Hands-on on Neutrinos

Basics on neutrino's oscillations

We will now start the hands on part

You will have a series of questions to answer online

You have 5 min for question and then we will discuss the answers together

A little bit of quantum mechanics

Quantum systems are described by wave functions that can be a superposition of states.
Therefore, a neutrino of a given flavor ***X*** as being represented by a state $|\nu_X\rangle$ = "flavor eigenstates".

Let us take the case of only two flavors, ν_e and ν_μ, associated to quantum states $|\nu_e\rangle$ and $|\nu_\mu\rangle$ and further assume that these states are not mass eigenstates.

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It means that they do not coincide with the eigenstates of the Hamiltonian for a free particle with mass m_i and energy $E_i^2 = p_i^2c^2 + m_i^2c^4$
However, this means that we can also have "mass eigenstates" for the neutrinos. Let's call them $|\nu_1\rangle$ and $|\nu_2\rangle$ which are eigenstates of the free particle's hamiltonian.

Can you think of a way to write the flavour eigenstates as a combination of the mass eigenstates?

Hints:

- Think of the simples parametrisation
- This superposition should obey the probability conservation law of quantum dynamics
- The two states should be orthogonal

Solution 1

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Hints:

- Think of the simples parametrisation
- This superposition should obey the probability conservation law of quantum dynamics
- The two states should be orthogonal

Solution:

- The simples way to write the flavour eigenstates as a function of the mass ones, is by applying the rotation matrix with one mixing angle.
- The matrix is unitary and the two eigenstates are orthogonal one to the other: $\langle \nu_e | \nu_\mu \rangle = \delta_{e\mu}$ and $\langle \nu_1 | \nu_2 \rangle = \delta_{12}$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

A little bit of quantum mechanics

Suppose now that at an electron neutrino described by the state $|\nu_e\rangle$ is produced in the Sun as a result of some some nuclear reaction.
Taking into account that the propagation of mass eigenstates follows the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial|\nu_i(t)\rangle}{\partial t} = H|\nu_i(t)\rangle = E|\nu_i(t)\rangle$$

Obtain $|\nu_e(t)\rangle$, which represents your flavor state **e** at any instant of time t.

- Use the natural units! $c = \hbar = 1$

Solution 2

Obtain $|\nu_e(t)\rangle$, which represents your flavor state **e** at any instant of time t.

Solution:

- Let's start by writing the flavour eigenstate as a function of the mass ones at the time t=0:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$|\nu_e(t=0)\rangle = \cos\theta|\nu_1(t=0)\rangle + \sin\theta|\nu_2(t=0)\rangle$$

- Now let's add the time dependence of the mass eigenstates

$$i\hbar\frac{\partial|\nu_i(t)\rangle}{\partial t} = E|\nu_i(t)\rangle \rightarrow |\nu_i(t)\rangle = |\nu_i(0)\rangle e^{-iEt}$$

- Where $E_i^2 = p_i^2c^2 + m_i^2c^4$
- Finally, let's add things together:

$$|\nu_e(t)\rangle = \cos\theta e^{-iE_1t}|\nu_1(0)\rangle + \sin\theta e^{-iE_2t}|\nu_2(0)\rangle$$

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What is the probability that, at a time t_1 your electron neutrino has oscillated into a muon neutrino?

Hint:

- The probability is defined as

$$P(\nu_e \to \nu_\mu) = |A(\nu_e \to \nu_\mu)|^2 = |\langle\nu_\mu|\nu_e(t)\rangle|^2$$

- Use the natural units! $c = \hbar = 1$
- Remember that: $E_i^2 = p_i^2c^2 + m_i^2c^4$ and generally p>>m

Solution 3

What is the probability that, at a time ***t₁*** your electron neutrino has oscillated into a muon neutrino?

Solution:

- The probability is defined as

$$P(\nu_e \to \nu_\mu) = |A(\nu_e \to \nu_\mu)|^2 = |\langle \nu_\mu | \nu_e(t) \rangle|^2 = |\langle \nu_\mu | (\cos\theta e^{-iE_1 t} | \nu_1(0) \rangle + \sin\theta e^{-iE_2 t} | \nu_2(0) \rangle)|^2$$

- Now let's write $|\nu_1(0)\rangle$ and $|\nu_2(0)\rangle$ as a function of $|\nu_e(0)\rangle$ and $|\nu_\mu(0)\rangle$ (we just need to reverse the rotation matrix!)

$$|\nu_1\rangle = \cos\theta |\nu_e\rangle - \sin\theta |\nu_\mu\rangle, |\nu_2\rangle = \sin\theta |\nu_e\rangle + \cos\theta |\nu_\mu\rangle$$

- Which by using the orthogonality between the states, gives:

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \nu_e(t) \rangle|^2 = (\cos\theta \sin\theta)^2 |e^{-iE_1 t} - e^{-iE_2 t}|^2$$

$$(\cos\theta \sin\theta)^2 \to \frac{\sin^2 2\theta}{4}$$

$$|e^{-iE_1 t} - e^{-iE_2 t}|^2 \to 2 \times [1 - \cos[t(E_1 - E_2)]] = 4\sin^2\left((E_1 - E_2)\frac{t}{2}\right)$$

- Finally, using c = 1 and p >> m for neutrinos, we can substitute t with L and $E_i = E\left(1 + \frac{m_i^2}{2E^2}\right)$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left((m_2^2 - m_1^2)\frac{L}{4E}\right)$$

A little bit of quantum mechanics

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What are the necessary conditions for neutrino oscillations to occur?

Solution 4

What are the necessary conditions for neutrino oscillations to occur?

Solution:

- A mass different from zero
- Different mass values
- A distance with respect to the source

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How do you express the probability that at a distance ***L*** from the source, the neutrinos of a given energy E, can be detected in the same flavor as they were produced?

Solution 5

How do you express the probability that at a distance ***L*** from the source, the neutrinos of a given energy E, can be detected in the same flavor as they were produced?

$$P(\nu_e \rightarrow \nu_e, L) = 1 - \sin^2 2\theta \sin^2\left(1.27\Delta m^2[eV^2]\frac{L[m]}{E[MeV]}\right)$$

Congratulations! You have just done Nobel Prize Physics

Experimental neutrino physics: (Anti)neutrinos produced in a reactor

Electron anti-neutrinos from a nuclear reactor were the first ones to be detected.

Their detector is extremely challenging since neutrinos have an extremely small interaction cross section with matter = $\sim 10^{-42}$ cm^2.

In order to detect neutrinos we need therefore a large target mass. One of the most common and cheap materials is WATER = density = 1 g/cm^3.

Finally we need a large neutrino flux. A nuclear reactor has generally 10 GW of power. The energy is given by uranium and plutonium fission chains, each releasing ~200 MeV of energy and 2 electron-anti-neutrinos. The energy spectrum of the emitted neutrinos peaks at 4 MeV.

How many anti-neutrinos per second are produced?

Solution 6

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$$I_{\bar{\nu}} = \frac{P}{E_{reaction}} \times n_{nu} = 6 \times 10^{20} \bar{\nu}/s$$

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How many will reach a detector located at a distance of 100 m from the reactor?

Solution 7

How many will reach a detector located at a distance of 100 m from the reactor?

$$\Phi_{\bar{\nu}} = \frac{I_{\bar{\nu}}}{A} = \frac{I_{\bar{\nu}}}{4\pi r^2} = 5 \times 10^{15} \bar{\nu}/m^2/s$$

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How many will produce signals in a 10 m^3 detector? Assume the interaction is with protons

This is called the near detector and is generally used to study the characteristics of the emitted neutrino flux

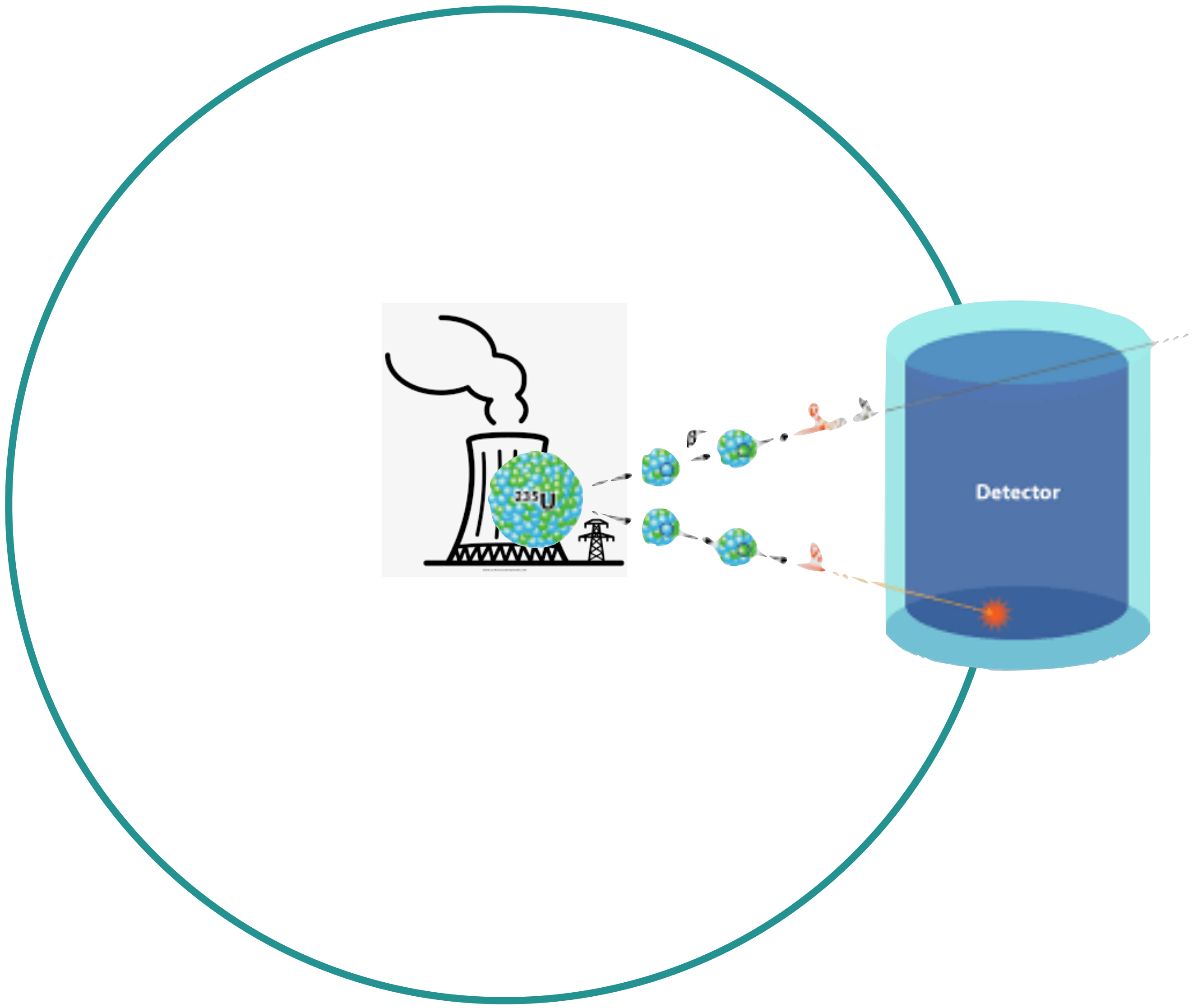
Can you guess why?

Hint: think about the oscillation effect you just study and its dependence with the distance

$$P_{osc} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 c^4}{4\hbar c}\frac{L}{E}\right) = \sin^2(2\theta)\sin^2\left(1.27\frac{\Delta m^2[eV^2]L[m]}{E[MeV]}\right), \Delta m^2 = m_2^2 - m_1^2 = 8 \times 10^{-5} eV^2$$

Solution 8

How many will produce signals in a 10 m³ detector? Assume the interaction is with protons



$$\Phi_{\bar{\nu}} = \frac{I_{\bar{\nu}}}{A} = \frac{I_{\bar{\nu}}}{4\pi r^2} = 5 \times 10^{15} \bar{\nu}/m^2/s$$

Interaction rate:

$$R = N_T \sigma \Phi_{\bar{\nu}} = 1.7/s$$

Target atoms N_T:

$$N_T = N_A \frac{\rho \cdot V}{m_T} n_p = 3.3 \times 10^{27} p/cm^3$$

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What it the best distance L at which to place another detector in order to measure the oscillation and check if it can be described by the solar oscillation parameters

$$\Delta m^2 = 8 \times 10^{-5} eV^2 \text{ and } \sin^2 2\theta = 0.856$$

This is called the far detector and is used to study if neutrinos do actually oscillate

Solution 9

What it the best distance L at which to place another detector in order to measure the oscillation and check if it can be described by the solar oscillation parameters

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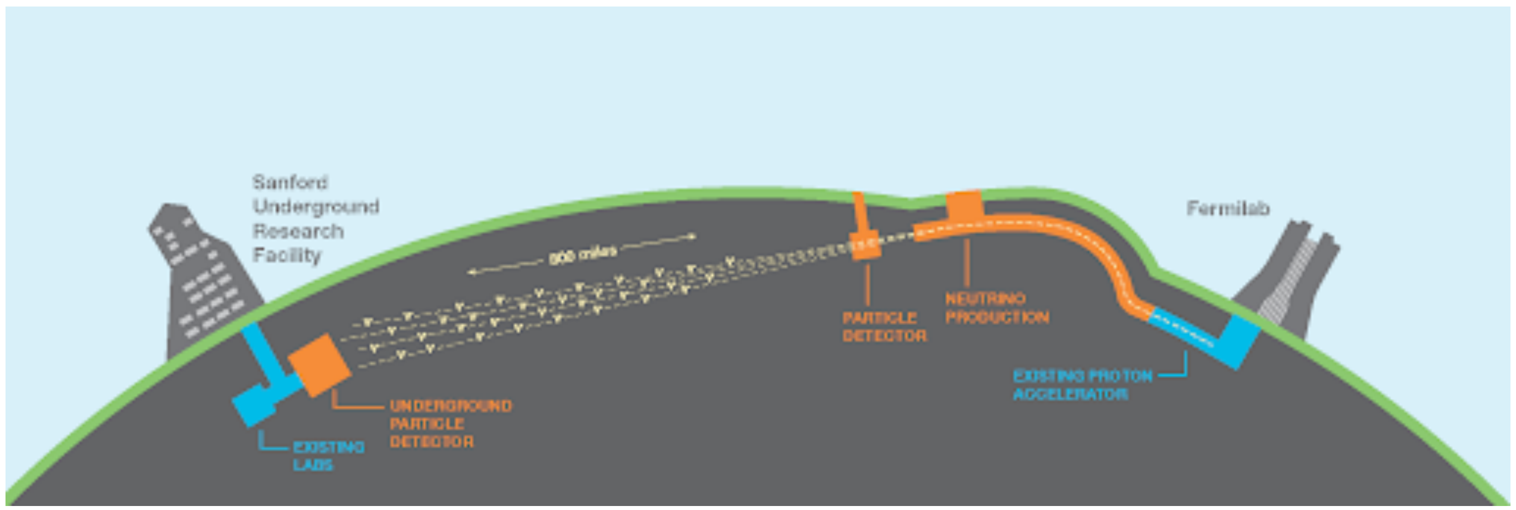


The maximum oscillation is reached when the argument of the sin is max.

$$\frac{\pi}{2} = \frac{1.27 \Delta m^2 L}{E} \rightarrow L = 62 km$$

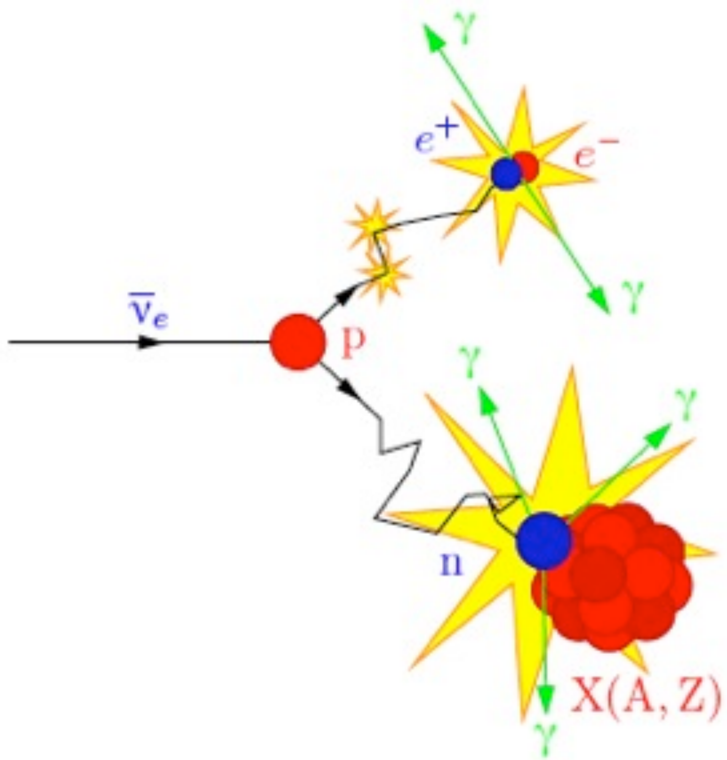
For a 4 MeV anti-neutrino

Experimental neutrino physics: The interaction

The main reaction of anti-neutrinos with water is the called INVERSE BETA DECAY: $\bar{\nu}_e + p \rightarrow n + e^+$

In the final state a neutron and a positron are emitted.

The neutron is used to tag anti-neutrino interactions.
The positron kinetic energy is used to measure the anti-neutrino energy, related to it by $E_{\nu_e} = E_{e^+} + \Delta$



What is the minimum energy threshold Δ, for this interaction to occur?

Solution 10

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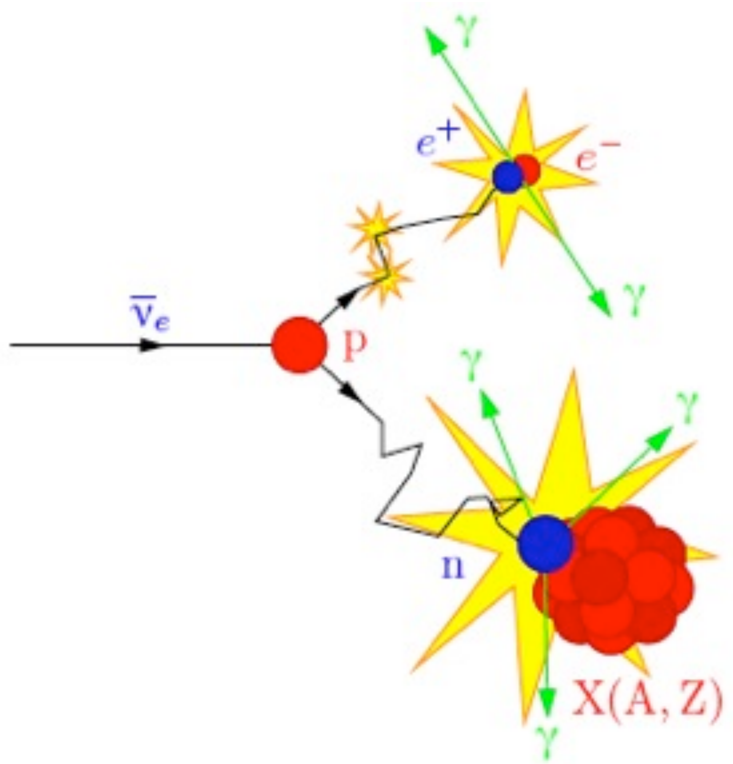

$$\bar{\nu}_e + p \rightarrow n + e^+$$

In the inverse beta decay reaction, it is necessary a minimum antineutrino energy to produce the neutron and positron at rest

$$E_{\bar{\nu}} = (m_n - m_p + m_e) = 1.8 MeV$$

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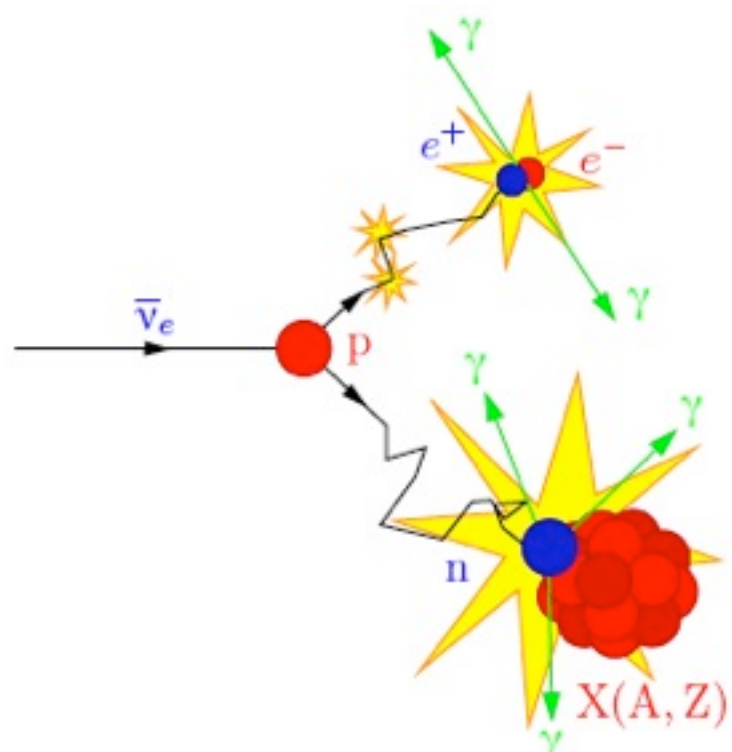

What is the energy threshold if we have muon anti-neutrinos?

$$\bar{\nu}_\mu + p \rightarrow n + \mu^+$$

Solution 10

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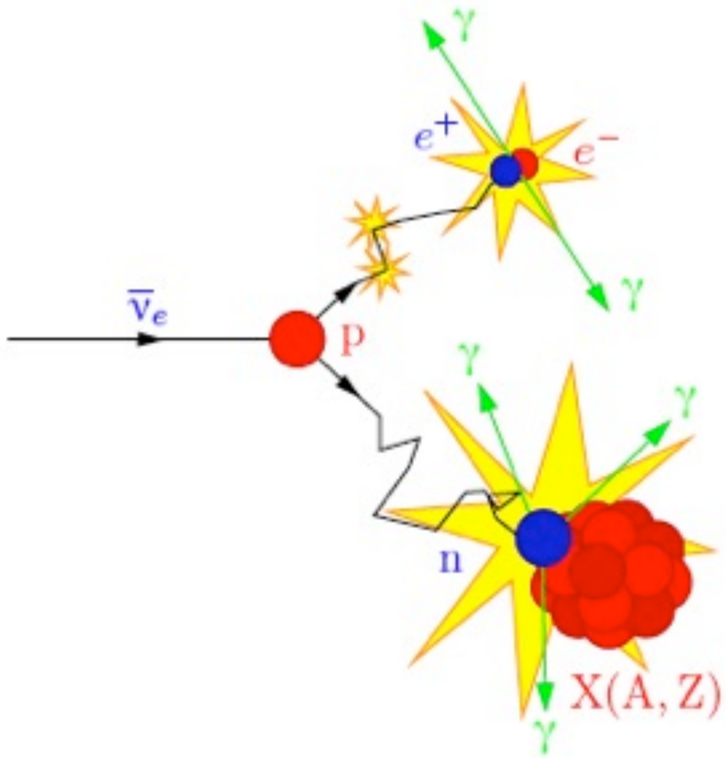

$$\bar{\nu}_\mu + p \rightarrow n + \mu^+$$

In the inverse beta decay reaction with muon antineutrinos, the necessary minimum energy for the reaction to happen is much larger, due to the larger muon mass compared to the one of the electron (nearly 200x)

$$E_{\bar{\nu}} = (m_n - m_p + m_\mu) = 107 MeV$$