Diluting Quark Flavour Hierarchies Using Dihedral Symmetry

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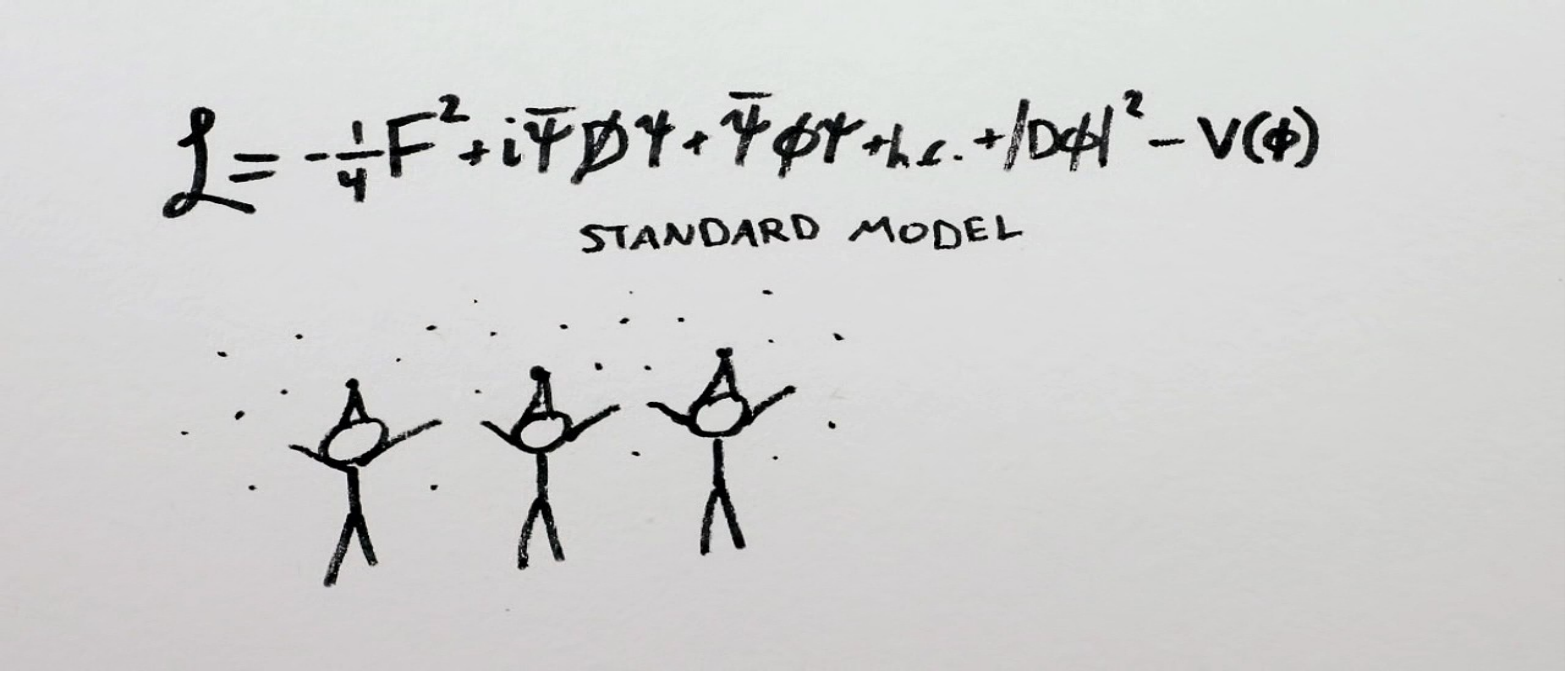
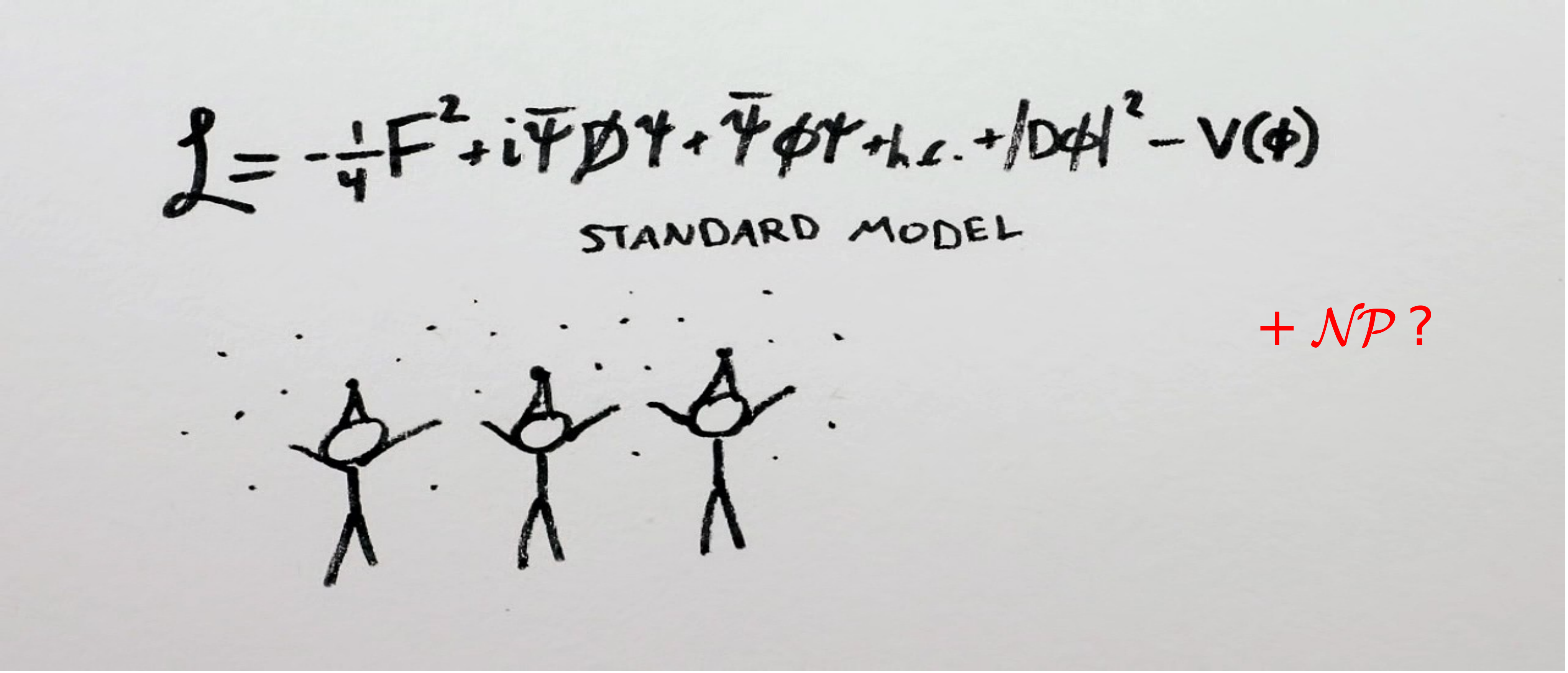
Outline

- Motivation
  - The SM and its Freedom
  - Flavour Symmetries
- Quark Flavour Hierarchies
  - Quark Masses
  - The Wolfenstein Parametrization
- D_4 Symmetry
  - For the masses
  - For the Mixings
- Summary

The SM and its Freedom



The SM and its Freedom



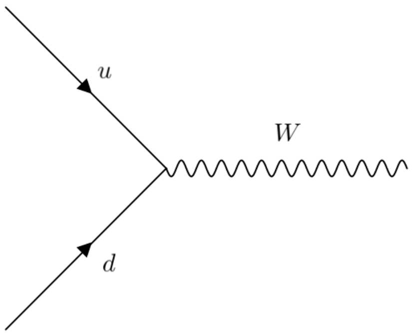
$+\,\mathcal{NP}\,?$

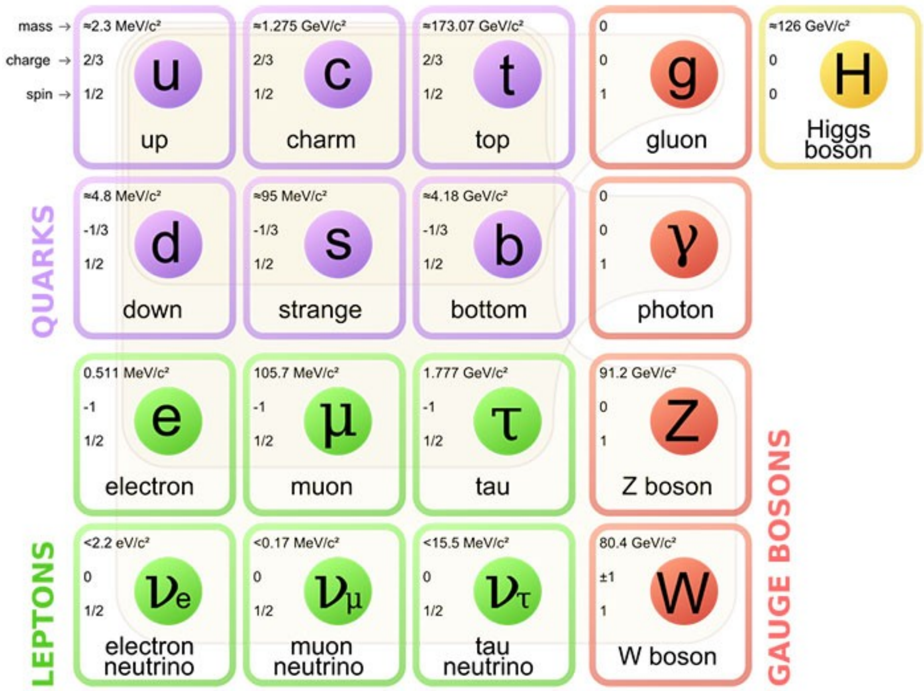
The SM and its Freedom

Flavour Symmetries

$\mathcal{Q}_L \equiv (u_L, c_L, t_L) \sim \mathbf{3}$

Gauge Symmetries





The SM and its Freedom

Free Parameters	Gauge	Flavour
SM	3+1	Masses: 3+3+3 Mixings: 3+1 Scalar: 2
SM+ν_R	3+1	Masses: 3+3+3+3 Mixings: (3+1)x2 Scalar: 2

The SM and its Freedom

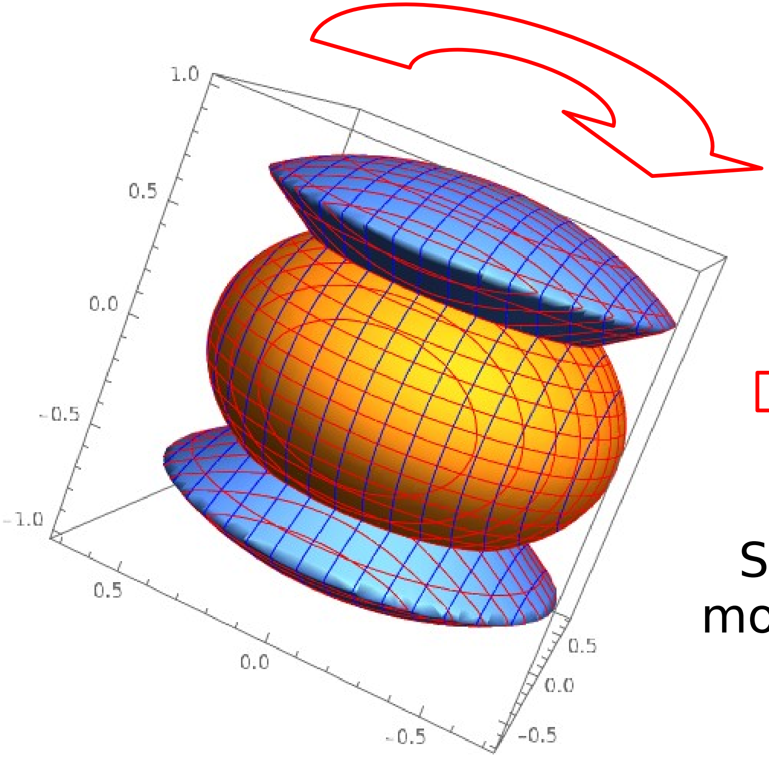
$$\mathcal{L} \supset Y_u \overline{Q}_L \tilde{\phi} u_R + Y_d \overline{Q}_L \phi d_R$$

$$M_{u,d} = vY_{u,d} \equiv \begin{pmatrix} \text{x} & \text{x} & \text{x} \\ \text{x} & \text{x} & \text{x} \\ \text{x} & \text{x} & \text{x} \end{pmatrix}$$

$$U^\dagger M M^\dagger U = \mathsf{Diag}(m_u, m_c, m_t)\,, \qquad V^\dagger M M^\dagger V = \mathsf{Diag}(m_d, m_s, m_b)$$

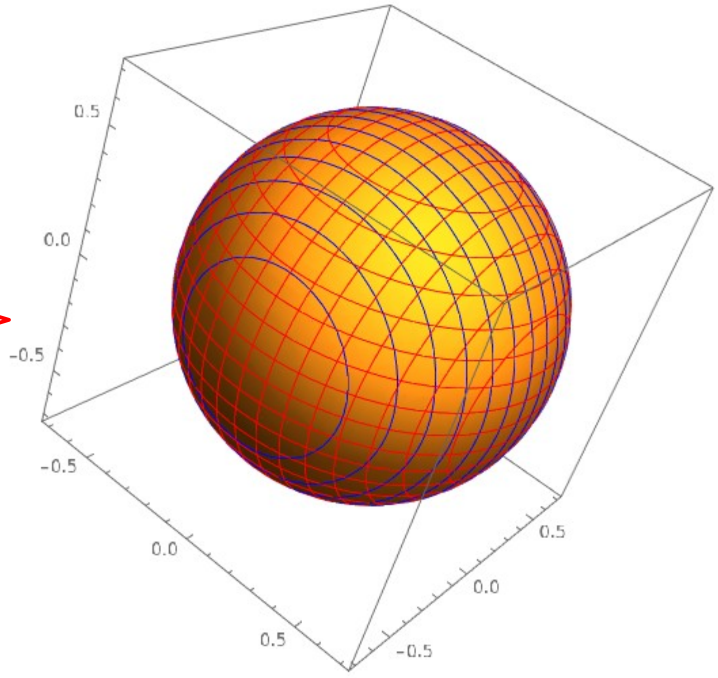
$$V_{\text{CKM}} = U^\dagger V \equiv \begin{pmatrix} \text{x} & \text{x} & \text{x} \\ \text{x} & \text{x} & \text{x} \\ \text{x} & \text{x} & \text{x} \end{pmatrix}$$

Flavour Symmetries



“SO(2)”

Enlarging the
Symmetry leads to
more restrictive cases



“SO(3)”

Flavour Symmetries

$$\mathcal{L} \supset Y_u \, \overline{Q}_L \, \tilde{\phi} \, u_R + Y_d \, \overline{Q}_L \, \phi \, d_R$$

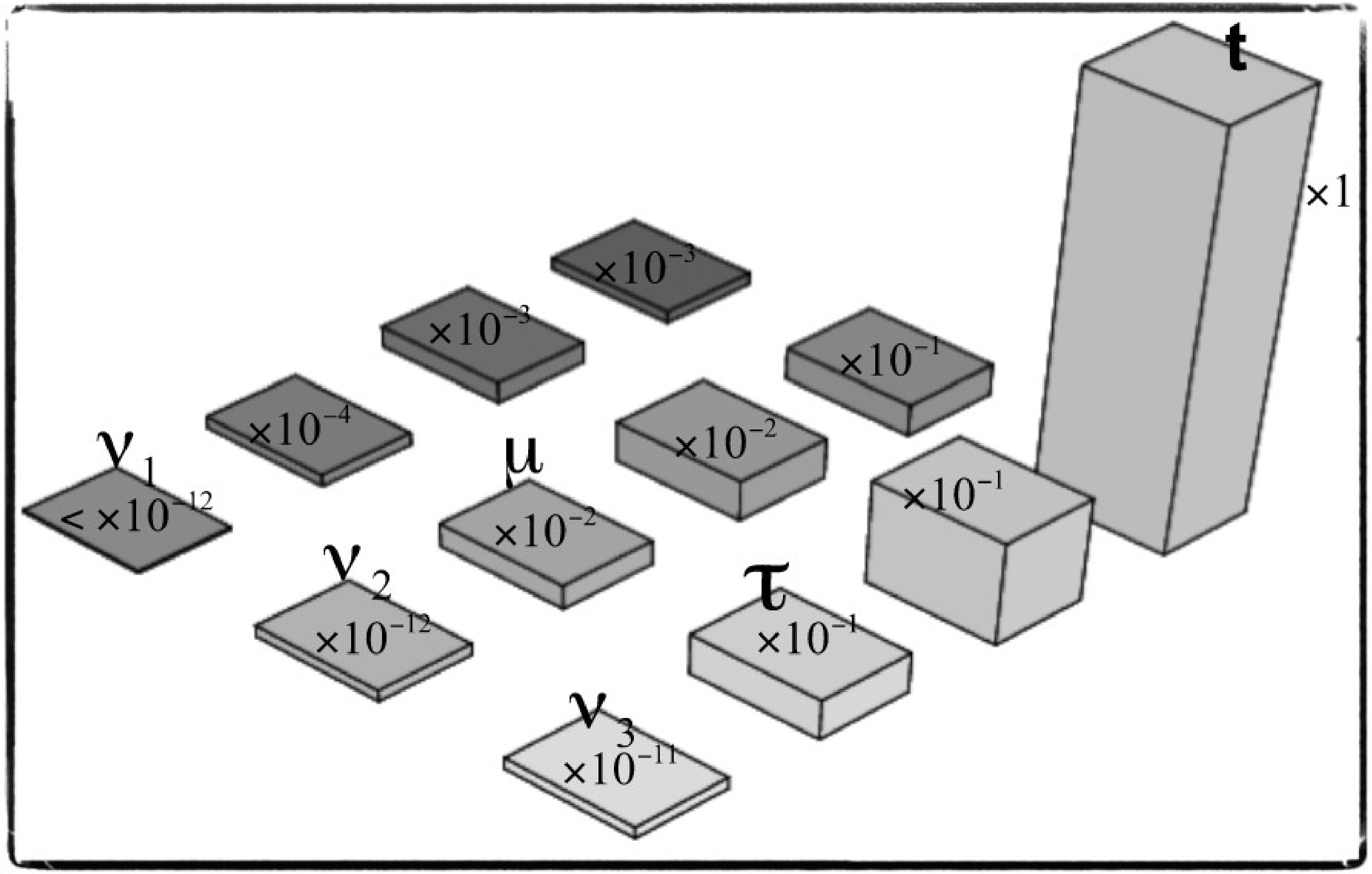
$$M_{u,d} = vY_{u,d} \equiv \begin{pmatrix} m_{u,d} & 0 & 0 \\ 0 & m_{u,d} & 0 \\ 0 & 0 & m_{u,d} \end{pmatrix}$$

$$U^\dagger M M^\dagger U = \mathsf{Diag}(m_u, m_c, m_t) \,, \qquad V^\dagger M M^\dagger V = \mathsf{Diag}(m_d, m_s, m_b)$$

$$V_{\mathrm{CKM}} = U^\dagger V \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Quark Flavour Hierarchies

Masses



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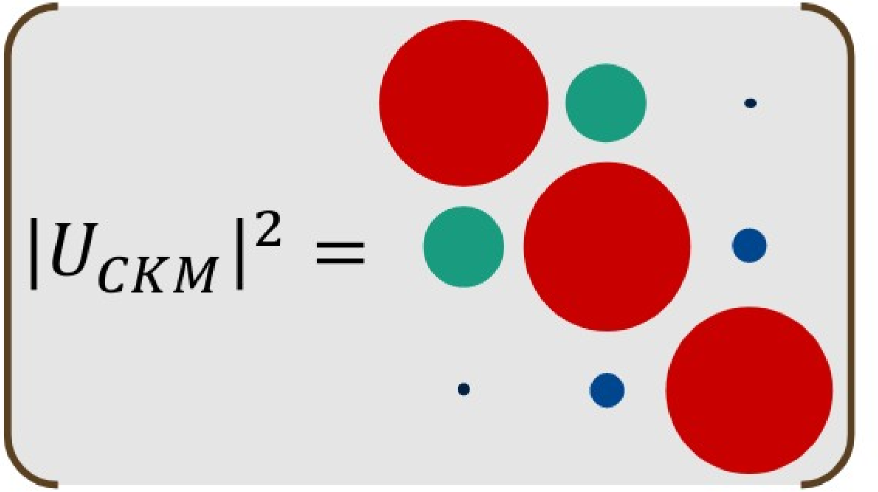
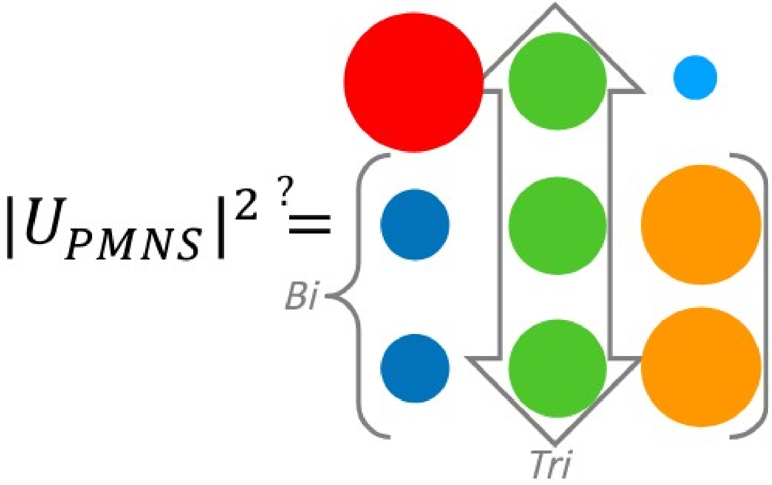
Mixing Hierarchies

CKM

PMNS





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Quark Flavour Hierarchies

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Parametrization of the Kobayashi-Maskawa Matrix

Lincoln Wolfenstein
Department of Physics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 22 August 1983)

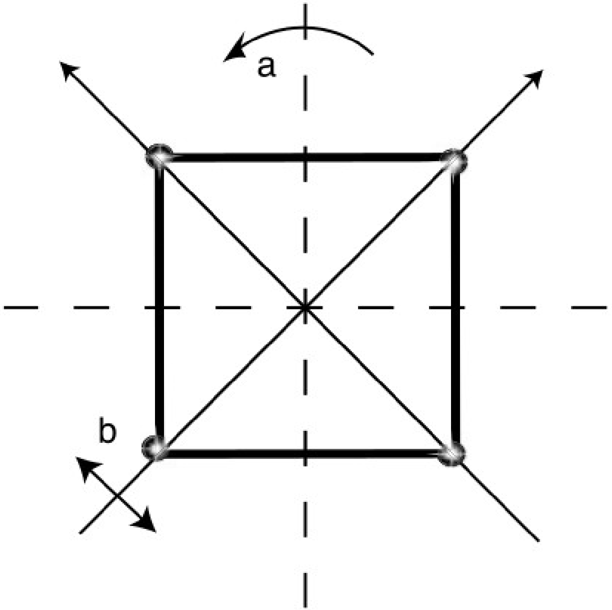
The quark mixing matrix (Kobayashi-Maskawa matrix) is expanded in powers of a small parameter λ equal to $\sin\theta_c = 0.22$. The term of order λ^2 is determined from the recently measured B lifetime. Two remaining parameters, including the CP-nonconservation effects, enter only the term of order λ^3 and are poorly constrained. A significant reduction in the limit on ϵ'/ϵ possible in an ongoing experiment would tightly constrain the CP-nonconservation parameter and could rule out the hypothesis that the only source of CP nonconservation is the Kobayashi-Maskawa mechanism.

PACS numbers: 11.30.Er, 12.10.Ck, 13.25.+m

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & -\lambda & \mathcal{O}(\lambda^3) \\ \lambda & 1-\lambda^2/2 & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1 \end{pmatrix} \qquad \lambda \approx 0.22$$

Group Theory

$$a = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$



$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\mathbf{2}} \otimes \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{\mathbf{2}} = [x_1y_1 + x_2y_2]_{\mathbf{1}_{++}} \oplus [x_1y_2 - x_2y_1]_{\mathbf{1}_{--}}$$
$$\oplus [x_1y_2 + x_2y_1]_{\mathbf{1}_{-+}}$$
$$\oplus [x_1y_1 - x_2y_2]_{\mathbf{1}_{+-}},$$

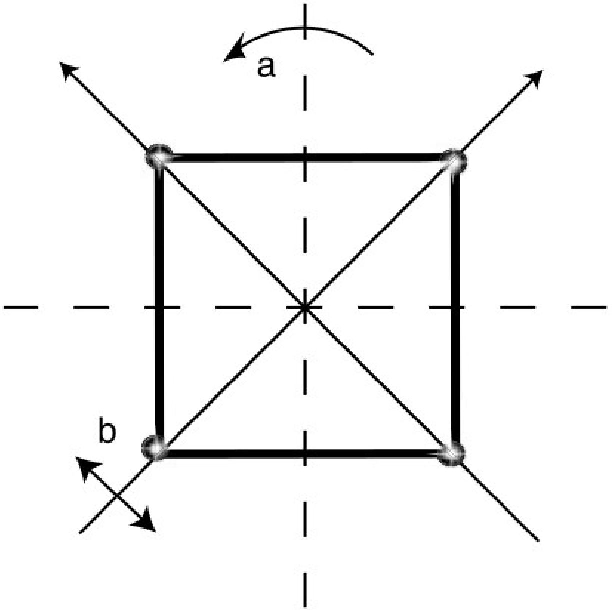
$$\mathbf{1}_{r,s} \otimes \mathbf{1}_{r',s'} = \mathbf{1}_{r\cdot r',s\cdot s'} .$$

Group Theory

$$a = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\mathbf{2}} \otimes \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{\mathbf{2}} = [x_1 y_1 + x_2 y_2]_{\mathbf{1}_{++}} \oplus [x_1 y_2 - x_2 y_1]_{\mathbf{1}_{--}}$$
$$\oplus [x_1 y_2 + x_2 y_1]_{\mathbf{1}_{-+}}$$
$$\oplus [x_1 y_1 - x_2 y_2]_{\mathbf{1}_{+-}},$$

$$\mathbf{1}_{r,s} \otimes \mathbf{1}_{r',s'} = \mathbf{1}_{r \cdot r', s \cdot s'}.$$



D_4 Symmetry

Model

$$\mathbf{2}: \begin{bmatrix} Q_{1L} \\ Q_{2L} \end{bmatrix}, \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \tag{4a}$$

$$\mathbf{1}_{++}: n_{1R}, \quad \mathbf{1}_{--}: n_{2R}, n_{3R}, \phi_u, \quad \mathbf{1}_{-+}: p_{2R}, p_{3R}, \phi_d,$$
$$\mathbf{1}_{+-}: Q_{3L}, p_{1R}. \tag{4b}$$

D_4 Symmetry

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D_4-Symmetric 4HDM

D_4 Symmetry

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$$\mathbf{2}: \begin{bmatrix} Q_{1L} \\ Q_{2L} \end{bmatrix}, \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \tag{4a}$$

$$\mathbf{1}_{++}: n_{1R}\,, \quad \mathbf{1}_{--}: n_{2R}\,, n_{3R}\,, \phi_u\,, \quad \mathbf{1}_{-+}: p_{2R}\,, p_{3R}\,, \phi_d\,,$$

$$\mathbf{1}_{+-}: Q_{3L}\,, p_{1R}\,. \tag{4b}$$

No Extra Energy Scales: All contribute to EWSB

$$\phi_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \phi_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \qquad \phi_u = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \phi_d = \begin{pmatrix} 0 \\ v_d \end{pmatrix},$$

$$v_1^2 + v_2^2 + v_u^2 + v_d^2 = v_{\text{SM}}^2\,, \qquad v_{12}^2 = v_1^2 + v_2^2\,, \qquad \frac{v_2}{v_1} = \tan\beta$$

Model

$$\mathbf{2}: \begin{bmatrix} Q_{1L} \\ Q_{2L} \end{bmatrix}, \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \tag{4a}$$

$$\mathbf{1}_{++}: n_{1R}, \quad \mathbf{1}_{--}: n_{2R}, n_{3R}, \phi_u, \quad \mathbf{1}_{-+}: p_{2R}, p_{3R}, \phi_d,$$
$$\mathbf{1}_{+-}: Q_{3L}, p_{1R}. \tag{4b}$$

$$\begin{aligned}
-\mathscr{L}_u = {} & A_u(\overline{Q}_{1L}\tilde{\phi}_1 - \overline{Q}_{2L}\tilde{\phi}_2)p_{1R} + B_u(\overline{Q}_{1L}\tilde{\phi}_2 \\
& + \overline{Q}_{2L}\tilde{\phi}_1)p_{2R} + C_u(\overline{Q}_{1L}\tilde{\phi}_2 + \overline{Q}_{2L}\tilde{\phi}_1)p_{3R} \\
& + X_u\overline{Q}_{3L}\phi_u p_{2R} \\
& + Y_u\overline{Q}_{3L}\phi_u p_{3R}, \\
-\mathscr{L}_d = {} & A_d(\overline{Q}_{1L}\phi_1 + \overline{Q}_{2L}\phi_2)n_{1R} \\
& + B_d(\overline{Q}_{1L}\phi_2 - \overline{Q}_{2L}\phi_1)n_{2R} \\
& + C_d(\overline{Q}_{1L}\phi_2 - \overline{Q}_{2L}\phi_1)n_{3R} \\
& + X_d\overline{Q}_{3L}\phi_d n_{2R} + Y_d\overline{Q}_{3L}\phi_d n_{3R},
\end{aligned}$$

$$M_u = \begin{pmatrix} A_u v_1 & B_u v_2 & C_u v_2 \\ -A_u v_2 & B_u v_1 & C_u v_1 \\ 0 & X_u v_u & Y_u v_u \end{pmatrix},$$

$$M_d = \begin{pmatrix} A_d v_1 & B_d v_2 & C_d v_2 \\ A_d v_2 & -B_d v_1 & -C_d v_1 \\ 0 & X_d v_d & Y_d v_d \end{pmatrix},$$

Model

$$\mathbf{2}: \begin{bmatrix} Q_{1L} \\ Q_{2L} \end{bmatrix}, \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \tag{4a}$$

$$\mathbf{1}_{++}: n_{1R}\,, \quad \mathbf{1}_{--}: n_{2R}\,, n_{3R}\,, \phi_u\,, \quad \mathbf{1}_{-+}: p_{2R}\,, p_{3R}\,, \phi_d\,,$$
$$\mathbf{1}_{+-}: Q_{3L}\,, p_{1R}\,. \tag{4b}$$

$$\begin{aligned} -\mathscr{L}_u = &\, A_u(\overline{Q}_{1L}\tilde{\phi}_1 - \overline{Q}_{2L}\tilde{\phi}_2)p_{1R} + B_u(\overline{Q}_{1L}\tilde{\phi}_2 \\ &+ \overline{Q}_{2L}\tilde{\phi}_1)p_{2R} + C_u(\overline{Q}_{1L}\tilde{\phi}_2 + \overline{Q}_{2L}\tilde{\phi}_1)p_{3R} \\ &+ X_u\overline{Q}_{3L}\phi_u p_{2R} \\ &+ Y_u\overline{Q}_{3L}\phi_u p_{3R}\,, \\ -\mathscr{L}_d = &\, A_d(\overline{Q}_{1L}\phi_1 + \overline{Q}_{2L}\phi_2)n_{1R} \\ &+ B_d(\overline{Q}_{1L}\phi_2 - \overline{Q}_{2L}\phi_1)n_{2R} \\ &+ C_d(\overline{Q}_{1L}\phi_2 - \overline{Q}_{2L}\phi_1)n_{3R} \\ &+ X_d\overline{Q}_{3L}\phi_d n_{2R} + Y_d\overline{Q}_{3L}\phi_d n_{3R}\,, \end{aligned}$$

$$M_u = \begin{pmatrix} A_u v_1 & B_u v_2 & C_u v_2 \\ -A_u v_2 & B_u v_1 & C_u v_1 \\ 0 & X_u v_u & Y_u v_u \end{pmatrix},$$

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β

D_4 Symmetry

Block-Diagonalization

$$M_u = \begin{pmatrix} A_u v_1 & B_u v_2 & C_u v_2 \\ -A_u v_2 & B_u v_1 & C_u v_1 \\ 0 & X_u v_u & Y_u v_u \end{pmatrix},$$

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$$O_\beta = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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$$O_\beta = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(M_u^2)_{\text{Block}} \equiv O_\beta M_u M_u^\dagger O_\beta^\dagger = \begin{pmatrix} A_u^2 v_{12}^2 & 0 & 0 \\ 0 & (B_u^2 + C_u^2) v_{12}^2 & (C_u Y_u + B_u X_u) v_{12} v_u \\ 0 & (C_u Y_u + B_u X_u) v_{12} v_u & (Y_u^2 + X_u{}^2) v_u^2 \end{pmatrix},$$

$$(M_d^2)_{\text{Block}} \equiv O_\beta^\dagger M_d M_d^\dagger O_\beta = \begin{pmatrix} A_d^2 v_{12}^2 & 0 & 0 \\ 0 & (B_d^2 + C_d^2) v_{12}^2 & -(C_d Y_d + B_d X_d) v_{12} v_d \\ 0 & -(C_d Y_d + B_d X_d) v_{12} v_d & (Y_d^2 + X_d{}^2) v_d^2 \end{pmatrix}$$

D_4 Symmetry

Full Diagonalization

$$(M_u^2)_{\text{Block}} \equiv O_\beta M_u M_u^\dagger O_\beta^\dagger = \begin{pmatrix} A_u^2 v_{12}^2 & 0 & 0 \\ 0 & (B_u^2 + C_u^2) v_{12}^2 & (C_u Y_u + B_u X_u) v_{12} v_u \\ 0 & (C_u Y_u + B_u X_u) v_{12} v_u & (Y_u^2 + {X_u}^2) v_u^2 \end{pmatrix},$$

$$(M_d^2)_{\text{Block}} \equiv O_\beta^\dagger M_d M_d^\dagger O_\beta = \begin{pmatrix} A_d^2 v_{12}^2 & 0 & 0 \\ 0 & (B_d^2 + C_d^2) v_{12}^2 & -(C_d Y_d + B_d X_d) v_{12} v_d \\ 0 & -(C_d Y_d + B_d X_d) v_{12} v_d & (Y_d^2 + {X_d}^2) v_d^2 \end{pmatrix}$$

D_4 Symmetry

Full Diagonalization

$$
\begin{aligned}
(M_u^2)_{\text{Block}} \equiv O_\beta M_u M_u^\dagger O_\beta^\dagger &= \begin{pmatrix} A_u^2 v_{12}^2 & 0 & 0 \\ 0 & (B_u^2 + C_u^2) v_{12}^2 & (C_u Y_u + B_u X_u) v_{12} v_u \\ 0 & (C_u Y_u + B_u X_u) v_{12} v_u & (Y_u^2 + X_u{}^2) v_u^2 \end{pmatrix}, \\
(M_d^2)_{\text{Block}} \equiv O_\beta^\dagger M_d M_d^\dagger O_\beta &= \begin{pmatrix} A_d^2 v_{12}^2 & 0 & 0 \\ 0 & (B_d^2 + C_d^2) v_{12}^2 & -(C_d Y_d + B_d X_d) v_{12} v_d \\ 0 & -(C_d Y_d + B_d X_d) v_{12} v_d & (Y_d^2 + X_d{}^2) v_d^2 \end{pmatrix}
\end{aligned}
$$

$$
O_\theta^{u,d} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{u,d} & -\sin\theta_{u,d} \\ 0 & \sin\theta_{u,d} & \cos\theta_{u,d} \end{pmatrix},
$$

Full Diagonalization

$$(M_u^2)_{\text{Block}} \equiv O_\beta M_u M_u^\dagger O_\beta^\dagger = \begin{pmatrix} A_u^2 v_{12}^2 & 0 & 0 \\ 0 & (B_u^2 + C_u^2) v_{12}^2 & (C_u Y_u + B_u X_u) v_{12} v_u \\ 0 & (C_u Y_u + B_u X_u) v_{12} v_u & (Y_u^2 + {X_u}^2) v_u^2 \end{pmatrix},$$

$$(M_d^2)_{\text{Block}} \equiv O_\beta^\dagger M_d M_d^\dagger O_\beta = \begin{pmatrix} A_d^2 v_{12}^2 & 0 & 0 \\ 0 & (B_d^2 + C_d^2) v_{12}^2 & -(C_d Y_d + B_d X_d) v_{12} v_d \\ 0 & -(C_d Y_d + B_d X_d) v_{12} v_d & (Y_d^2 + {X_d}^2) v_d^2 \end{pmatrix}$$

$$O_\theta^{u,d} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{u,d} & -\sin\theta_{u,d} \\ 0 & \sin\theta_{u,d} & \cos\theta_{u,d} \end{pmatrix},$$

$$\tan 2\theta_u = \frac{2(C_u Y_u + B_u X_u) v_{12} v_u}{(Y_u^2 + {X_u}^2) v_u^2 - (B_u^2 + C_u^2) v_{12}^2},$$

$$\tan 2\theta_d = -\frac{2(C_d Y_d + B_d X_d) v_{12} v_d}{(Y_d^2 + {X_d}^2) v_d^2 - (B_d^2 + C_d^2) v_{12}^2}.$$

$$D_u^2 = O_\theta^u O_\beta (M_u M_u^\dagger) O_\beta^\dagger O_\theta^{u\dagger} \equiv \text{diag}(m_u^2, m_c^2, m_t^2),$$

$$D_d^2 = O_\theta^d O_\beta^\dagger (M_d M_d^\dagger) O_\beta O_\theta^{d\dagger} \equiv \text{diag}(m_d^2, m_s^2, m_b^2).$$

Goal:

$$V_{\mathrm{CKM}} = \begin{pmatrix} \cos 2\beta & -\cos\theta_d \sin 2\beta & -\sin 2\beta \sin\theta_d \\ \cos\theta_u \sin 2\beta & \cos 2\beta \cos\theta_d \cos\theta_u + \sin\theta_d \sin\theta_u & \cos 2\beta \cos\theta_u \sin\theta_d - \cos\theta_d \sin\theta_u \\ \sin 2\beta \sin\theta_u & -\cos\theta_u \sin\theta_d + \cos 2\beta \cos\theta_d \sin\theta_u & \cos\theta_d \cos\theta_u + \cos 2\beta \sin\theta_d \sin\theta_u \end{pmatrix}$$

$$+$$

$$\sin 2\beta = \lambda\,, \qquad \sin\theta_{u,d} \approx \mathcal{O}\left(\lambda^2\right)\,, \qquad \cos\theta_{u,d} \approx \mathcal{O}\left(1\right)\,.$$

$$=$$

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & -\lambda & \mathcal{O}\left(\lambda^3\right) \\ \lambda & 1-\lambda^2/2 & \mathcal{O}\left(\lambda^2\right) \\ \mathcal{O}\left(\lambda^3\right) & \mathcal{O}\left(\lambda^2\right) & 1 \end{pmatrix}$$

Can we do it?

$$m_u^2 = A_u^2 v_{12}^2\,,$$

$$m_c^2 \approx \frac{(B_u Y_u - C_u X_u)^2}{(Y_u^2 + X_u{}^2)} v_{12}^2\,.$$

$$m_t^2 \approx (Y_u^2 + X_u{}^2) v_u^2\,.$$

$$\frac{m_c}{m_t} \approx \frac{v_{12}}{v_u} \sim \mathcal{O}\left(\lambda^2\right)$$

$$\theta_u \approx \frac{(C_u Y_u + B_u X_u)}{(Y_u^2 + X_u{}^2)} \frac{v_{12}}{v_u} \approx \mathcal{O}\left(\frac{v_{12}}{v_u}\right)\,,$$

$$\theta_d \approx -\frac{(C_d Y_d + B_d X_d)}{(Y_d^2 + X_d{}^2)} \frac{v_{12}}{v_d} \approx \mathcal{O}\left(\frac{v_{12}}{v_d}\right)\,,$$

D_4 Symmetry

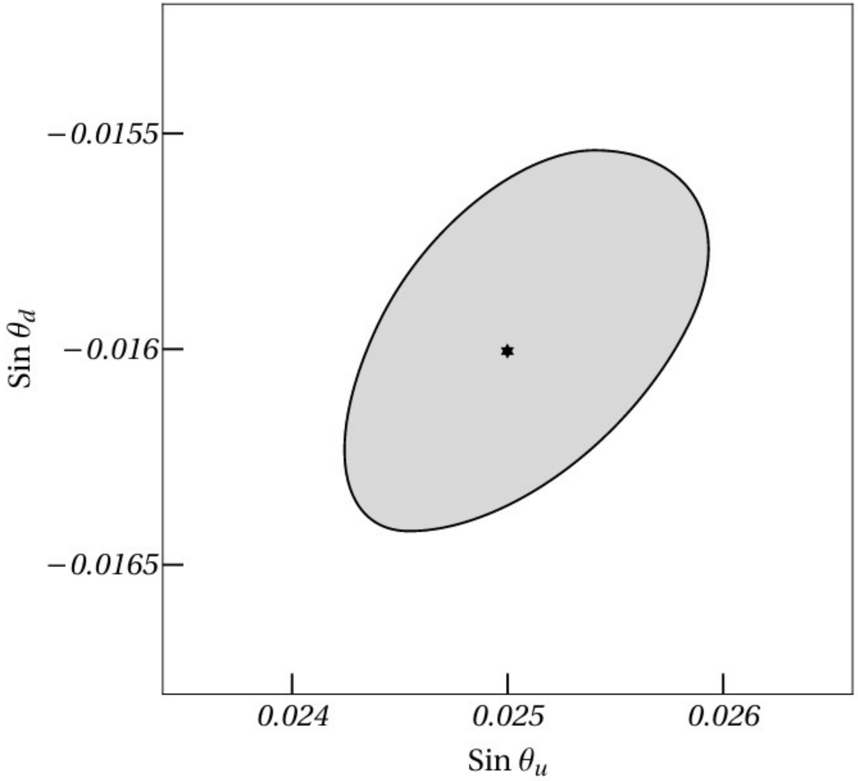
Can we do it?

$$m_u^2 = A_u^2 v_{12}^2 \,,$$

$$m_c^2 \approx \frac{(B_u Y_u - C_u X_u)^2}{(Y_u^2 + {X_u}^2)} v_{12}^2 \,.$$

$$m_t^2 \approx (Y_u^2 + {X_u}^2) v_u^2 \,.$$

$$\frac{m_c}{m_t} \approx \frac{v_{12}}{v_u} \sim \mathcal{O}\left(\lambda^2\right)$$



D_4 Symmetry

Can we do it?

yes

D_4 Symmetry

Can we do it?

Well, yes
But Why?

Summary

Why we did it:

Aesthetics:
- Dillution of the Quark Yukawa hierarchies
- Relating Quark mass hierarchies to CKM hierarchies
- Connected Quark Masses and Mixings to Scalar Sector

Observable Consequences:
- FCNCs
- Extra Higgs Phenomenology
- Small Parametric Space

The End?

Thank You For
Your Attention!

The End?

Back-Up Slides

FCNCs

$$N_d^1 \approx \frac{1}{\sqrt{2}v_{12}} \begin{pmatrix} m_d \cos\beta & -m_s \sin\beta & m_b\theta_d \sin\beta \\ -m_d \sin_\beta & -m_s \cos\beta & m_b\theta_d \cos\beta \\ -m_d\theta_d \sin\beta & -m_s\theta_d \cos\beta & m_b\theta_d^2 \cos\beta \end{pmatrix}$$

$$N_d^2 \approx \frac{1}{\sqrt{2}v_{12}} \begin{pmatrix} m_d \sin\beta & m_s \cos\beta & -m_b\theta_d \cos\beta \\ m_d \cos\beta & -m_s \sin\beta & m_b\theta_d \sin\beta \\ m_b\theta_d \cos\beta & -m_s\theta_d \sin\beta & m_b\theta_d^2 \sin\beta \end{pmatrix}$$

$$N_d^d \approx \frac{1}{\sqrt{2}v_d} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -m_b\theta_d \\ 0 & 0 & m_b \end{pmatrix} .$$

Scalar Potential

$$V(\phi) = V_{\text{quadratic}} + V_{\text{quartic}},$$

$$\begin{aligned}
V_{\text{quartic}} &= \lambda_1 \left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2\right)^2 + \lambda_2 \left(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1\right)^2 + \lambda_3 \left(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1\right)^2 + \lambda_4 \left(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2\right)^2 \\
&+\lambda_5 \left(\phi_u^\dagger\phi_u\right)^2 + \lambda_6 \left(\phi_d^\dagger\phi_d\right)^2 + \lambda_7 \left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2\right)\left(\phi_d^\dagger\phi_d\right) + \lambda_8 \left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2\right)\left(\phi_u^\dagger\phi_u\right) \\
&+\lambda_9 \left(\phi_u^\dagger\phi_u\right)\left(\phi_d^\dagger\phi_d\right) + \lambda_{10} \left[\left(\phi_u^\dagger\phi_1\right)\left(\phi_1^\dagger\phi_u\right) + \left(\phi_u^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_u\right)\right] \\
&+\lambda_{11} \left[\left(\phi_d^\dagger\phi_1\right)\left(\phi_1^\dagger\phi_d\right) + \left(\phi_d^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_d\right)\right] \\
&+\lambda_{12} \left[\left(\phi_u^\dagger\phi_1\right)^2 + \left(\phi_u^\dagger\phi_2\right)^2 + \left(\phi_1^\dagger\phi_u\right)^2 + \left(\phi_2^\dagger\phi_u\right)^2\right] \\
&+\lambda_{13} \left[\left(\phi_d^\dagger\phi_1\right)^2 + \left(\phi_d^\dagger\phi_2\right)^2 + \left(\phi_1^\dagger\phi_d\right)^2 + \left(\phi_2^\dagger\phi_d\right)^2\right] \\
&+\lambda_{14} \left[\left(\phi_u^\dagger\phi_1\right)\left(\phi_d^\dagger\phi_1\right) + \left(\phi_u^\dagger\phi_2\right)\left(\phi_d^\dagger\phi_2\right) + \left(\phi_1^\dagger\phi_u\right)\left(\phi_1^\dagger\phi_d\right) + \left(\phi_2^\dagger\phi_u\right)\left(\phi_2^\dagger\phi_d\right)\right] \\
&+\lambda_{15} \left[\left(\phi_u^\dagger\phi_1\right)\left(\phi_1^\dagger\phi_d\right) + \left(\phi_u^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_d\right) + \left(\phi_1^\dagger\phi_u\right)\left(\phi_d^\dagger\phi_1\right) + \left(\phi_2^\dagger\phi_u\right)\left(\phi_d^\dagger\phi_2\right)\right] \\
&+\lambda_{16} \left(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2\right)\left[\left(\phi_u^\dagger\phi_d\right) + \left(\phi_d^\dagger\phi_u\right)\right], \qquad (35)
\end{aligned}$$

$$\begin{aligned}
V_{\text{quadratic}} &= \mu_{11}^2 \left(\phi_1^\dagger\phi_1\right) + \mu_{22}^2 \left(\phi_2^\dagger\phi_2\right) + \mu_{uu}^2 \left(\phi_u^\dagger\phi_u\right) + \mu_{dd}^2 \left(\phi_d^\dagger\phi_d\right) \\
&+\mu_{12}^2 \left(\phi_1^\dagger\phi_2 + \text{h.c.}\right) + \mu_{1u}^2 \left(\phi_1^\dagger\phi_u + \text{h.c.}\right) + \mu_{1d}^2 \left(\phi_1^\dagger\phi_d + \text{h.c.}\right) \\
&+\mu_{2u}^2 \left(\phi_2^\dagger\phi_u + \text{h.c.}\right) + \mu_{2d}^2 \left(\phi_2^\dagger\phi_d + \text{h.c.}\right) + \mu_{ud}^2 \left(\phi_u^\dagger\phi_d + \text{h.c.}\right). \qquad (36)
\end{aligned}$$

Escaping Bounds

$$
\begin{aligned}
V_{\mathrm{MS}} \;=\;& \mu_{11}^2\, \phi_1^\dagger \phi_1 + \mu_{22}^2\, \phi_2^\dagger \phi_2 + \mu_{uu}^2\, \phi_u^\dagger \phi_u + \mu_{dd}^2\, \phi_d^\dagger \phi_d \\
&+ \left(\mu_{12}^2\, \phi_1^\dagger \phi_2 + \mu_{1u}^2\, \phi_1^\dagger \phi_u + \mu_{1d}^2\, \phi_1^\dagger \phi_d + \mu_{2u}^2\, \phi_2^\dagger \phi_u + \mu_{2d}^2\, \phi_2^\dagger \phi_d + \mu_{ud}^2\, \phi_u^\dagger \phi_d + \mathrm{h.c.} \right) \\
&+ \lambda \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_u^\dagger \phi_u + \phi_d^\dagger \phi_d \right)^2 ,
\end{aligned}
$$

- The alignment limit emerges automatically (there is a mass eigenstate which is SM-like).
- The nonstandard masses are disentangled from the EW scale (the nonstandard masses are tier-wise degenerate and decoupled).
- There is no restriction on the relative hierarchies of the VEVs.
- Unitarity and boundedness from below constraints are trivially satisfied, because there is only one quartic parameter λ, which is related to the SM-like Higgs mass as $m_h^2 = 4\lambda v^2$.
- The ρ-parameter is trivially satisfied by the tier-wise degeneracy of the masses [31, 32].