TÉCNICO LISBOA

A Toy Monte Carlo Model for the Simulation of Extensive Proton Air Showers

Astroparticle Multi-Messengers

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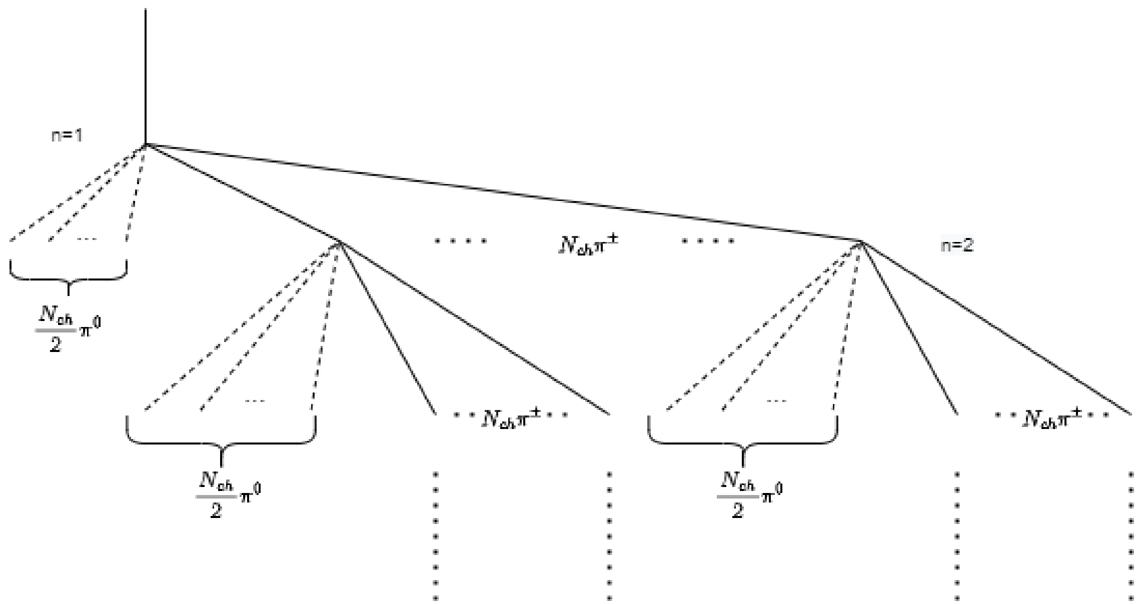
Matthews Model for Hadronic Cascades

- Inspired by Heitler Model for electromagnetic cascades
- Layers of atmosphere with thickness $\lambda_I \ln(2)$
- After travelling this distance, hadrons interact, producing $N_{ch}\pi^{\pm}$ and $\frac{N_{ch}}{2}\ \pi^0$
- Only $\pi^{\pm}$'s give rise to new cascades, while π^0's decay to photons, producing EM cascades
- Energy equally distributed for all the particles in an interaction
- When the energy of $\pi^{\pm}$ falls below a critical energy, E_c, the $\pi^{\pm}$'s decay to muons



Toy Monte Carlo and its objectives: how the work was divided

1. **Toy Monte Carlo results under the same conditions of Matthews model and comparison with the analytical expressions from the model**
   - ➢ Number of muons and maximum shower depth as a function of energy
   - ➢ Evolution of the number of muons with the multiplicity
   - ➢ Evolution of the number of muons with p, the fraction of energy that goes into hadronic showers in each interaction (generalization of Matthews model for $p \neq 2/3$)

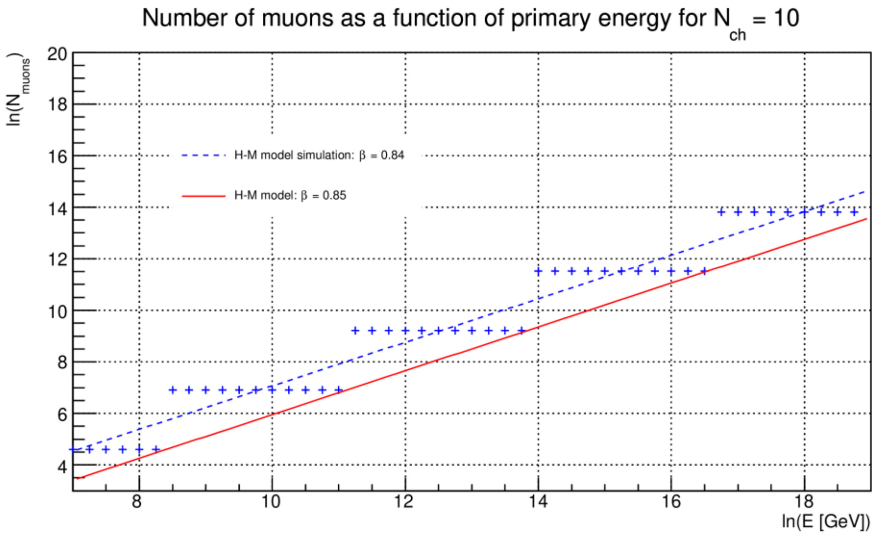
2. **Introduction of stochasticity: relaxing of some conditions of Matthews model**

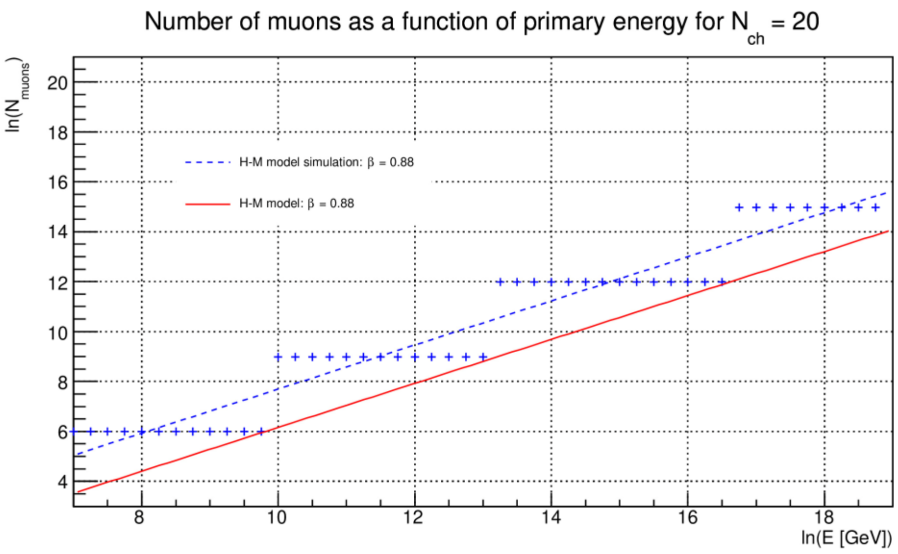
- Multiplicity distribution
- Comparison with Matthews model
  - ➢ Evolution of number of muons with primary energy
- Try to understand the correlation between the number of muons and the maximum depth of the shower
  - ➢ Study of the correlation of the number of muons and maximum depth of the shower with several parameters

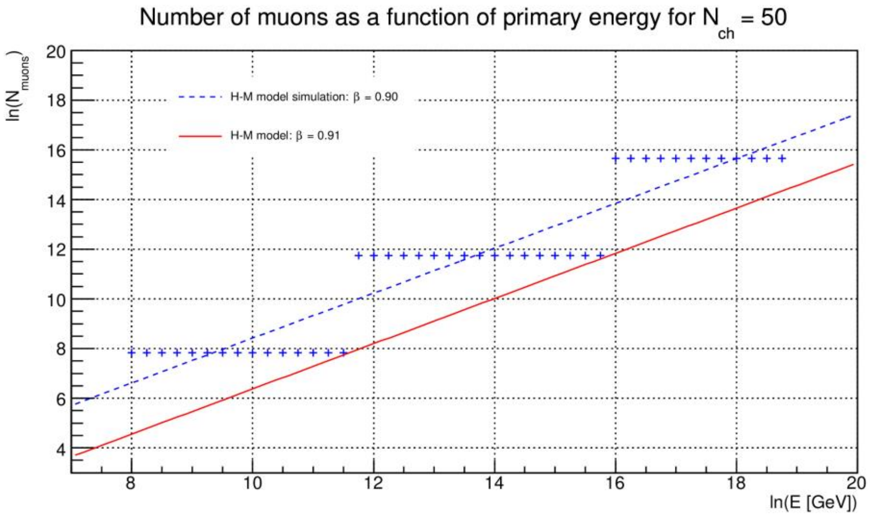
II. Toy Monte Carlo under Matthews conditions

Evolution of the number of muons with primary energy

Matthews model expression: $\ln N_\mu = \beta \ln\left(\frac{E_0}{E_c}\right)$, where $\beta = \frac{\ln(N_{ch})}{\ln(1.5 N_{ch})}$





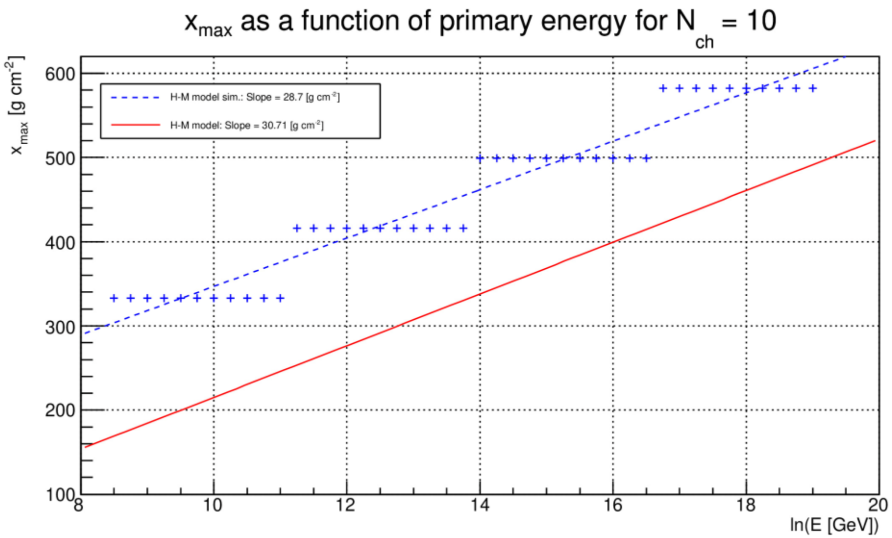


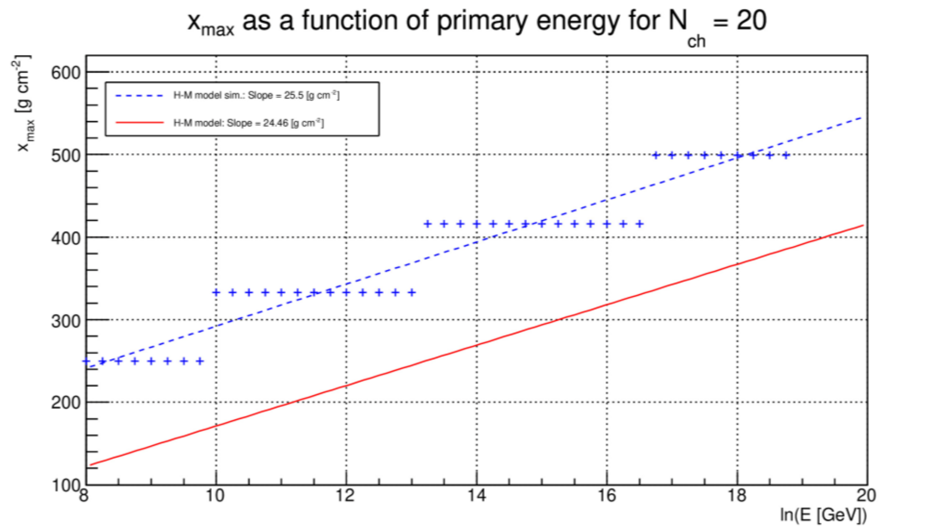
- Absolute values don´t coincide with the analytical Matthews model due to the step-character of the simulation
- Evolution with energy in agreement with Matthews model for several multiplicities

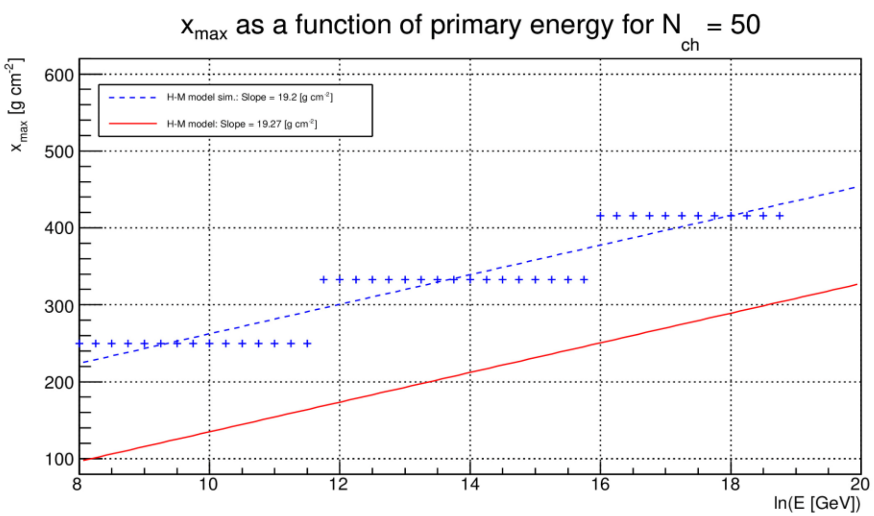
II. Toy Monte Carlo under Matthews conditions

Evolution of the maximum shower depth with primary energy

Matthews model expression: $x_{max} = dn_c = \frac{d}{\ln(1.5N_{ch})}\ln(\frac{E_0}{E_c})$ with $d = \lambda_I \ln(2) = 83.2\ gcm^{-2}$



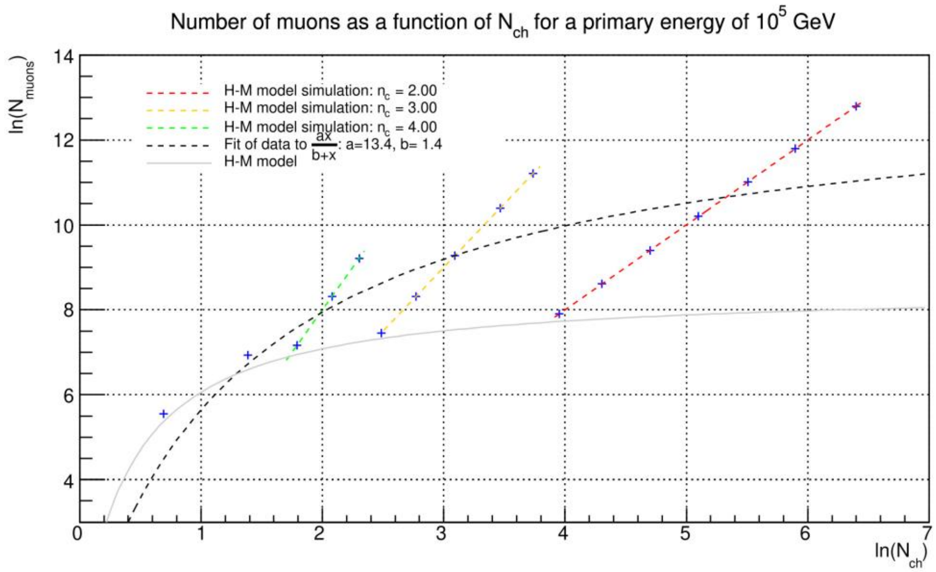




- Once again, absolute values differ from analytical Matthews model
- Evolution with energy in agreement with Matthews model for several multiplicities

Evolution of the number of muons with multiplicity

Matthews model expression: $\ln N_\mu = n_c lnN_{ch} = \ln(E_0/E_c)\frac{\ln(N_{ch})}{\ln(1.5N_{ch})}$

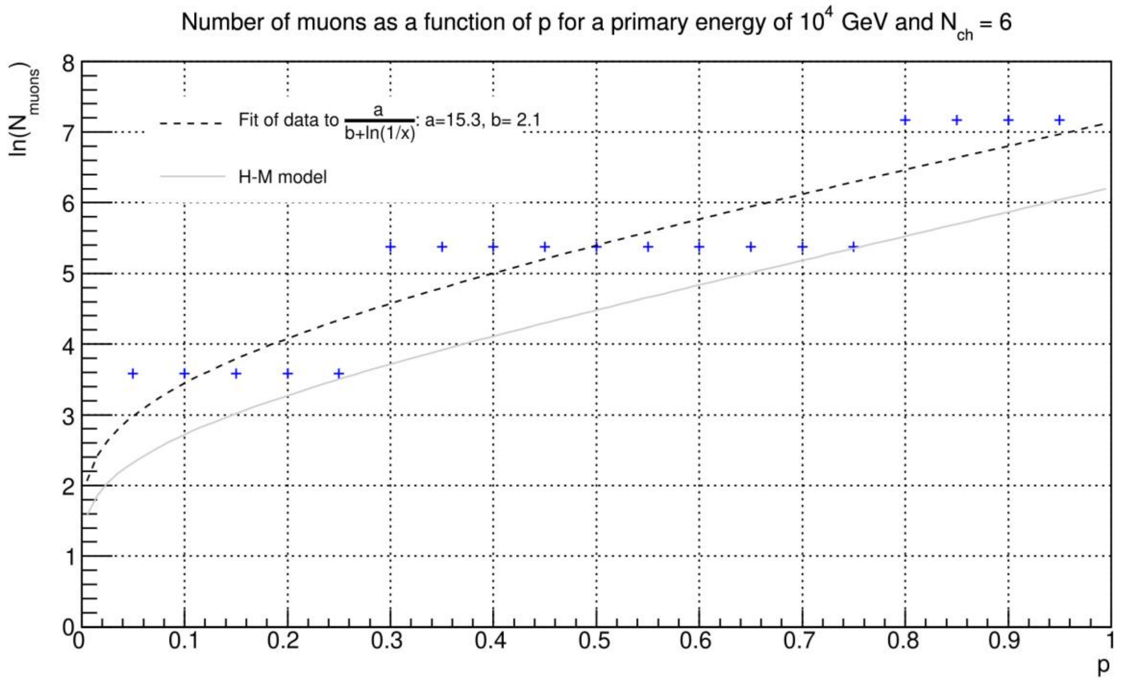


- Evolution of the number of muons with the multiplicity consistent with Matthews model

II. Toy Monte Carlo under Matthews conditions

Evolution of the number of muons with p

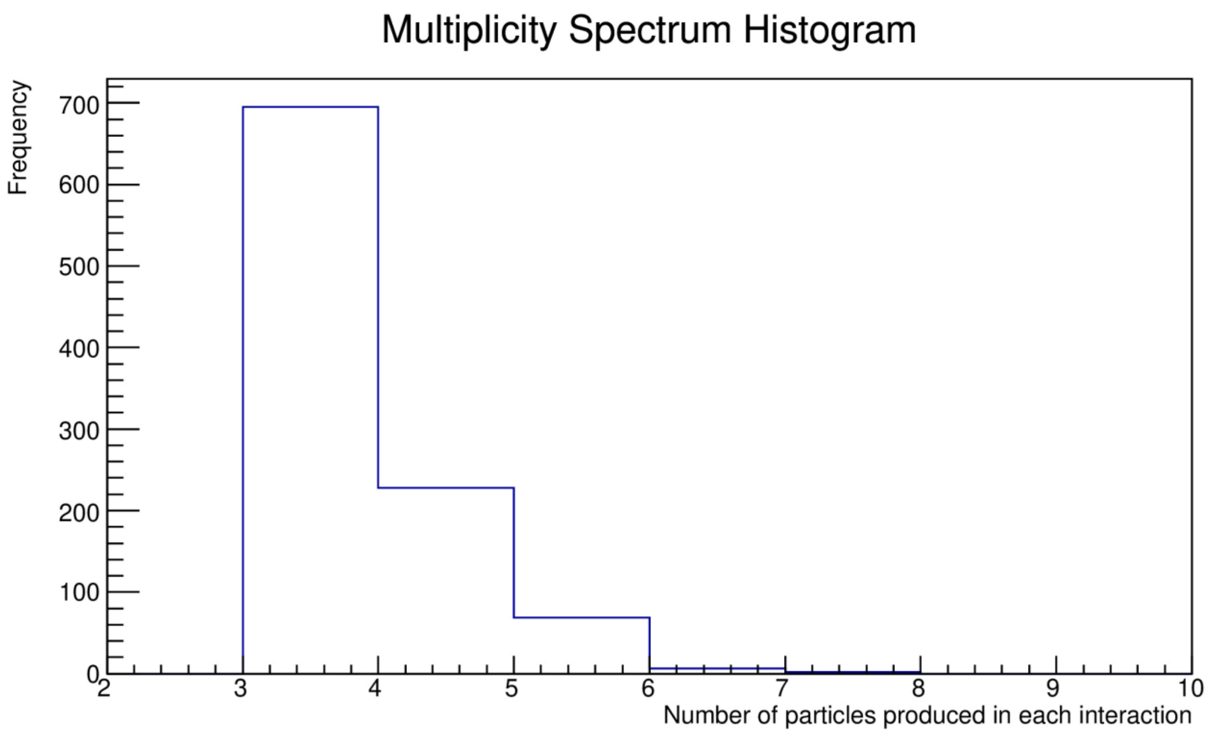
(Generalized) Matthews model expression: $\ln N_\mu = n_c ln N_{ch} = \frac{\ln(E_0/E_c)\ln(N_{ch})}{\ln(N_{ch})-\ln(p)}$



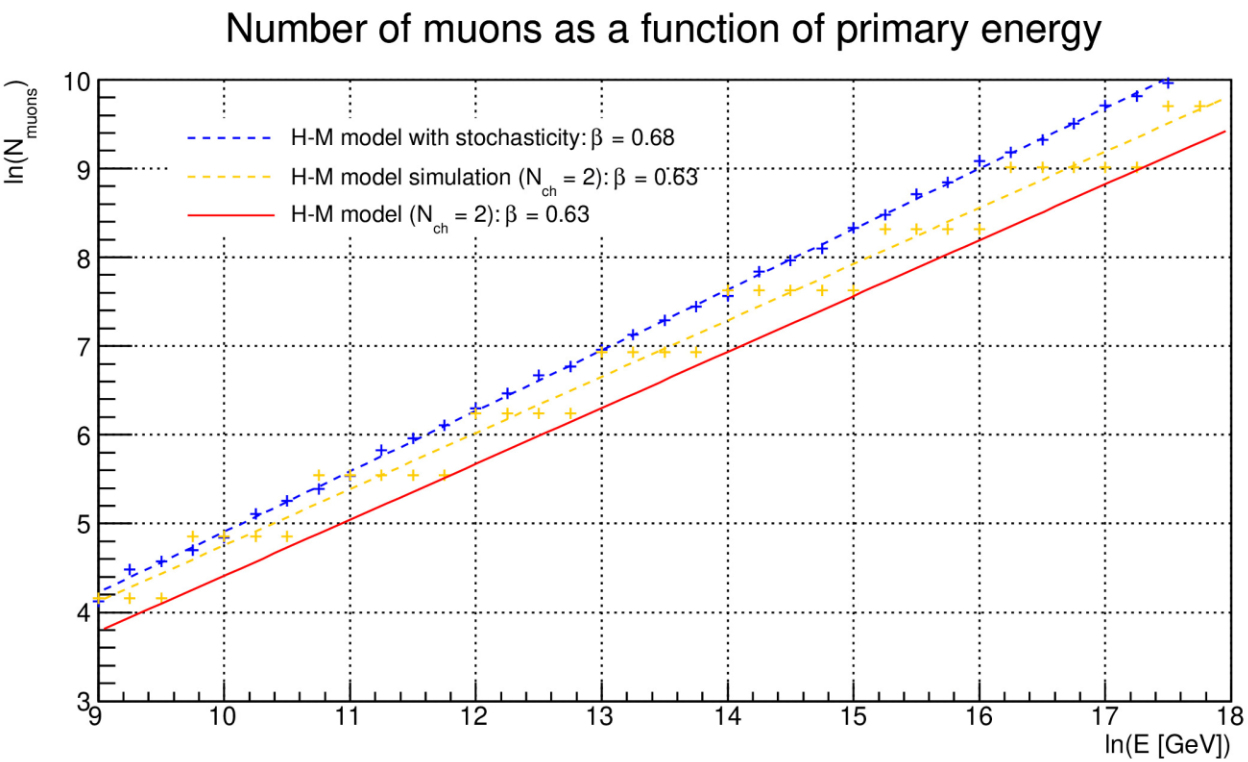
- Evolution of the number of muons with p matches the prediction from the analytical (generalized) Matthews model

Introduction

- Relaxing of equally distributed energy and constant multiplicity along the shower
  - In each interaction, energies are distributed uniformly
  - Multiplicity adjusted as the distribution of energy occurs
  - $\pi^{\pm}/\pi^{0}$ 2:1 ratio cannot always be guaranteed: increase of computational time and decrease of stochasticity
  - Choice: privilege $\pi^{\pm}$ over π^{0}



Evolution of the number of muons with primary energy

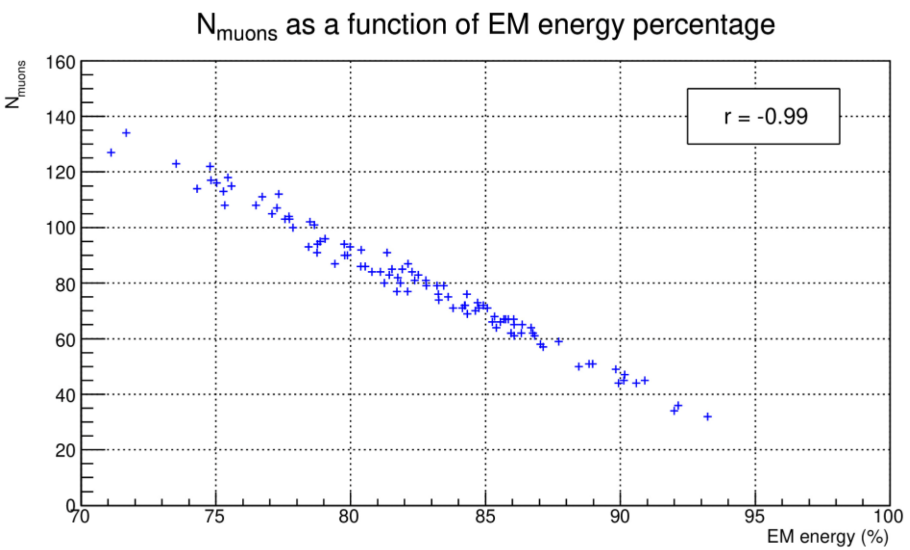


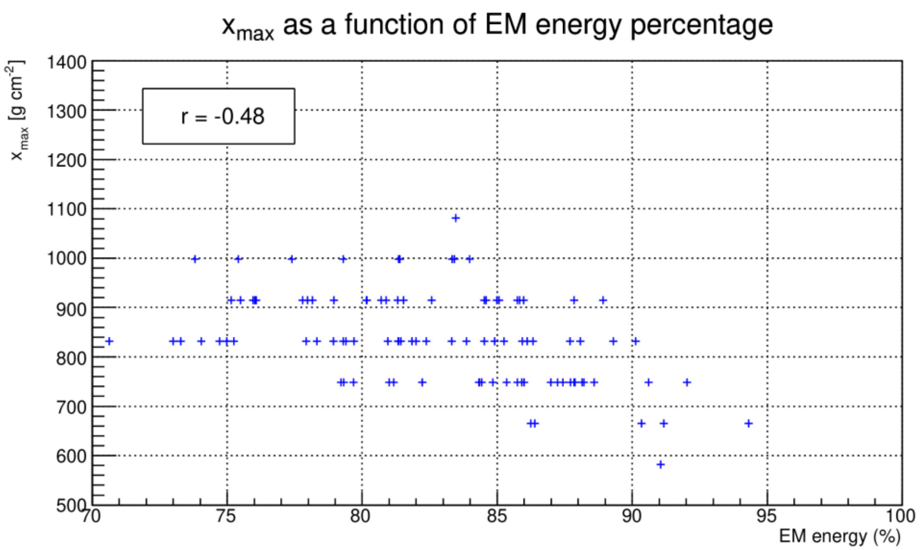
- Comparison with Matthews model for N_{ch}= 2
- Higher β mainly justified by a $\pi^{\pm}/\pi^{0}$ ratio greater than 2:1
- Evolution of number of muons with energy follows the same law as in Matthews model

III. Toy Monte Carlo with stochasticity

Sensitivity of N_{muons} and x_{max} to the energy percentage that goes into EM showers

- Expected negative correlation for both N_{muons} and x_{max}

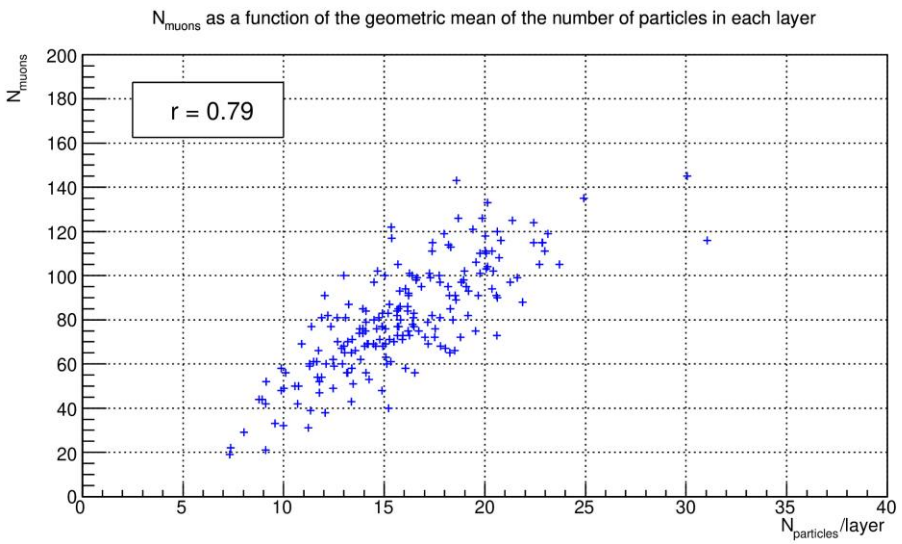


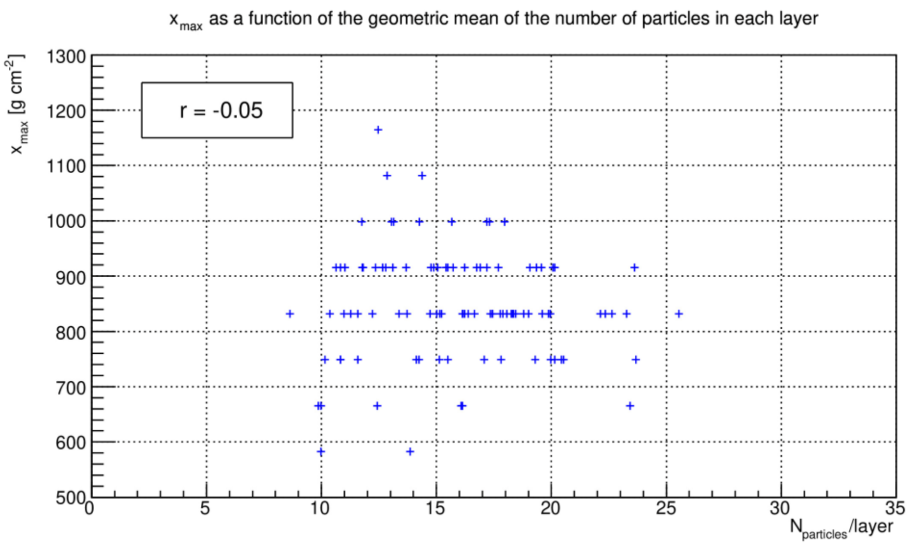


- Very strong negative linear correlation between N_{muons} and EM energy percentage
- Weaker correlation between x_{max} and EM energy percentage

Sensitivity of N_{muons} and x_{max} to multiplicity

- Multiplicity measured as the geometric mean of the number of particles per layer
- Correlation is harder to predict: a greater multiplicity increases the number of subshowers but decreases (on average) the energy per particle

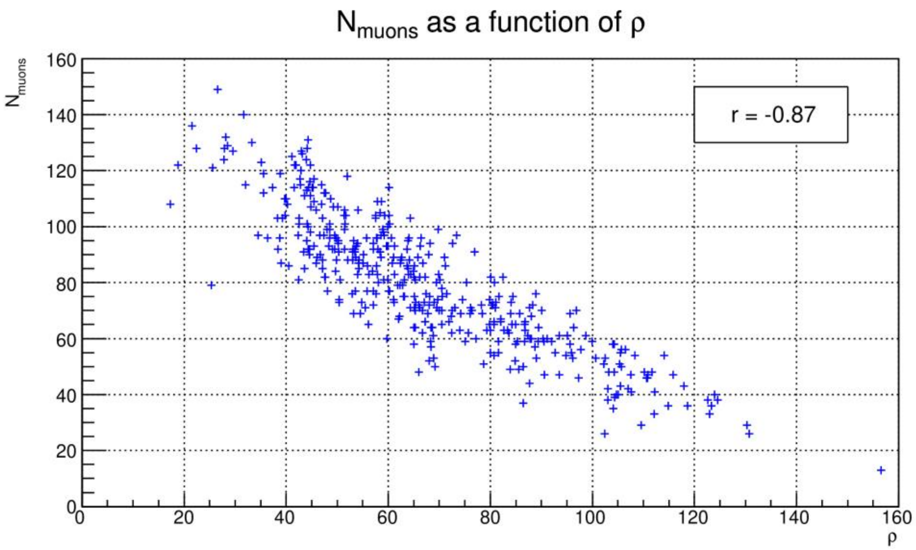


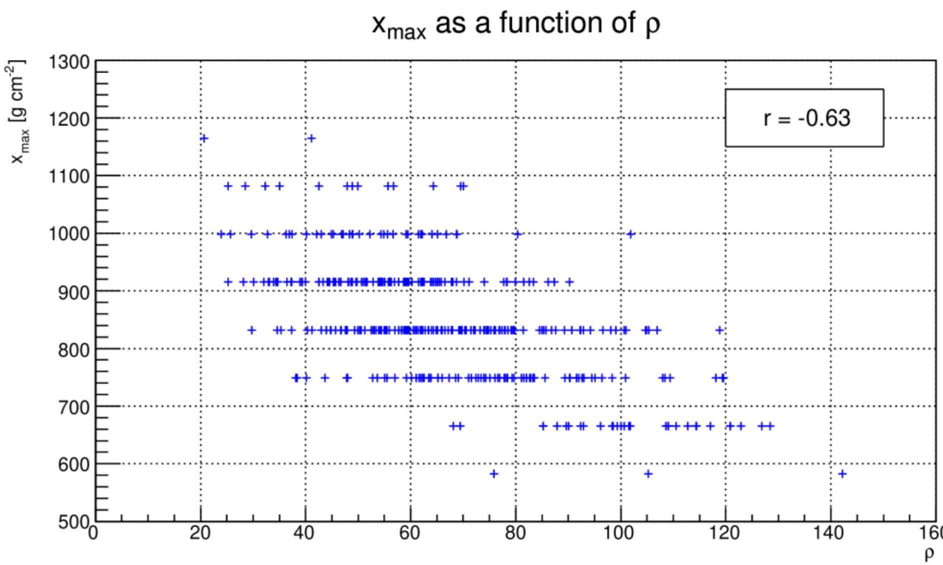


- Strong positive correlation between N_{muons} and multiplicity: despite the decrease of the average energy per particle, the increase in the number of subshowers increases N_{muons}
- No correlation between x_{max} and multiplicity (alone): find another parameter

Sensitivity of N_{muons} and x_{max} to ρ

- Basic idea: the greater the fraction of energy that goes into EM showers in the first levels, the smaller the energy available either to increase N_{muons} or x_{max}
- $\rho = \sum_{layer=1}^{4} \frac{E_{em_{layer}}(\%)}{\sqrt{layer}}$

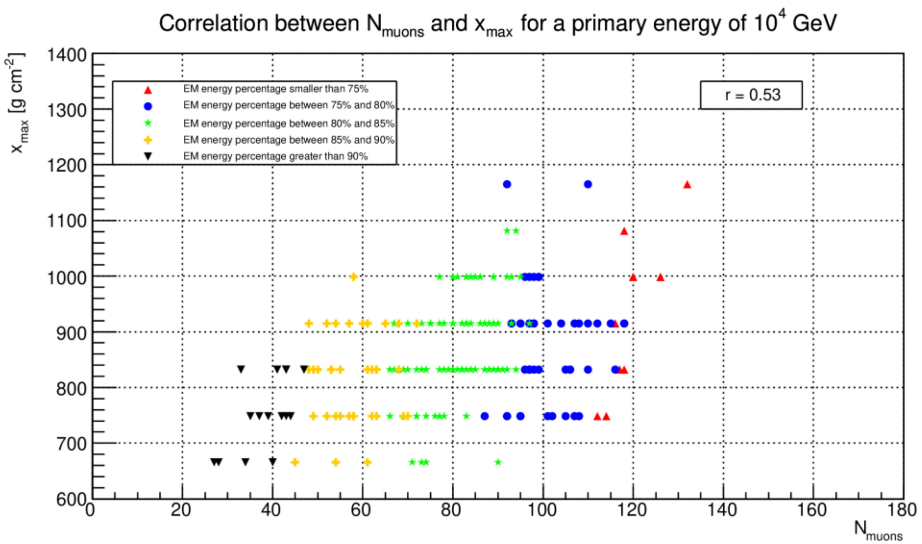


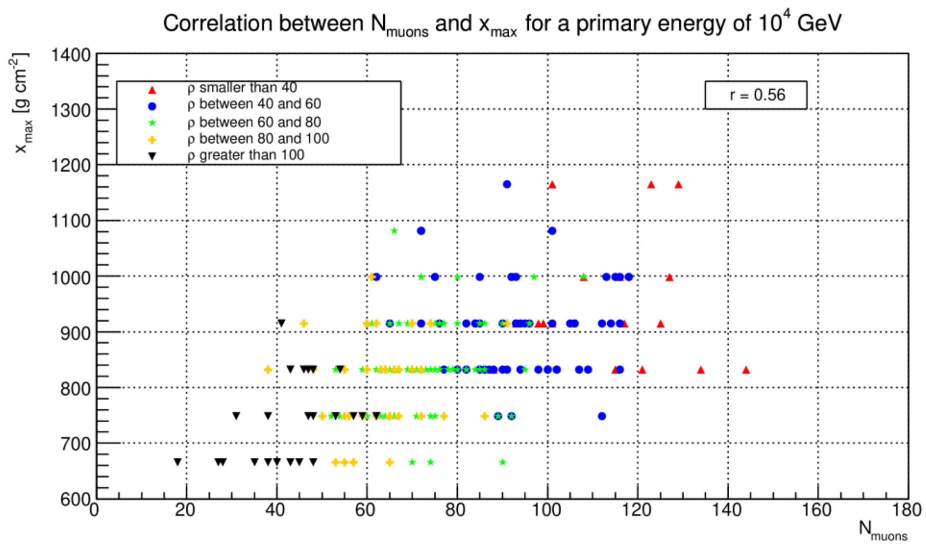


- Both N_{muons} and x_{max} show a negative correlation with ρ
- Correlation between x_{max} and ρ seems to be stronger than between x_{max} and the percentage of primary energy that goes into EM showers

Correlation between N_{muons} and x_{max}

- Use of EM energy percentage and ρ to study correlation between N_{muons} and x_{max}





- N_{muons} mainly governed by EM energy percentage, as expected
- Weak negative correlation between x_{max} and both EM energy percentage and ρ
- Weak positive correlation between N_{muons} and x_{max}

IV. Conclusions

- Matthews analytical model coincides in several aspects with the Toy Monte Carlo simulation under the model's assumptions:
  - Discrepancies attributed to the step-character of the simulation, in contrast with the continuous analytical expressions
  - Despite discrepancies in the absolute values, the evolution of several quantities with energy, multiplicity and p follows the analytical Matthews model

- Evolution of N_{muons} with energy when stochasticity is introduced follows what we expect from Matthews model

IV. Conclusions

- N_{muons} is strongly correlated with the parameters studied, mainly the EM energy percentage. Intuitive: when EM energy percentage decreases, there is more energy going into hadronic showers, producing more muons

- x_{max} is weakly correlated with the parameters studied. It seems to decrease slowly with the increase of the EM energy percentage and ρ, which matches the intuition

- N_{muons} and x_{max} were observed to be weakly and positively correlated. Despite understanding that when the EM energy percentage decreases, N_{muons} increases and x_{max} tends to increase, a deeper analysis of the variation of x_{max} with different parameters is needed