The smallest Warm Little Inflaton

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1 2 9 0 UNIVERSIDADE D COIMBRA

7th IDPASC/LIP PhD Students Workshop

FCT Fundação para a Ciência e a Tecnologia

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IDPASC

Cosmological Inflation

Accelerated expansion: $\ddot{a} > 0$ → Exotic matter: $p < -\frac{1}{3}\rho$

Inflaton:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

Slow-roll dynamics:

$$3H\dot{\phi} + V_{,\phi}(\phi) \simeq 0$$

Slow-roll parameters:

$$\epsilon_\phi = \frac{M_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1 \qquad |\eta_\phi| = M_p^2\left|\frac{V_{,\phi\phi}}{V}\right| \ll 1$$

Warm Inflation

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} = \xi$$

[Berera95]

Υ – Dissipation coefficient

$$\Upsilon = \Upsilon(T, \phi)$$

ξ – Noise term

$$\langle \xi(t, x)\xi(t', x') \rangle = \bar{\xi}^2(T, H)\delta(x - x')\delta(t - t')$$

Warm Inflation

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} = \xi$$

[Berera95]

$\Upsilon -$ Dissipation coefficient

$\xi -$ Noise term

$$\Upsilon = \Upsilon(T, \phi)$$

$$\langle \xi(t,x)\xi(t',x')\rangle = \bar{\xi}^2(T,H)\delta(x-x')\delta(t-t')$$

Sustain a thermal bath during inflation: $\dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\phi}^2$

Improved slow-roll conditions: $\epsilon_\phi \ll 1 + Q$ $\quad |\eta_\phi| \ll 1 + Q$

No reheating? $\dfrac{\rho_R}{V(\phi)} \simeq \dfrac{1}{2}\dfrac{\epsilon_\phi}{1+Q}\dfrac{Q}{1+Q}$

Two regimes:

$T > H, \quad \Gamma > H$

$$Q = \frac{\Upsilon}{3H}$$

Warm Little Inflaton

[Rosa et al '16]

We have a U(1) symmetry:

Two charged scalar fields: $\Phi_{1,2} = \frac{M}{\sqrt{2}} e^{\pm i\phi}$

Four Weyl fermions: $\psi_{L1,2}, \quad \psi_{R1,2}$

$$\mathscr{L}_Y = -\frac{g}{\sqrt{2}}(\Phi_1 + \Phi_2)\psi_{1,L}\overline{\psi_{1,R}} - i\frac{g}{\sqrt{2}}(\Phi_1 - \Phi_2)\psi_{2,L}\overline{\psi}_{2,R} + \text{h.c.}$$

$$= -gM\cos(\phi/M)\psi_{1,L}\overline{\psi_{1,R}} - gM\sin(\phi/M)\psi_{2,L}\overline{\psi}_{2,R}$$

Warm Little Inflaton

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i) $|m_\psi| < gM$

ii) $V_T \propto (m_1^2 + m_2^2)T^2 = (gM)^2T^2$

Υ – Recipe :

1) $\phi \to \psi_{1,2} \to \psi_\sigma, \sigma$

2) $\mathscr{L} = -h\sigma\overline{\psi}_\sigma(\psi_1 + \psi_2) + \text{h.c.}$

$$\Upsilon \simeq C_T T, \quad C_T = C_T(h, g)$$

The smallest Warm Little Inflaton

Only two Weyl fermions leads to

$$\mathscr{L}_Y = -\, gM\cos(\phi/M)\psi_{1,L}\overline{\psi_{1,R}}$$

such that

$$V_T = -\frac{7\pi^2}{180}T^4 + \frac{(gM)^2}{12}T^2\cos^2(\phi/M)$$

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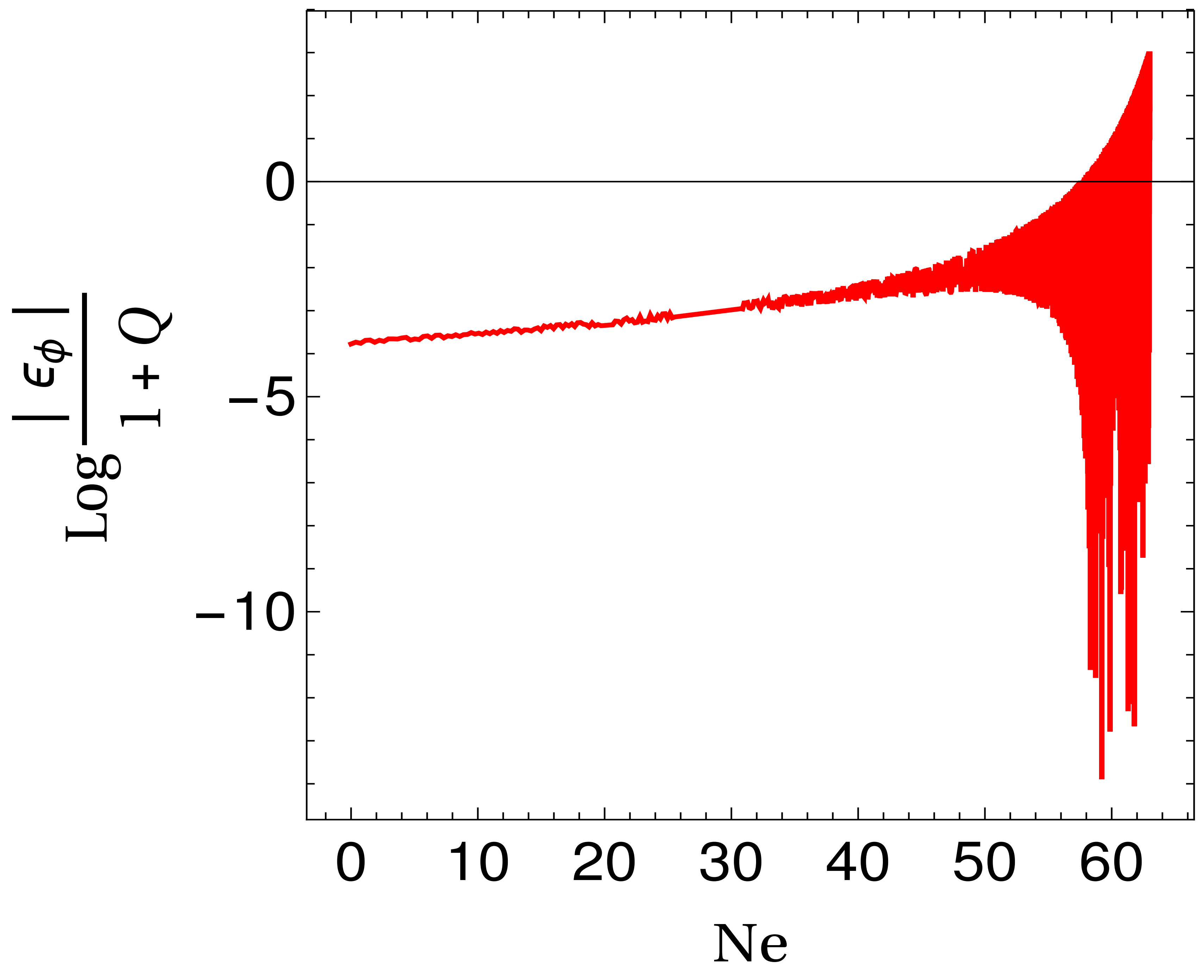
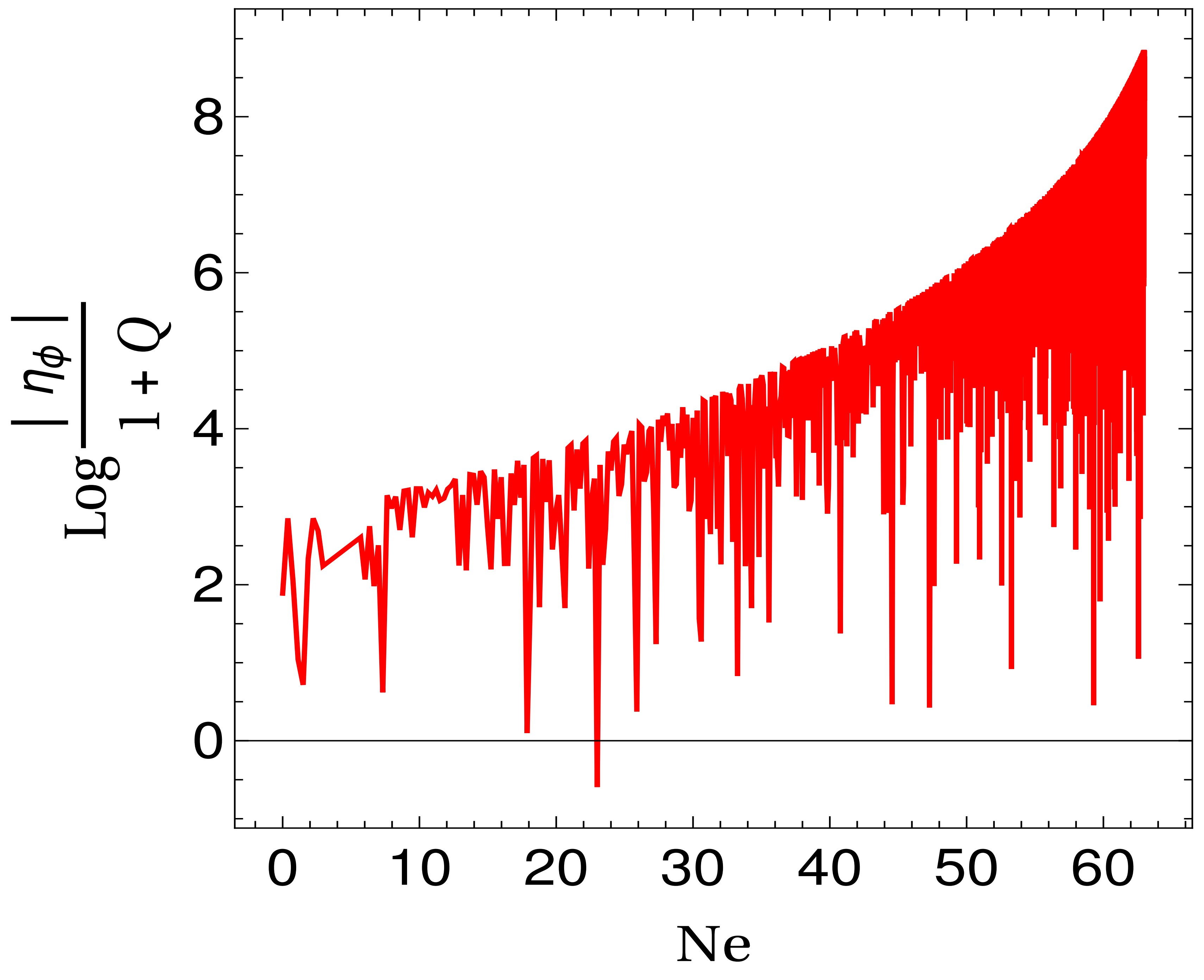
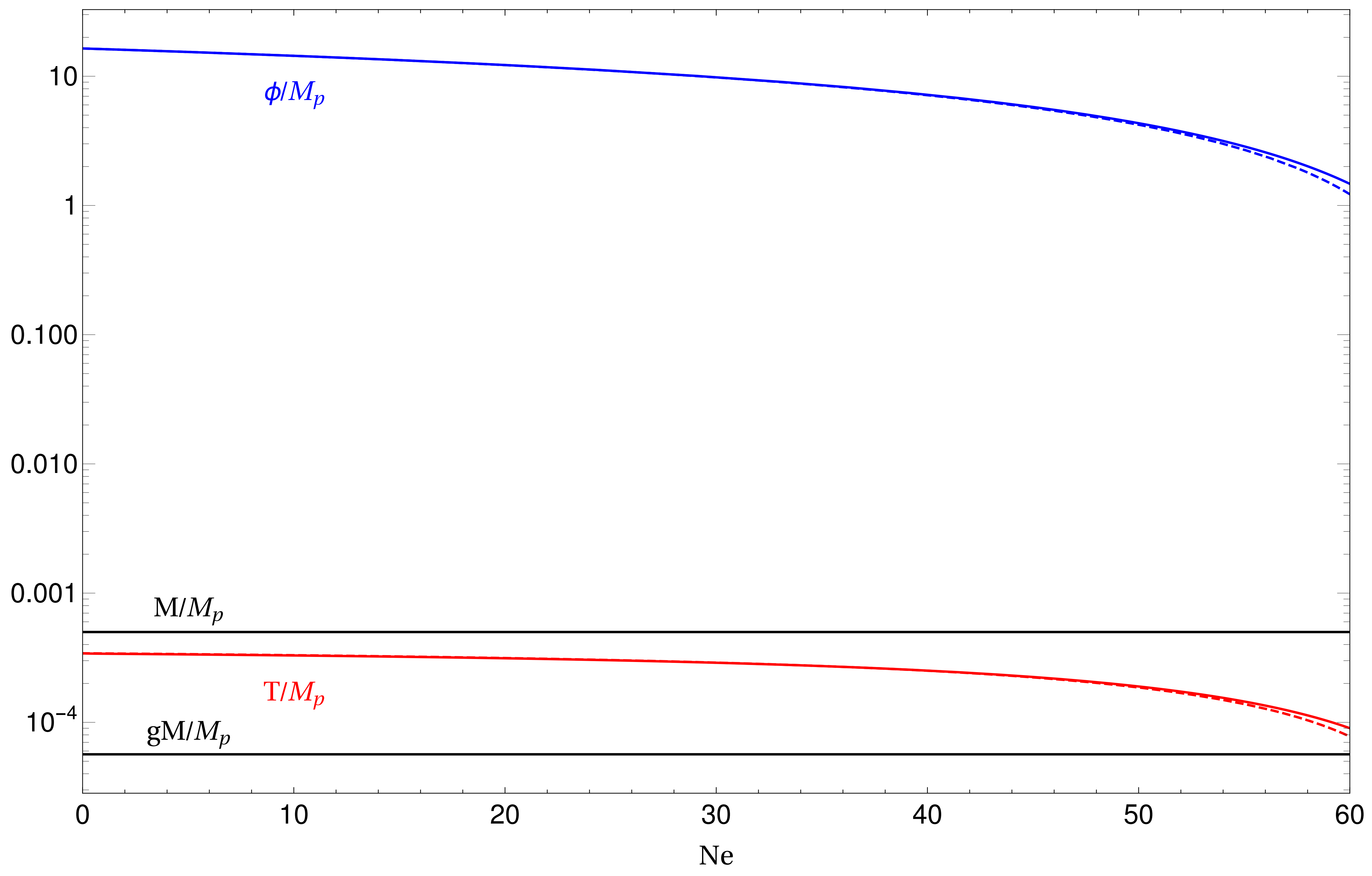
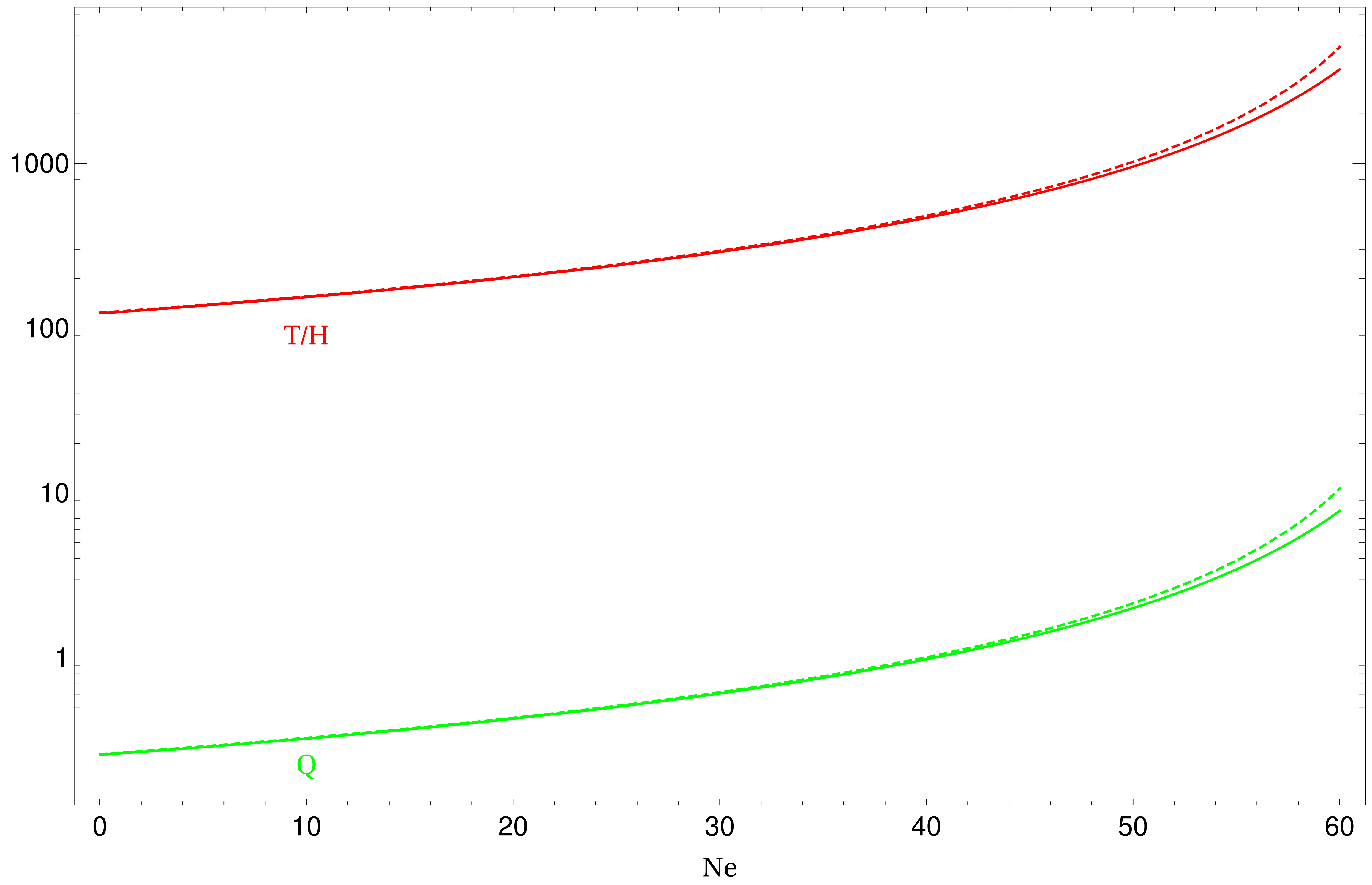




However, if we compute its dynamics:





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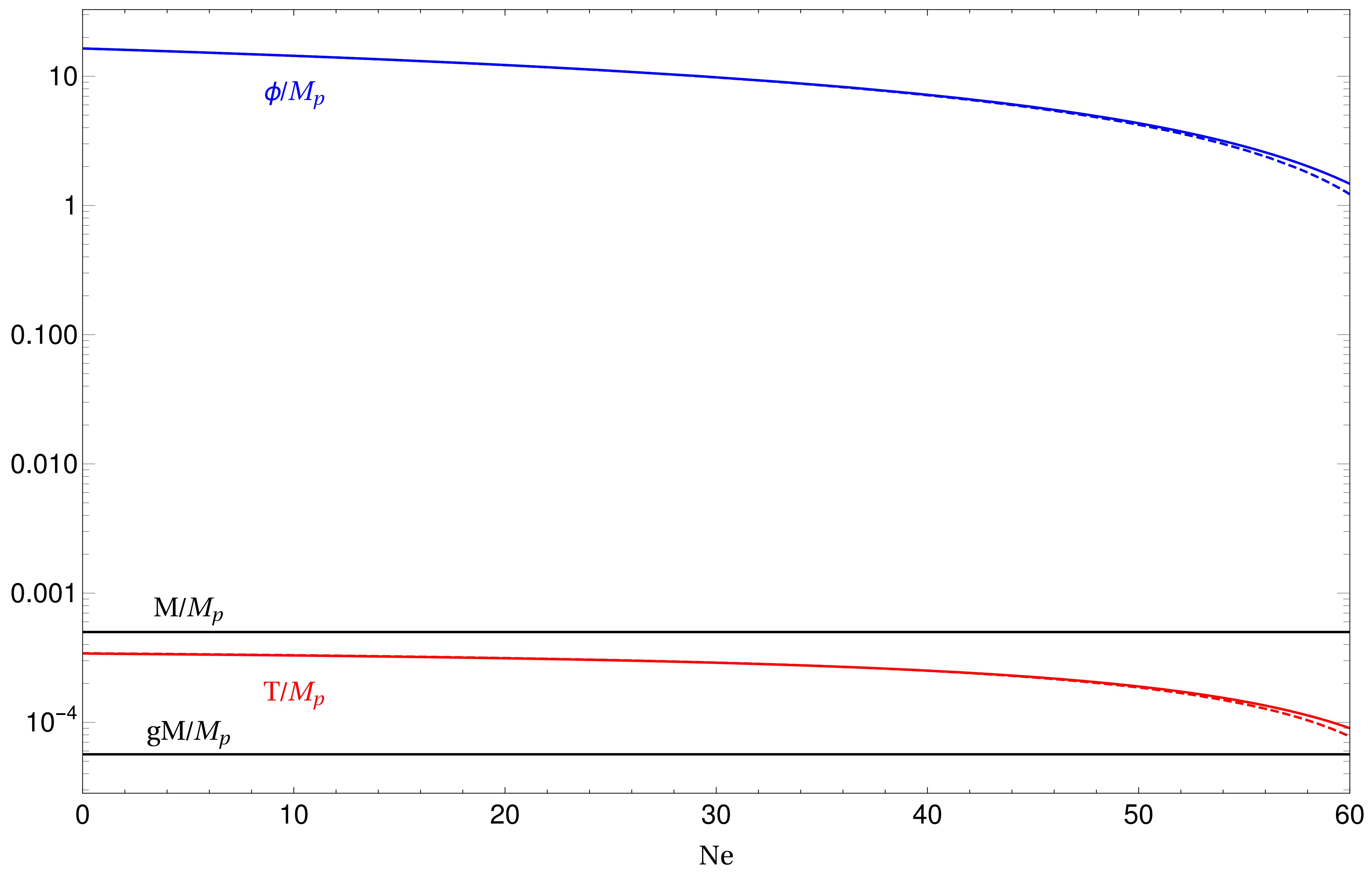

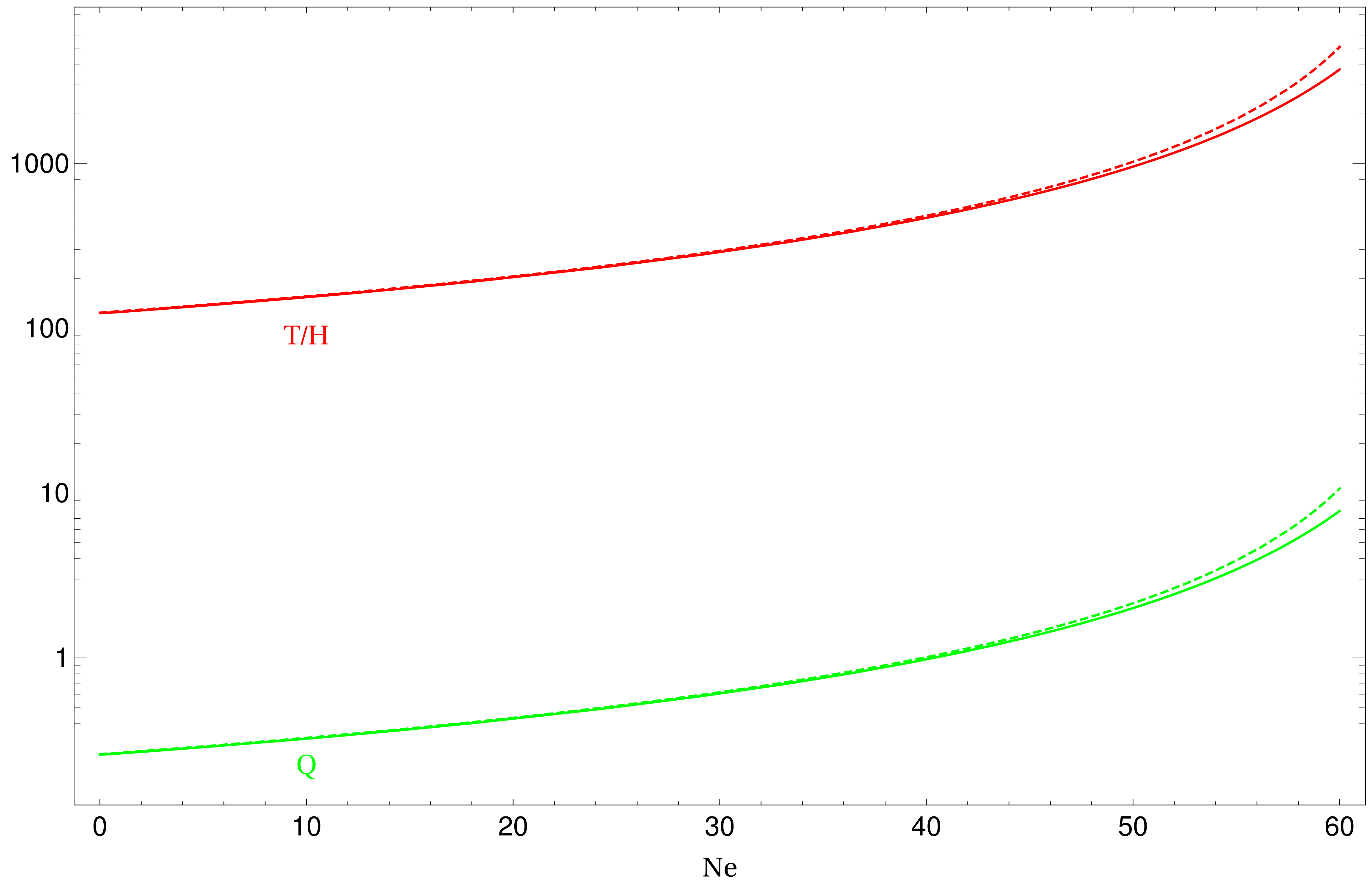

$$\Delta\eta_\phi \sim \langle \Delta\eta_\phi \rangle \simeq \frac{g^2}{18}\left(\frac{T}{H}\right)^2\left(\frac{M}{M_p}\right)\sqrt{\frac{\epsilon_\phi}{2}}\sin(2\phi/M)$$

Inflaton perturbations

$$\phi_k = \overline{\phi} + \delta\phi_k$$

$$\delta\ddot{\phi}_k + 3H(1+Q)\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} - \frac{g^2}{6}T^2\cos(2\phi/M)\right)\delta\phi_k = \xi_k$$

During inflation: $H \simeq T \simeq \dot{\phi} \simeq \text{constant}$

Change of variables: $\chi_k = a^{3/2}\delta\phi_k, \quad 2z = \alpha + \Omega t$

$$\partial_z^2 X_k + (A_k(z) - 2q\cos(2z))X_k = \tilde{\xi}_k$$

where $\Omega = 2\dot{\phi}/M, \quad A_k(z) = \dfrac{4k^2}{\Omega^2 a^2}, \quad q = \dfrac{g^2}{3}\dfrac{T^2}{\Omega^2} \ll 1$

We will ignore the noise term for now...

Inflaton perturbations

Solutions of Mathieu's equation: $\chi(z) = e^{\mu z} f(z)$

Floquet exponent: $\mu \simeq \frac{1}{2}\sqrt{|q|^2 - (A_k - 1)^2}$

→

Time a mode spends inside the resonance band:

$$\Delta z = z_f - z_i = \frac{\Omega}{4H} \log \frac{1+q}{1-q} \ll 1$$

Band centralised at:

$$k_c \simeq \frac{\Omega}{2} a \gg aH$$

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Expected behaviour of the solutions:

For $z \to -\infty$:

$$X_k \to X_k^{q=0}$$

$$A_k \gg 1$$

For z inside the band

$$X_k \propto e^{\int_{z_i}^{z} \mu dz}$$

$$A_k \sim 1$$

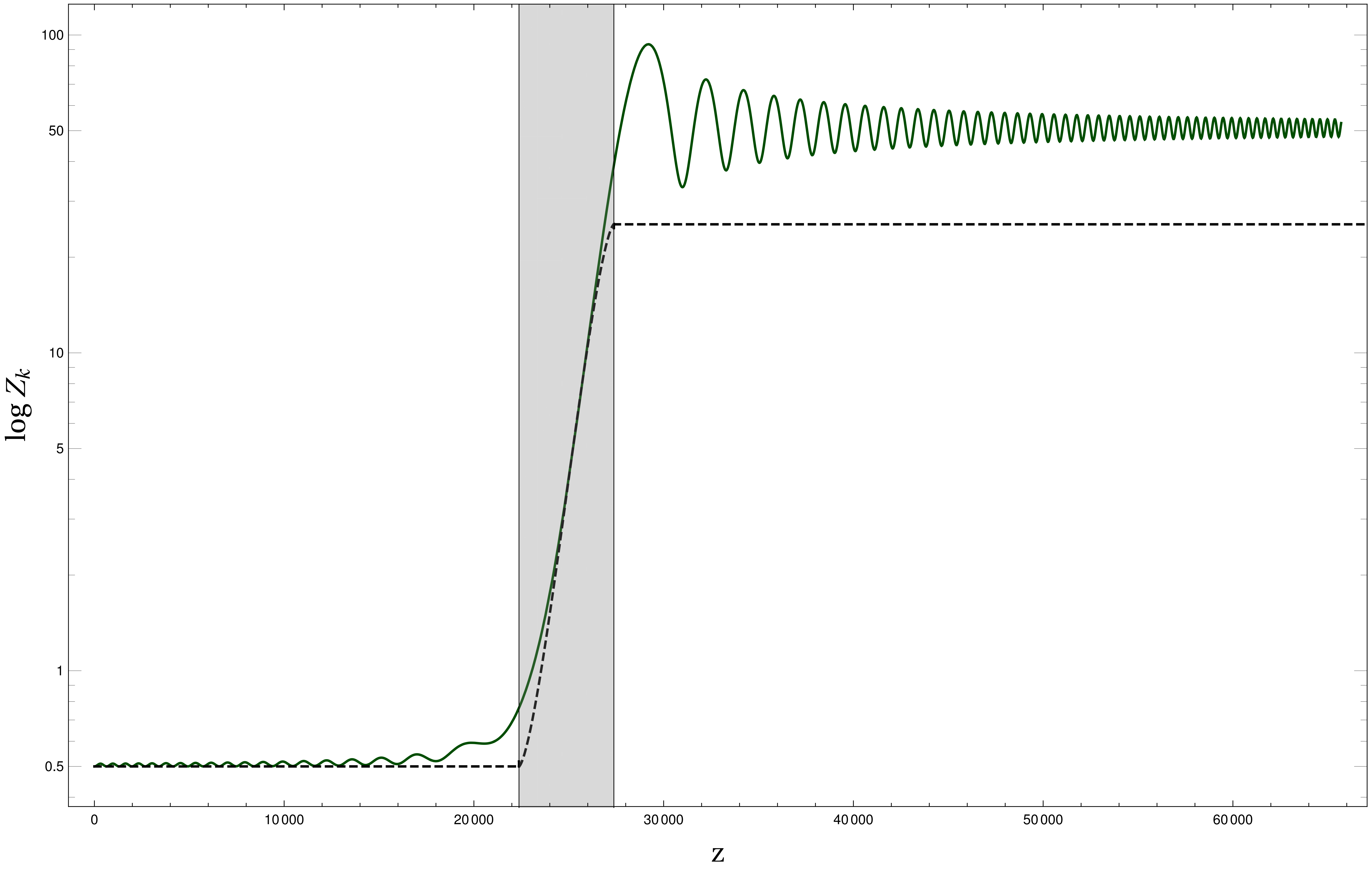
For $z \to +\infty$:

$$X_k \to X_k^{q=0} e^{\int_{z_i}^{z_f} \mu dz}$$

$$A_k < 1$$

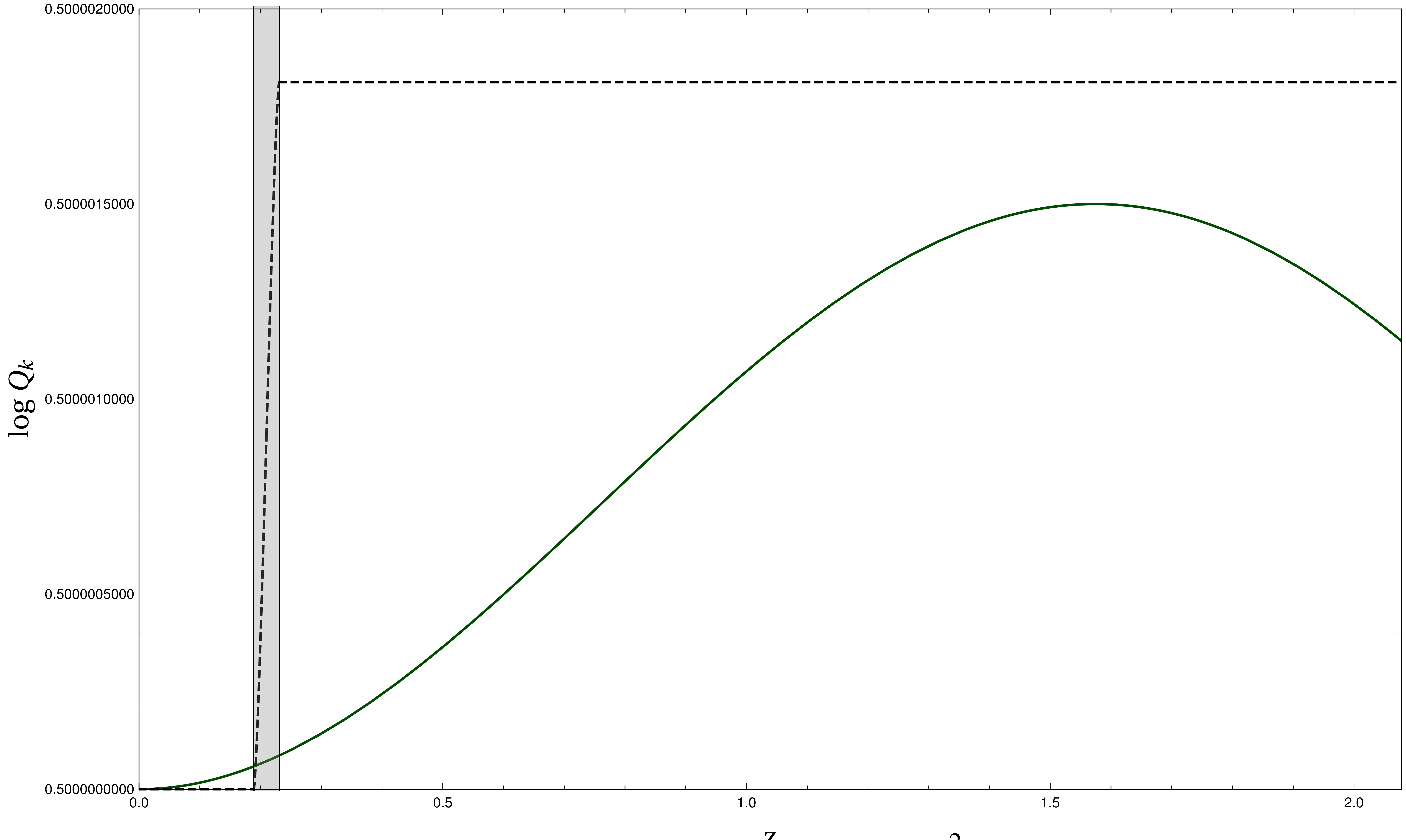
$$\omega_k^2(z) = A_k(z) - 2q\cos(2z)$$

Parametric resonance



$$Z_k = n_k + \frac{1}{2} = \frac{\omega_k}{2}\left(|X_k|^2 + \frac{|\partial_z X_k|^2}{\omega_k^2} \right)$$

Parametric resonance



$$Z_k = n_k + \frac{1}{2} = \frac{\omega_k}{2}\left(|X_k|^2 + \frac{|\partial_z X_k|^2}{\omega_k^2} \right)$$

Power spectrum

$$\mathscr{P}_{X_k} = \frac{k^3}{2\pi^2}|X_k|^2 \simeq \frac{k^3}{2\pi^2}|X_k^{q=0}|^2 \exp\int_{z_i}^{z_f} \mu dz = \mathscr{P}_{X_k^{q=0}} \exp(\frac{\Omega}{8H}|q|^2\pi) \simeq \mathscr{P}_{X_k^{q=0}}$$

At horizon crossing

$k = aH$

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At horizon crossing

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Can these oscillations be seen?

$$\mathscr{P}_R = \frac{H^2}{\dot{\phi}}\mathscr{P}_{\delta\phi}$$

Can we decouple noise from parametric resonance?

$$\delta\phi_k(t) = \int G(t,y)\xi(y)dy$$

Thank you!