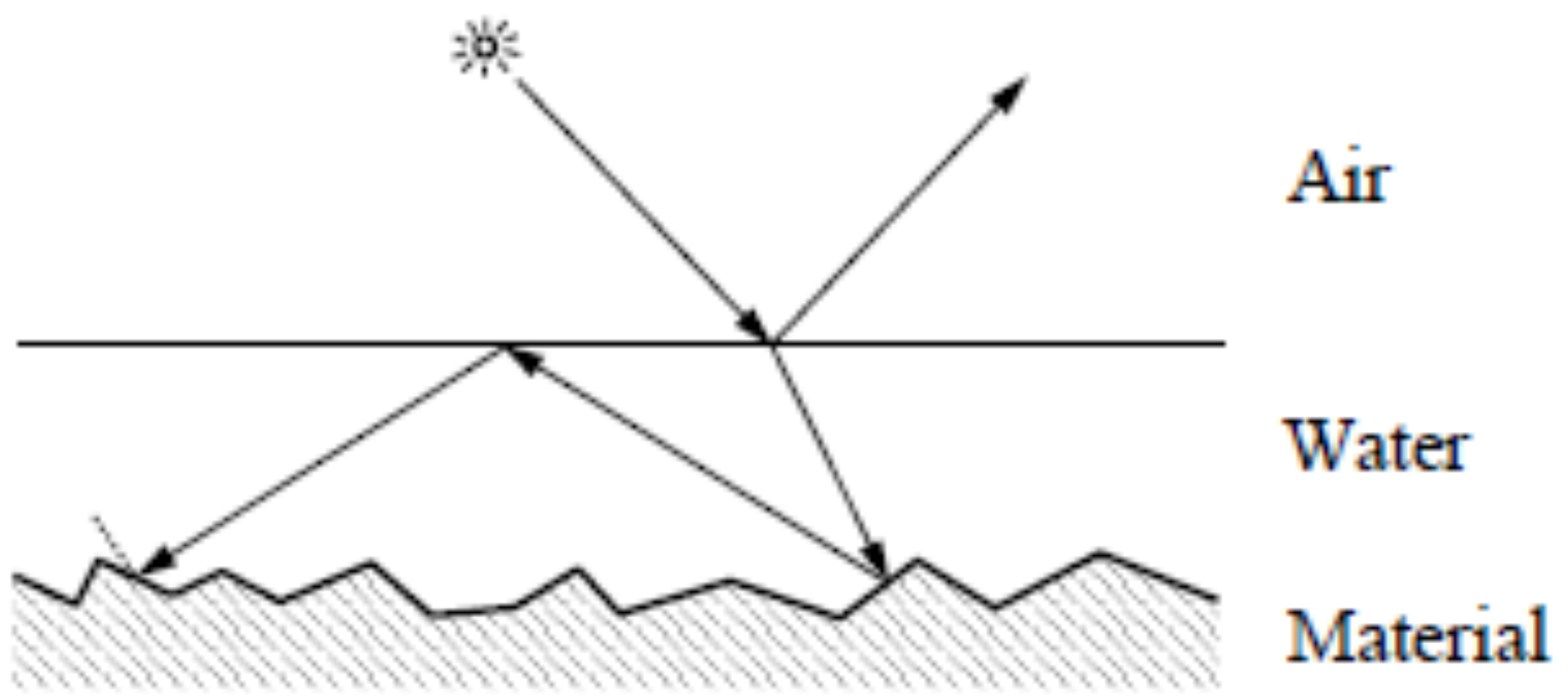


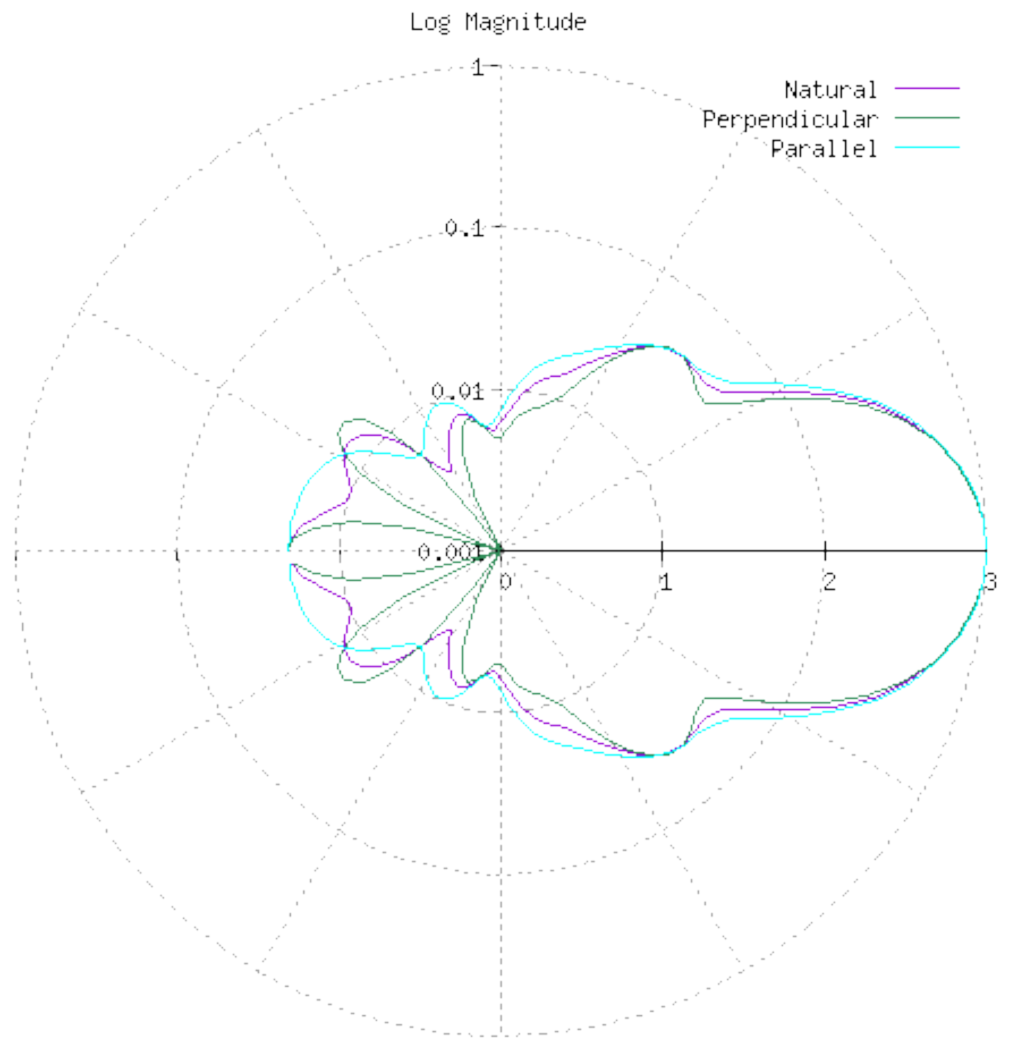
Diffuse Lobe - Effect of the Interface

But wet surfaces look darker!

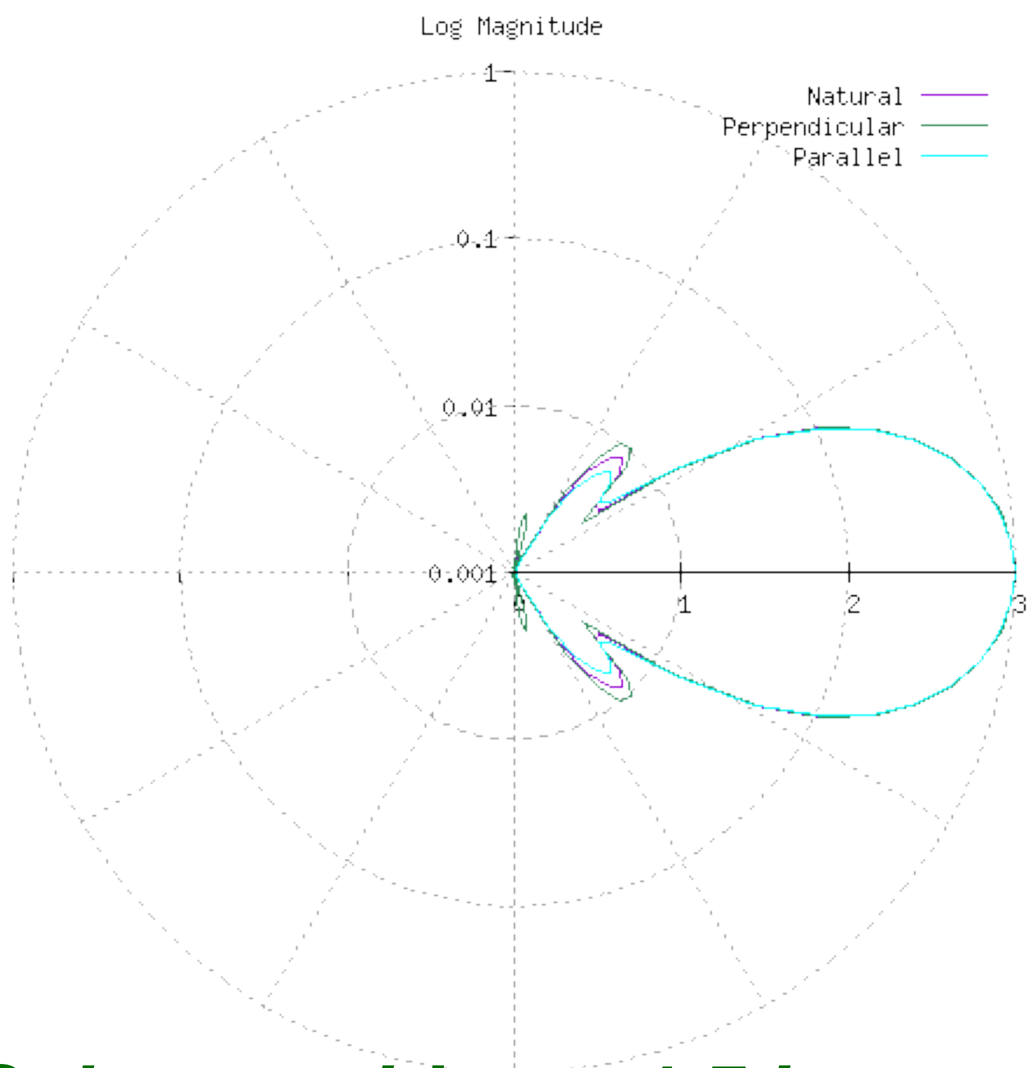
Diffuse Lobe - Wet Surfaces

- We do not observe the surface directly in the liquid interface. As such, we have to consider the refraction from the liquid to the air (and the respective multiple reflections)
- For some surfaces, such as soil and paper, the material absorbs the liquid, changing the value of ρ_m.

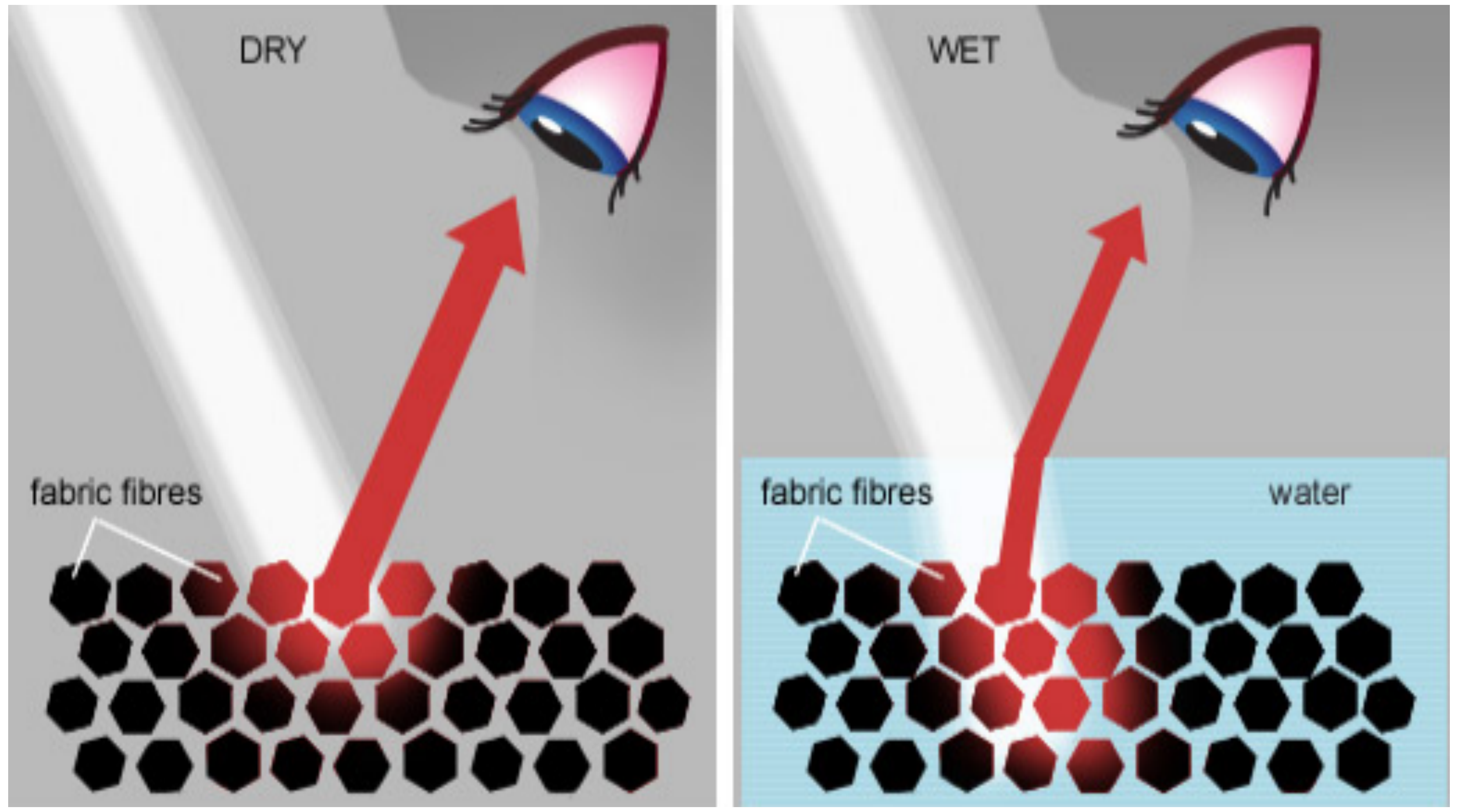




Spheres with n = 1.5 in a medium with n = 1.0



Spheres with n = 1.5 in a medium with n = 1.33



Computer Vision

- Some models used in the computer vision are not physically based. For example, one of the most used models is the Phong model:

$$I = k_d \cos\theta + k_s \left(\cos\alpha\right)^{n'}$$

- Where k_d controls the diffuse reflection and k_s controls the specular reflection.
- It might be worth to adapt some of the models that we have discussed to computer vision.

Conclusions:

- The reflectance has two main components - diffuse and specular.
- To describe the specular reflectance, we have to understand how the light scatters in a rough surface.
- We described the reflectance and transmittance of a diffuser using the radiative transfer model as described by the Chandrasekhar in 1950 and we compared it with the Monte-Carlo simulations.
- We added the effect of the interface in the diffuse reflection (Saunders correction).
- This reflectance model is partially available in ANTS2 (specular and diffuse reflectance of semi-infinite diffusers).

Future Work

- Find appropriated approximations to the functions X and Y (almost done);
- Calculate the inverse cumulative functions of the Chandrasekhar function H;
- Implement the reflectance and transmittance models for finite diffusers in ANTS3;
- Systematisation of the form of the specular lobe for different types and levels of roughness;
- Add the dependence with the wavelength in the reflectance model (currently the reflectance model is purely geometric).
- You are welcome to collaborate in this work!