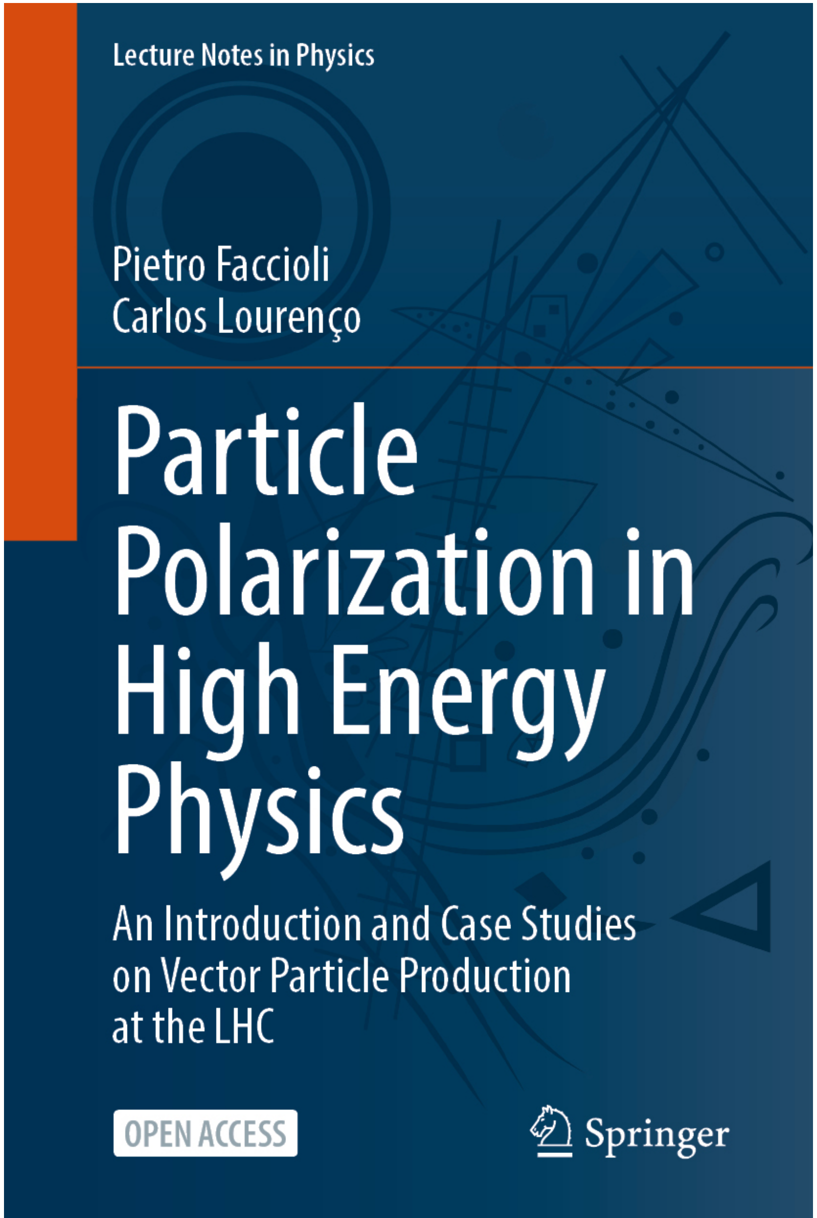
Measurements of particle polarization

- Dilepton decays of vector particles (Z, W, photon, J/ψ, etc.)
- The Lam-Tung relation

Pietro Faccioli

Course on "Physics at the LHC"
May 11th, 2022

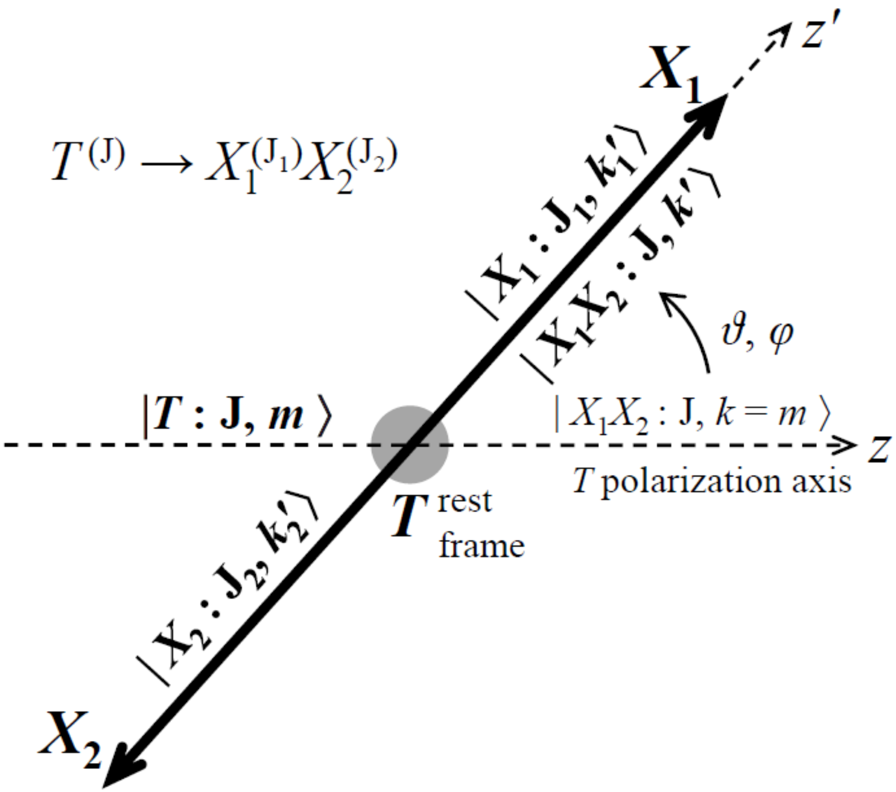
Coming soon:



Lecture Notes in Physics, Springer, 2022
ISBN: 978-3-031-08874-2
Open Access

Basics

Measure **polarization** of a particle = measure the (average) **angular momentum composition** in which the particle is produced, by studying the **angular distribution** of its **decay** in its rest frame



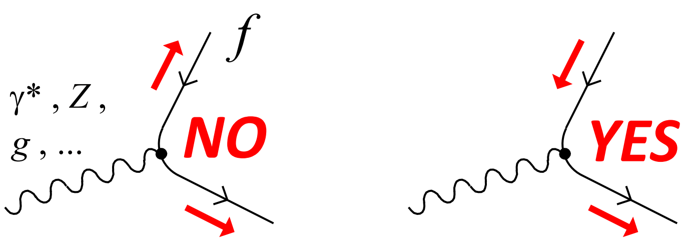
Polarization of vector particles

J* = 1** → three J_z eigenstates **| 1, +1 ⟩, | 1, 0 ⟩, | 1, -1 ⟩** wrt a certain ***z

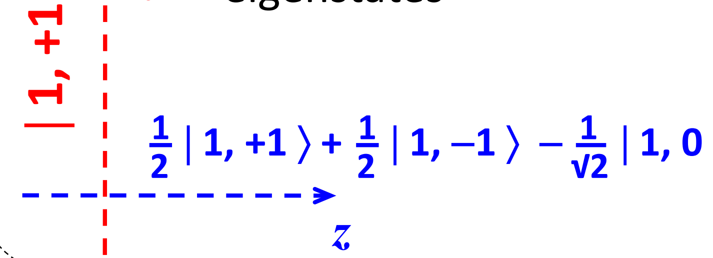
The decay **into a fermion-antifermion pair** is an especially clean case to be studied

The shape of the observable angular distribution is determined by a few basic principles:

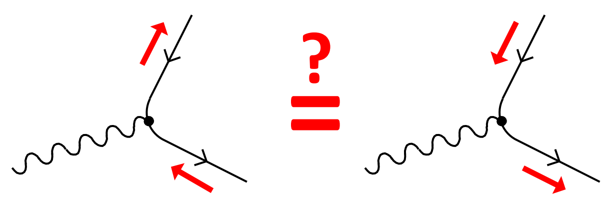
1) elementary coupling properties
"helicity conservation"



2) rotational covariance
of angular momentum eigenstates

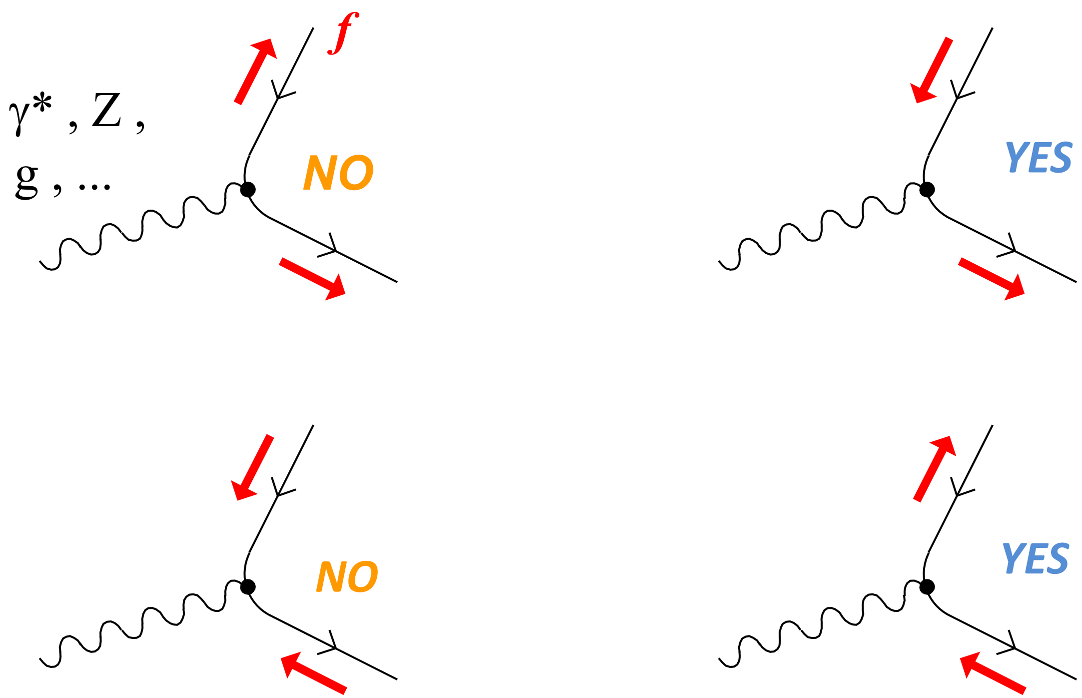


3) parity properties

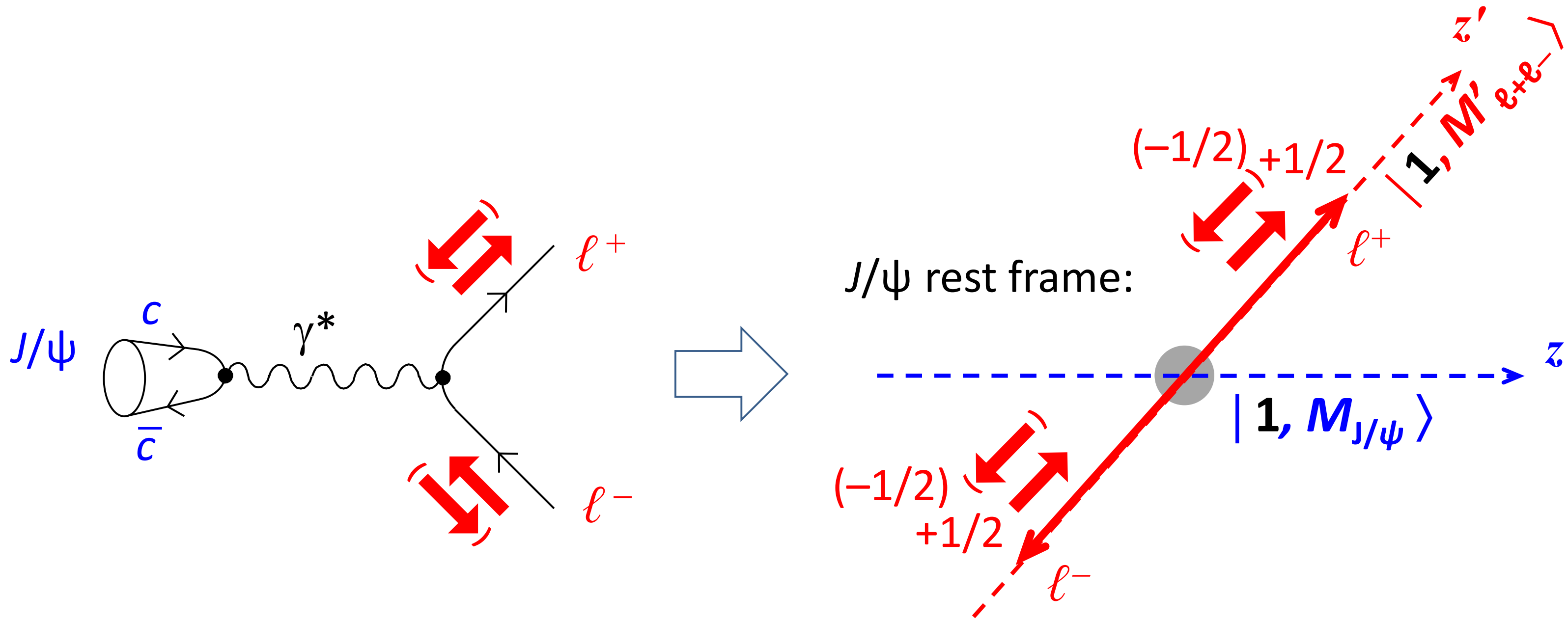


1: helicity conservation

Relevant property for cases considered here:
EW and strong forces preserve the *chirality* (L/R) of fermions.
In the relativistic (massless) limit, *chirality* = ***helicity* = spin-momentum alignment**
→ the **fermion spin never flips** in the coupling to gauge bosons:



example: dilepton decay of J/ψ



J/ψ angular momentum component along the polarization axis ***z:***

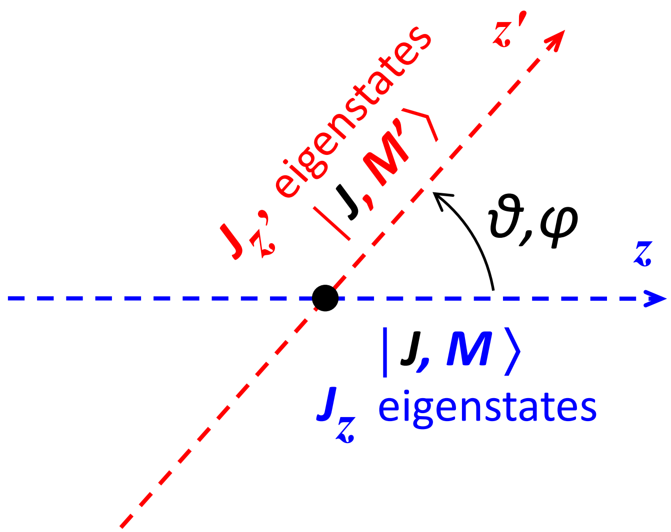
$M_{J/\psi}$ = **-1, 0, +1** (determined by *production mechanism*)

The **two leptons** can only have total angular momentum component

$M'_{\ell^+\ell^-} = +1$ or -1 along their common direction ***z'***

0 is forbidden

2: rotational covariance of angular momentum eigenstates



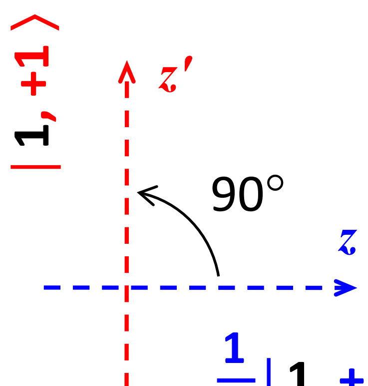
change of quantization frame:

$R(\vartheta,\varphi)$: $z \to z'$
$y \to y'$
$x \to x'$

$$|J, M'\rangle = \sum_{M=-J}^{+J} D^{J}_{MM'}(\vartheta,\varphi)\;|J, M\rangle$$

Wigner D-matrices

Example:



$$\frac{1}{2}|1,+1\rangle + \frac{1}{2}|1,-1\rangle - \frac{1}{\sqrt{2}}|1,0\rangle$$

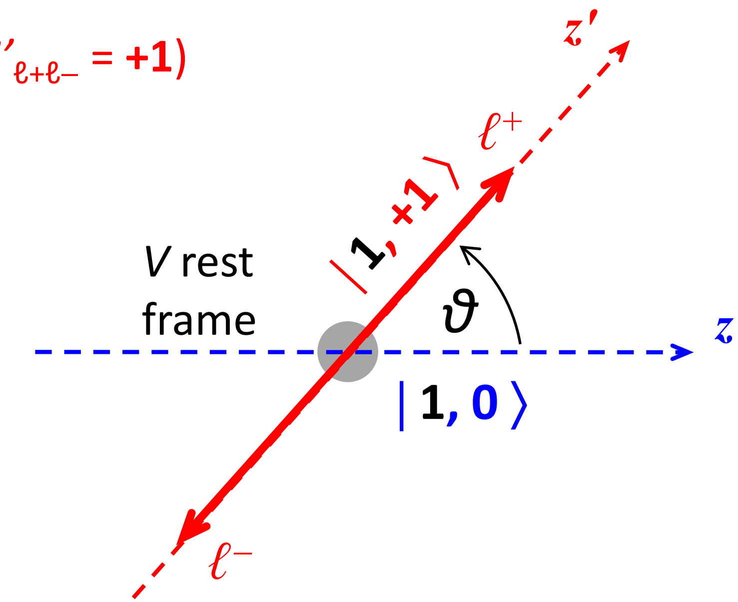
Classically, we would expect $|1, 0\rangle$

example: $M = 0$

$V\,(M_V = 0) \rightarrow \ell^+\ell^-(M'_{\ell^+\ell^-} = +1)$

$\downarrow$

$V = J/\psi \mid Z$



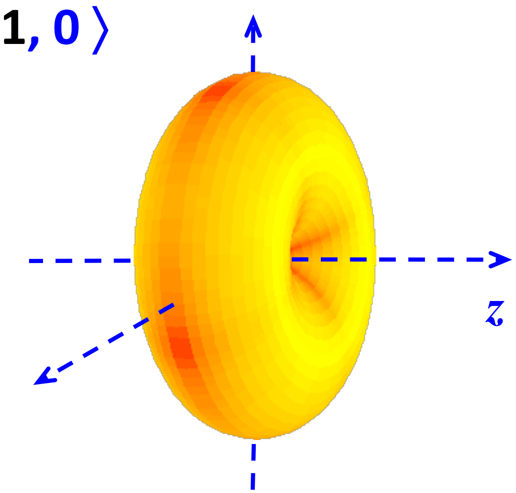
$$|\,1, +1\,\rangle = D^1_{-1,+1}(\vartheta,\varphi)\;|\,1, -1\,\rangle + D^1_{0,+1}(\vartheta,\varphi)\;|\,1, 0\,\rangle + D^1_{+1,+1}(\vartheta,\varphi)\;|\,1, +1\,\rangle$$

→ the $J_{z'}$ eigenstate $|\,1, +1\,\rangle$ "contains" the J_z eigenstate $|\,1, 0\,\rangle$ with component amplitude $D^1_{0,+1}(\vartheta,\varphi)$

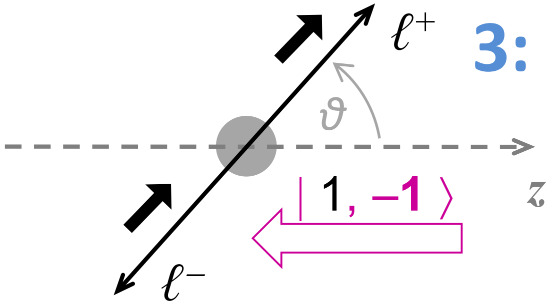
→ the decay distribution is

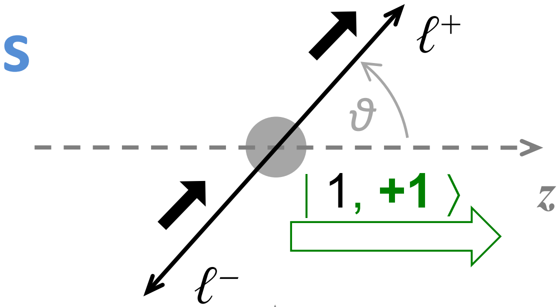
$$|\langle\,1, +1\,|\,\mathcal{O}\,|\,1, 0\,\rangle|^2 \propto |D^{1*}_{0,+1}(\vartheta,\varphi)|^2 = \frac{1}{2}\,(\,1 - \cos^2\vartheta\,)$$

$\ell^+\ell^- \leftarrow J/\psi$

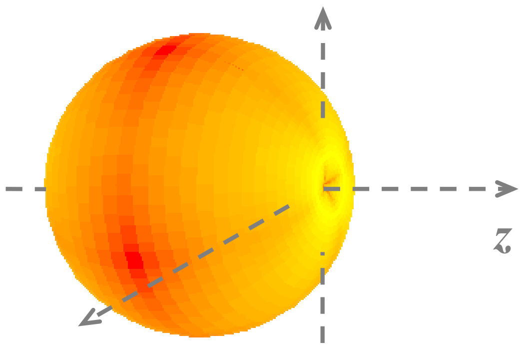


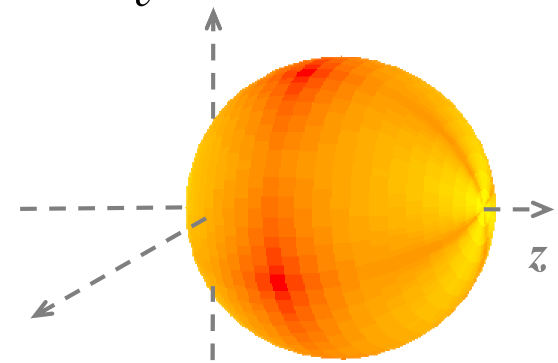
3: parity properties





| 1, –1 ⟩ and | 1, +1 ⟩ distributions are mirror reflections of one another

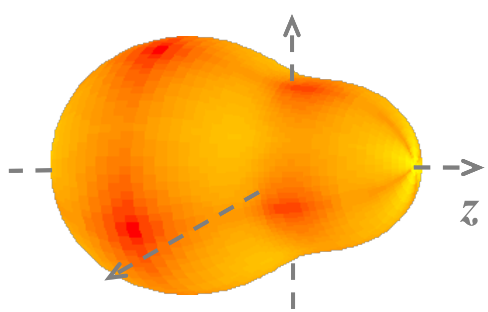


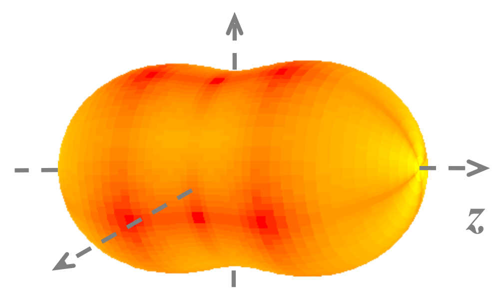


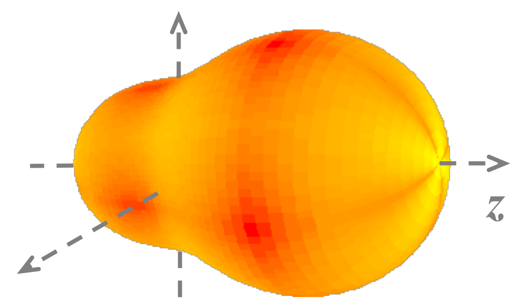
$$\frac{dN}{d\Omega} \propto |D^{1*}_{-1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta - \mathbf{2\cos\vartheta}$$

$$\frac{dN}{d\Omega} \propto |D^{1*}_{+1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta + \mathbf{2\cos\vartheta}$$

Are they equally probable?



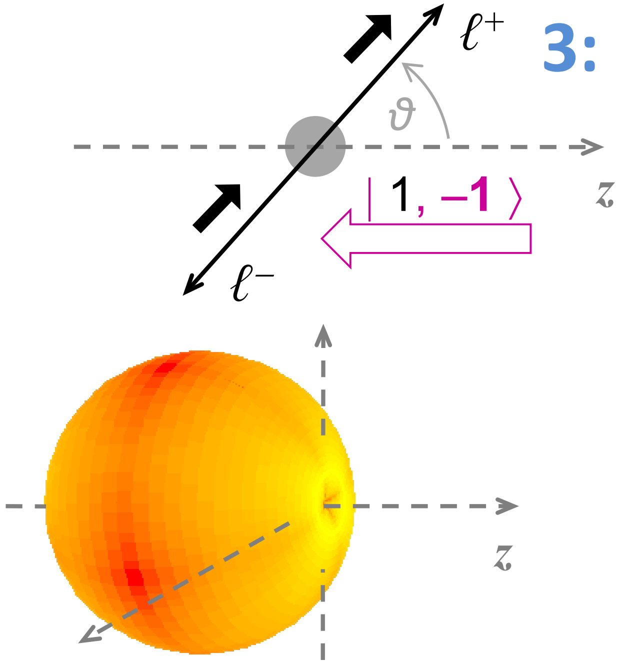




$\mathcal{P}(-1) > \mathcal{P}(+1)$ $\mathcal{P}(-1) = \mathcal{P}(+1)$ $\mathcal{P}(-1) < \mathcal{P}(+1)$

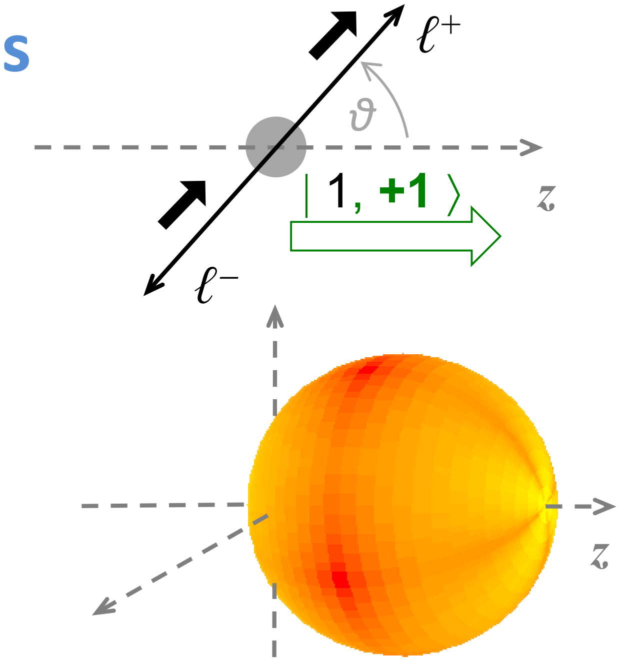
$$\frac{dN}{d\Omega} \propto 1 + \cos^2\vartheta + 2[\mathcal{P}(+1) - \mathcal{P}(-1)]\cos\vartheta$$

3: parity properties



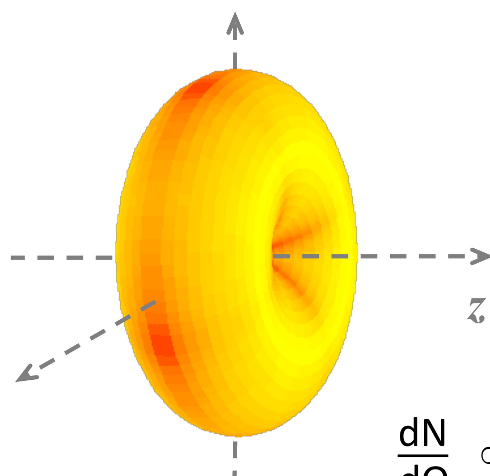
$$\frac{dN}{d\Omega} \propto |D^{1*}_{-1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta - \mathbf{2\cos\vartheta}$$

| 1, –1 ⟩ and | 1, +1 ⟩
distributions
are mirror reflections
of one another



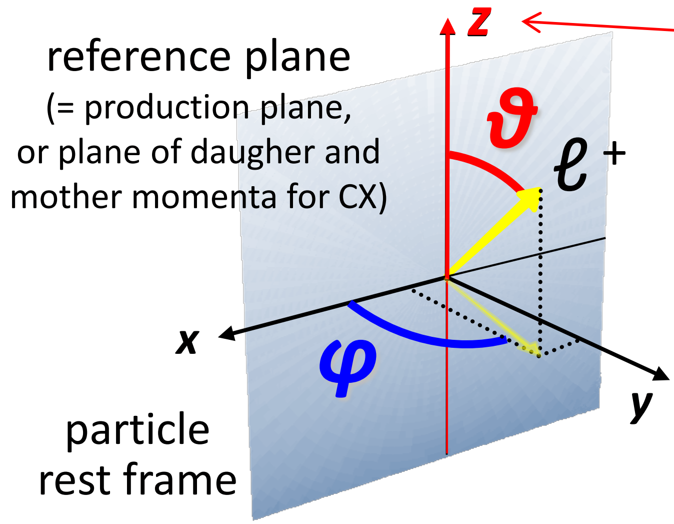
$$\frac{dN}{d\Omega} \propto |D^{1*}_{+1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta + \mathbf{2\cos\vartheta}$$

Decay distribution of | 1, 0 ⟩ state is always parity-symmetric:



$$\frac{dN}{d\Omega} \propto |D^{1*}_{0,+1}(\vartheta,\varphi)|^2 \propto 1 - \cos^2\vartheta$$

General distribution: reference frame



chosen polarization axis

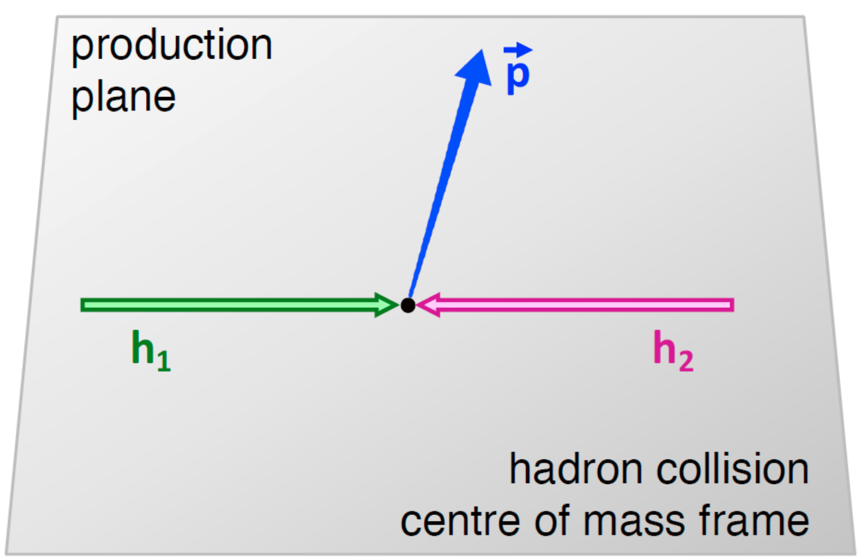
pp c.o.m. helicity (HX): particle direction wrt pp c.o.m.

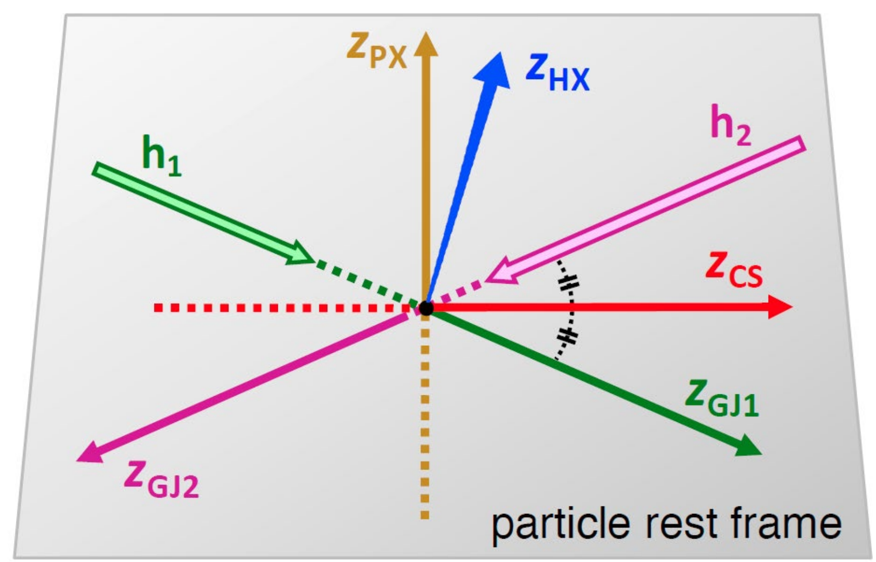
Gottfried-Jackson (GJ): direction of one or the other beam

Collins-Soper (CS): average of the two beam directions

perpendicular helicity (PX): perpendicular to CS

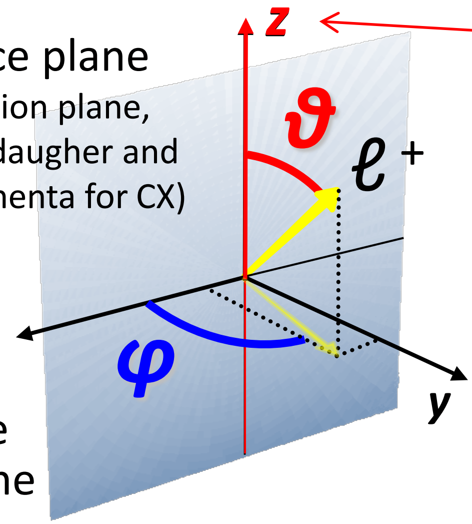
(etc.)





General distribution: shape

reference plane
(= production plane,
or plane of daugher and
mother momenta for CX)



particle
rest frame

chosen polarization axis

pp c.o.m. helicity (HX): particle direction wrt pp c.o.m.

Gottfried-Jackson (GJ): direction of one or the other beam

Collins-Soper (CS): average of the two beam directions

perpendicular helicity (PX): perpendicular to CS

(etc.)

average polar anisotropy — average azimuthal anisotropy — correlation polar - azimuthal

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\vartheta \cos^2\vartheta + \lambda_\varphi \sin^2\vartheta \cos 2\varphi + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos\varphi$$

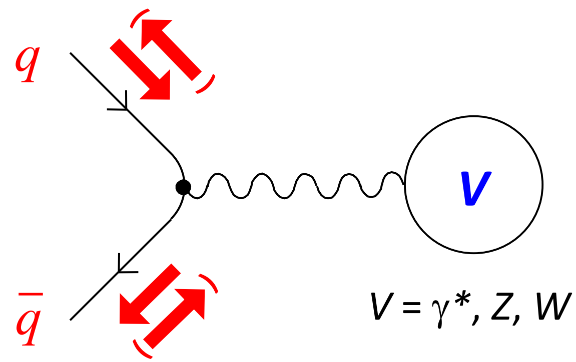
$$+ 2A_\vartheta \cos\vartheta + 2A_\varphi \sin\vartheta \cos\phi + \ldots$$

parity violating

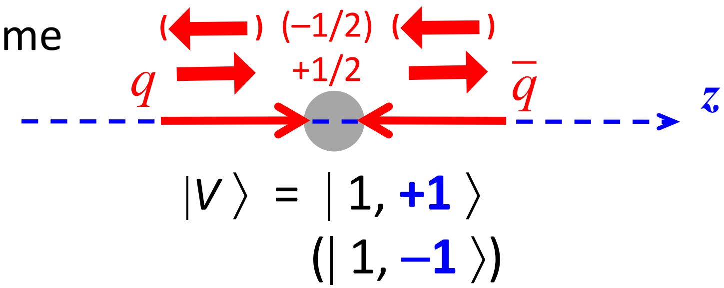
λ_ϑ, λ_φ, $\lambda_{\vartheta\varphi}$, etc. *depend* on the chosen frame [Faccioli *et al.*, EPJC 69, 657 (2010)]

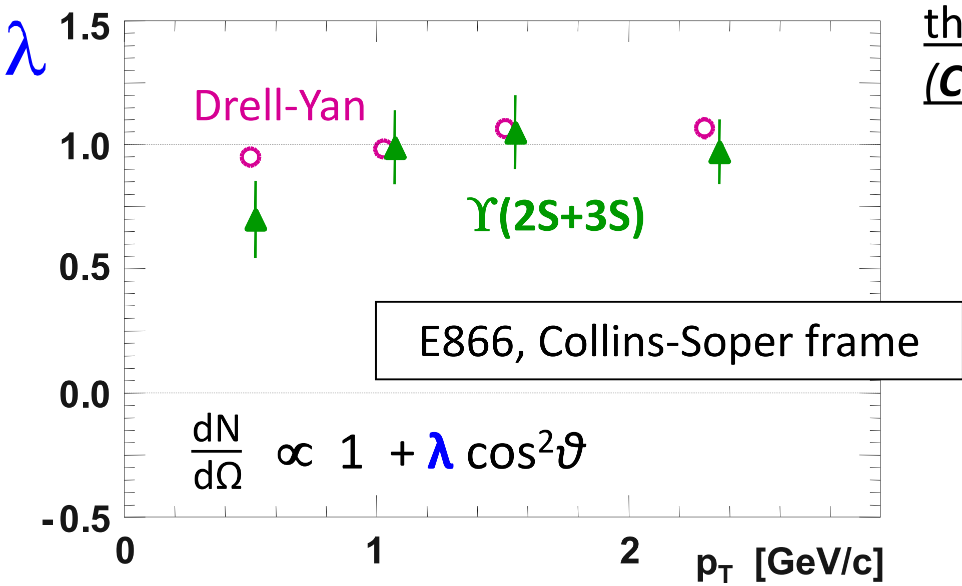
Why "transverse" (photon-like) polarizations are common

We can apply **helicity conservation at the *production* vertex** to predict that all *vector* states produced in ***fermion-antifermion annihilations*** **(q-$\bar{q}$ or e^+e^-)** at Born level have *transverse* polarization



q-$\bar{q}$ rest frame
= V rest frame





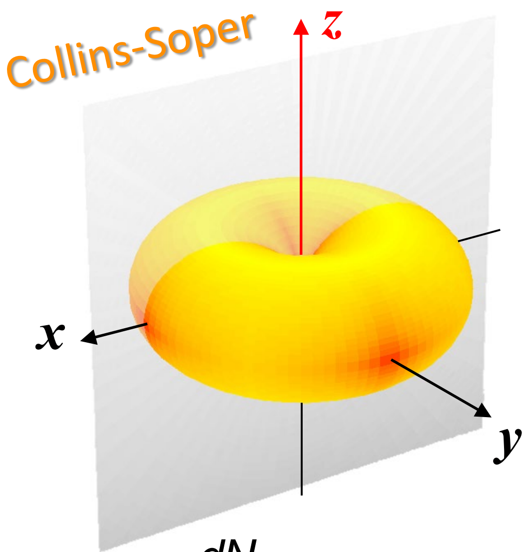
The "natural" polarization axis in this case is the *relative direction of the colliding fermions* ***(Collins-Soper axis)***

Drell-Yan is a paradigmatic case

But not the only one

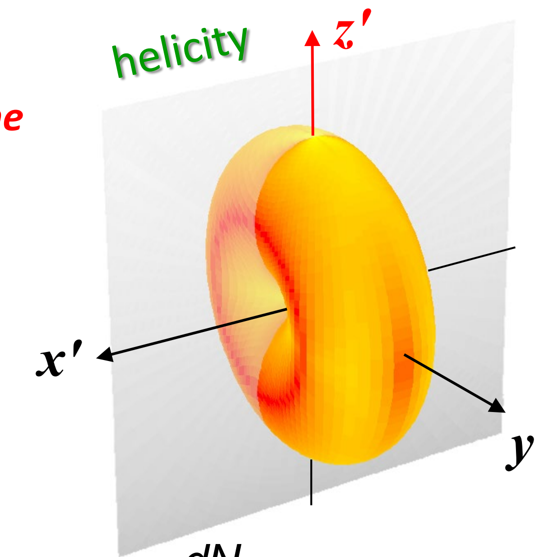
The observed polarization depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90º



***rotation** in the production plane!*

90º



$$\frac{dN}{d\Omega} \propto 1 - \cos^2\theta$$

longitudinal

$$|\psi\rangle = |0\rangle$$

(pure state)

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta \cos 2\varphi$$

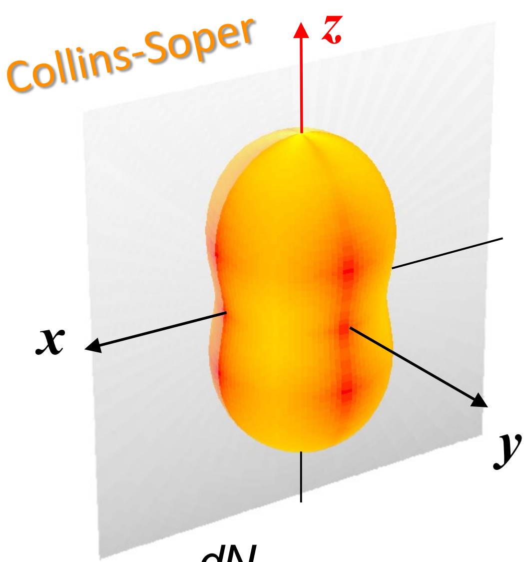
“transverse” (!)

$$|\psi\rangle = \frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$$

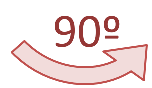
(mixed state)

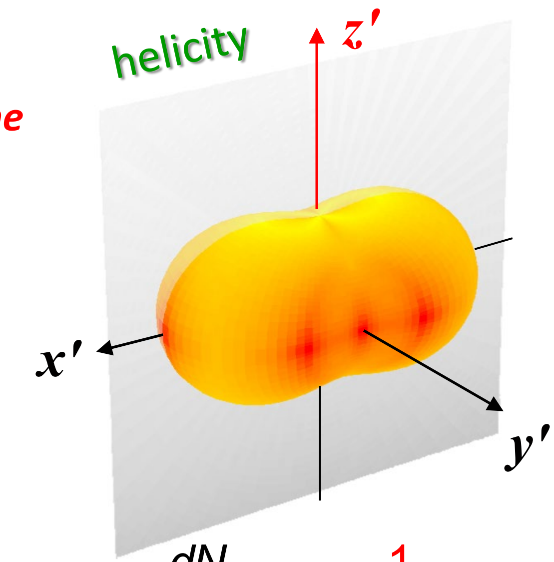
The observed polarization depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90º



rotation *in the production plane!*





$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta$$

transverse

$$\frac{dN}{d\Omega} \propto 1 - \frac{1}{3}\cos^2\theta + \frac{1}{3}\sin^2\theta\cos 2\varphi$$

moderately “longitudinal”

$$|\psi\rangle = |+1\rangle \text{ or } |-1\rangle$$

(pure state)

$$|\psi\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle \mp \frac{1}{\sqrt{2}}|0\rangle$$

(mixed state)

All reference frames are equal…
but some are more equal than others

What do different detectors measure with *arbitrary* frame choices?

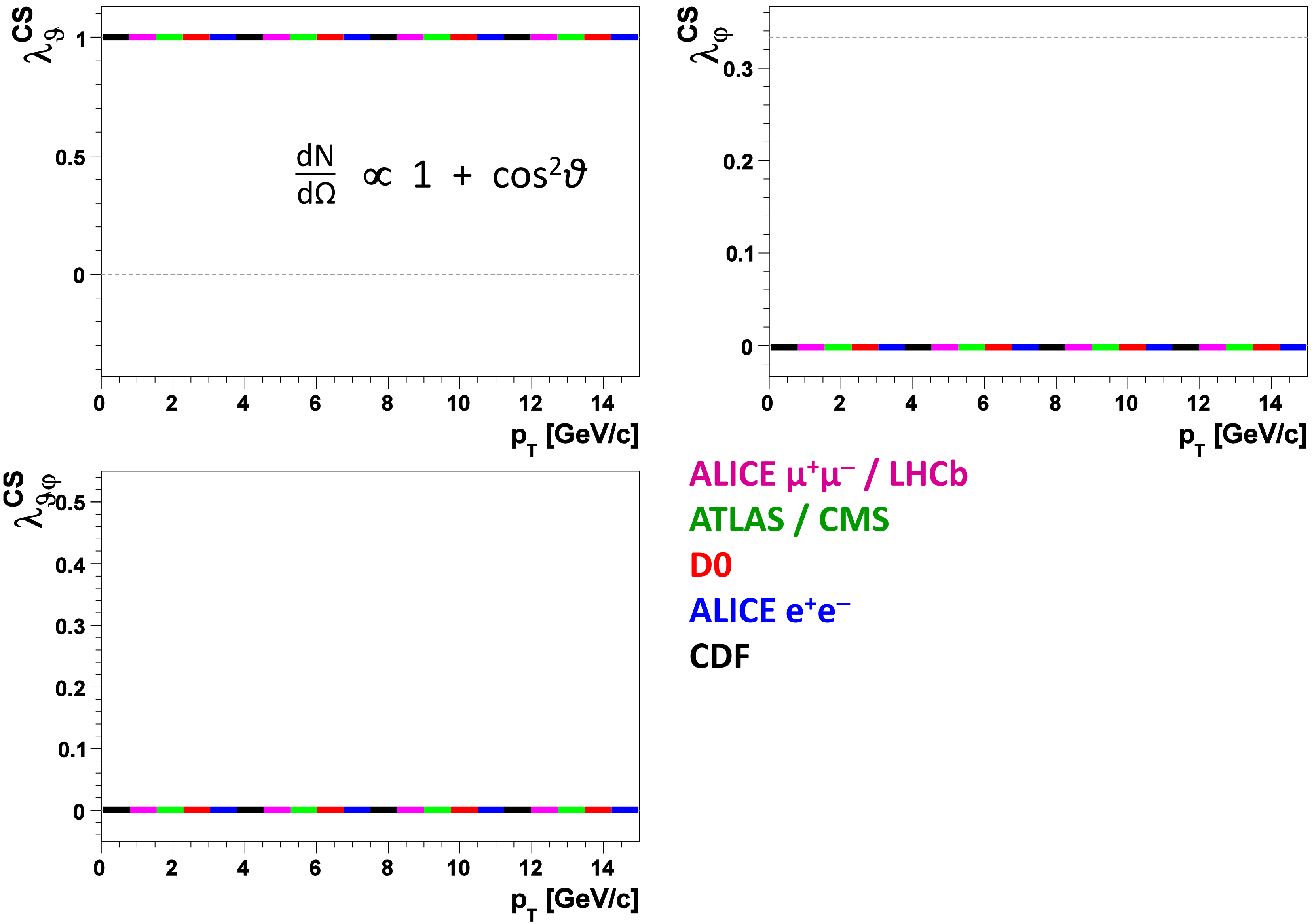
Gedankenscenario:

- **dileptons are fully transversely polarized in the CS frame**
- the decay distribution is measured at the $\Upsilon(1S)$ mass by 6 detectors with different **dilepton acceptances**:

CDF	$\|y\| < 0.6$
D0	$\|y\| < 1.8$
ATLAS & CMS	$\|y\| < 2.5$
ALICE e^+e^-	$\|y\| < 0.9$
ALICE $\mu^+\mu^-$	$2.5 < y < 4$
LHCb	$2 < y < 4.5$

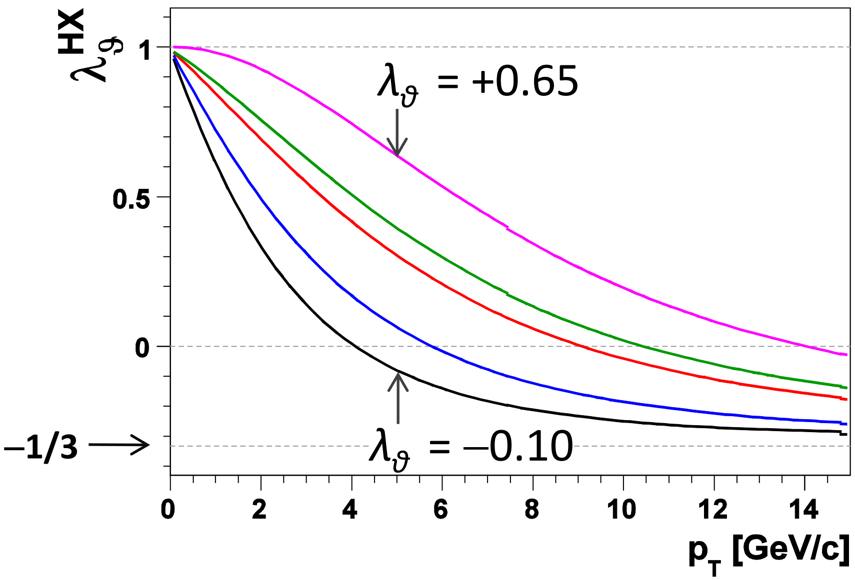
The lucky frame choice

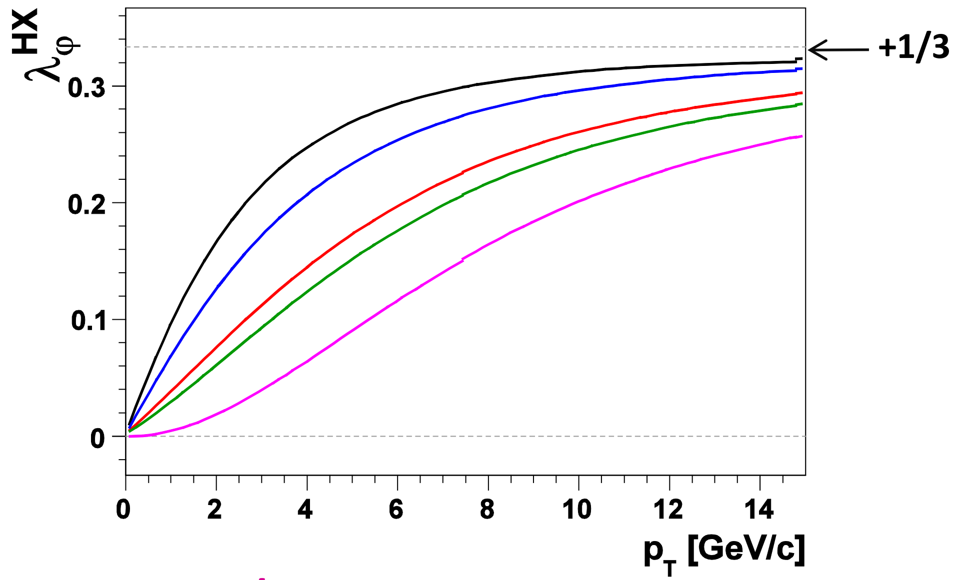
(CS in this case)

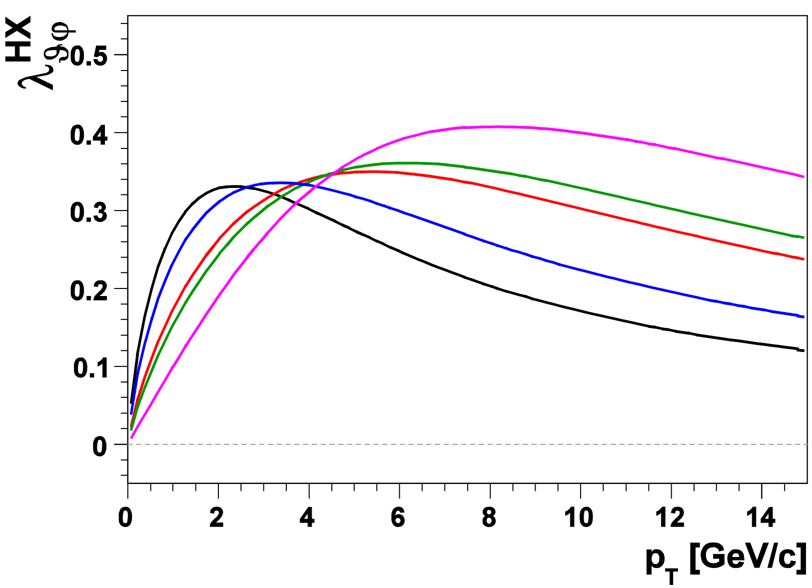


Less lucky choice

(HX in this case)







ALICE $\mu^+\mu^-$ / LHCb

ATLAS / CMS

D0

ALICE e^+e^-

CDF

artificial (experiment-dependent!) kinematic behaviour
→ measure in more than one frame!

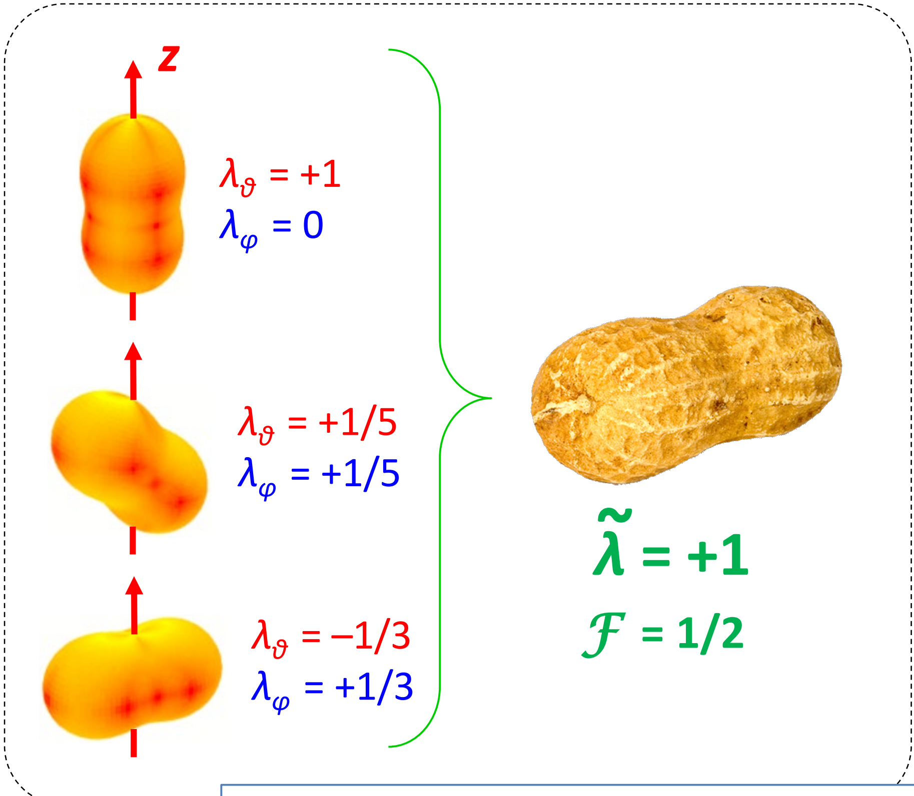
A complementary approach: frame-independent polarization

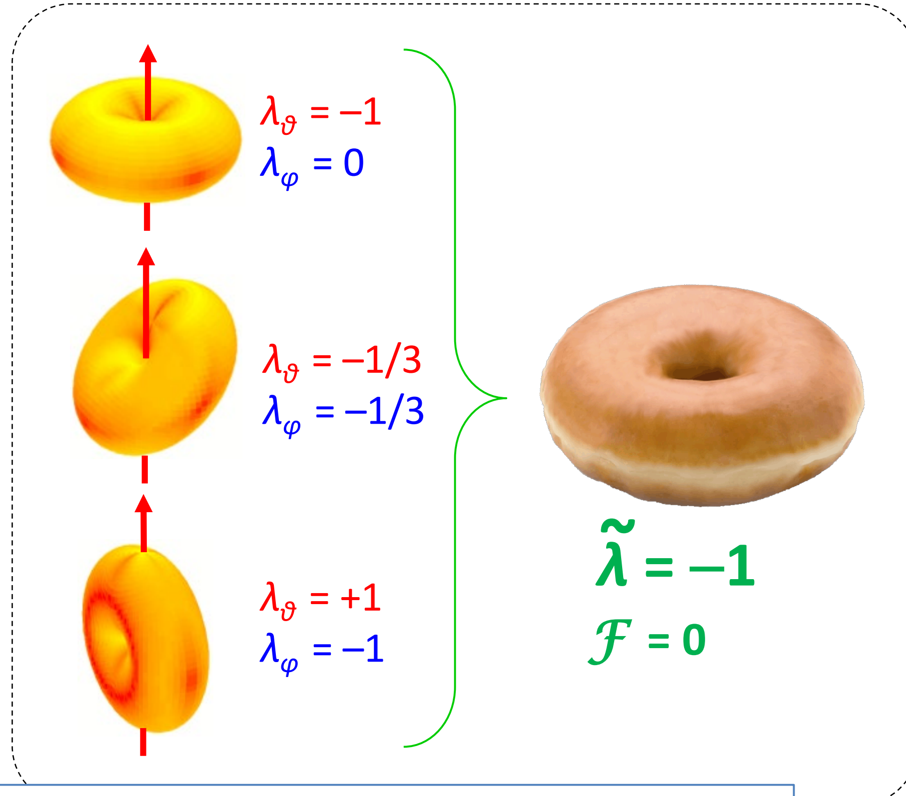
The *shape* of the distribution is (obviously) frame-invariant (= invariant by rotation)

→ it can be characterized by a frame-independent parameter, defined e.g. as

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi} \quad \text{or} \quad \mathcal{F} = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta} \quad \left(\mathcal{F} = \frac{1 + \tilde{\lambda}}{3 + \tilde{\lambda}} \right)$$

[Faccioli *et al.*, PRL 105, 061601 (2010)]





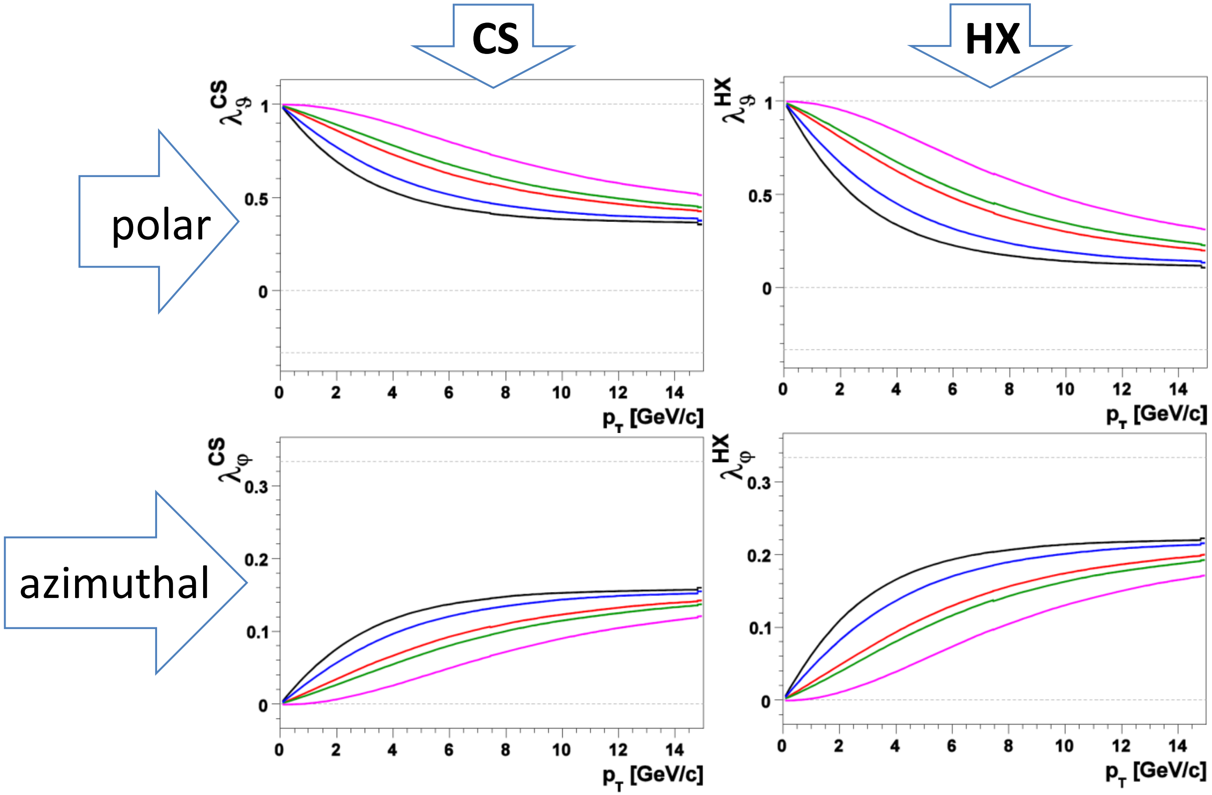
frame transformations HX ↔ CS ↔ GJ: all rotations *in the production plane!*

Example

Gedankenscenario: vector state produced in this subprocess admixture:

- **60%** processes with natural **transverse** polarization in the **CS** frame
- **40%** processes with natural **transverse** polarization in the **HX** frame

assumed indep. of kinematics, for simplicity



M = 10 GeV/c^2

CDF	**$\lvert y \rvert < 0.6$**
D0	**$\lvert y \rvert < 1.8$**
ATLAS/CMS	**$\lvert y \rvert < 2.5$**
ALICE e^+e^-	**$\lvert y \rvert < 0.9$**
ALICE $\mu^+\mu^-$	**$2.5 < y < 4$**
LHCb	**$2 < y < 4.5$**

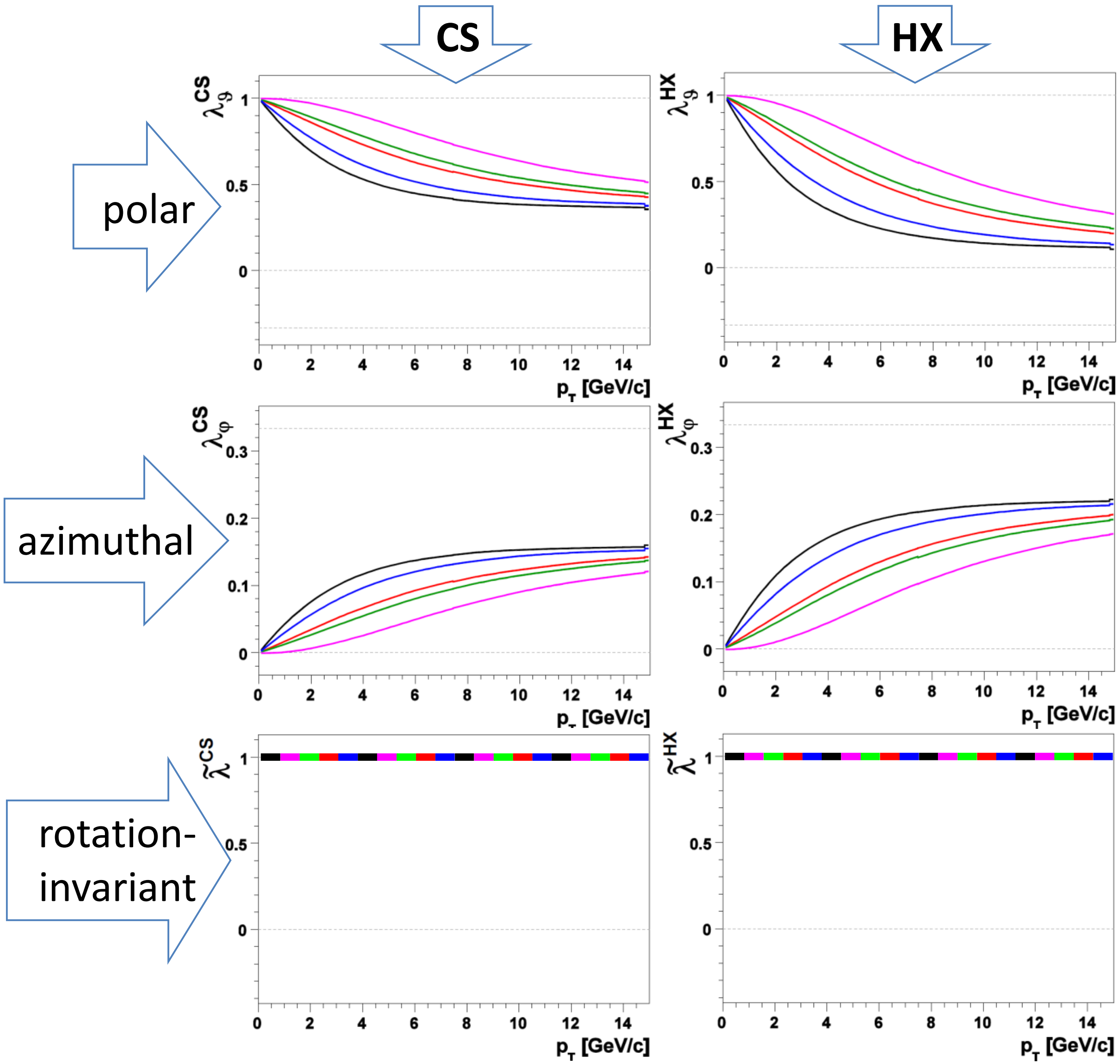
In neither frame we recognize that the natural polarization is always fully transverse!

Example

Gedankenscenario: vector state produced in this subprocess admixture:

- **60%** processes with natural **transverse** polarization in the **CS** frame
- **40%** processes with natural **transverse** polarization in the **HX** frame

assumed indep. of kinematics, for simplicity



M = 10 GeV/c^2

CDF	**|y| < 0.6**
D0	**|y| < 1.8**
ATLAS/CMS	**|y| < 2.5**
ALICE e^+e^-	**|y| < 0.9**
ALICE $\mu^+\mu^-$	**2.5 < y < 4**
LHCb	**2 < y < 4.5**

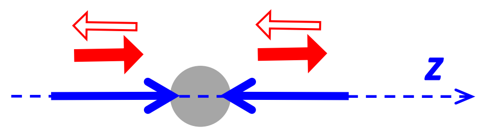
In neither frame we recognize that the natural polarization is always fully transverse!

- Immune to "extrinsic" kinematic dependences
  → *less acceptance-dependent*
  → *facilitates comparisons*
- *useful as closure test*

Frames for Drell-Yan, *Z* and *W* polarizations

- polarization is *always fully* ***transverse***...

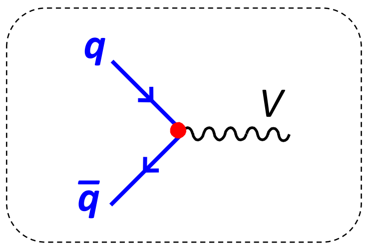
$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex,
$J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction ***z***

- ...but with respect to a ***subprocess-dependent quantization axis***

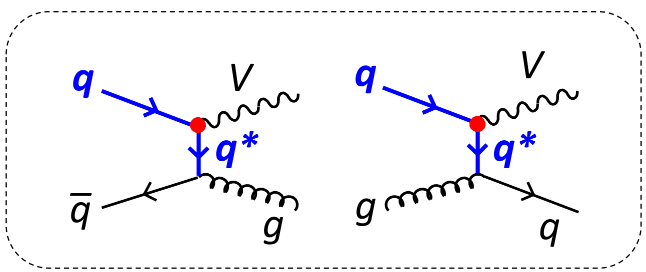
$O(\alpha_S^0)$



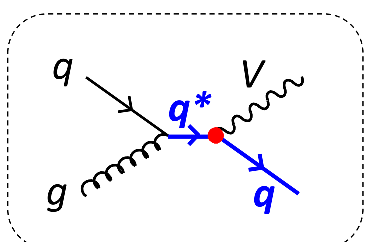
z = relative dir. of incoming *q* and *q*bar
(~ **Collins-Soper frame**)

important only up to $p_T = \mathcal{O}(\text{parton } k_T)$

$O(\alpha_S^1)$
QCD corrections



z = dir. of *one* incoming quark
(~ **Gottfried-Jackson frame**)

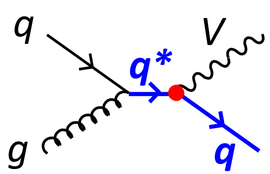


z = dir. of outgoing *q*
(= **parton-cms-helicity** ≈ **lab-cms-helicity**)

$$\sin^2 \delta_{HX-CS} = \frac{1}{1 + (M/p_T)^2 \tanh^2 \hat{y}} \qquad \frac{1}{1 + (M/p_T)^2 \tanh^2 y}$$

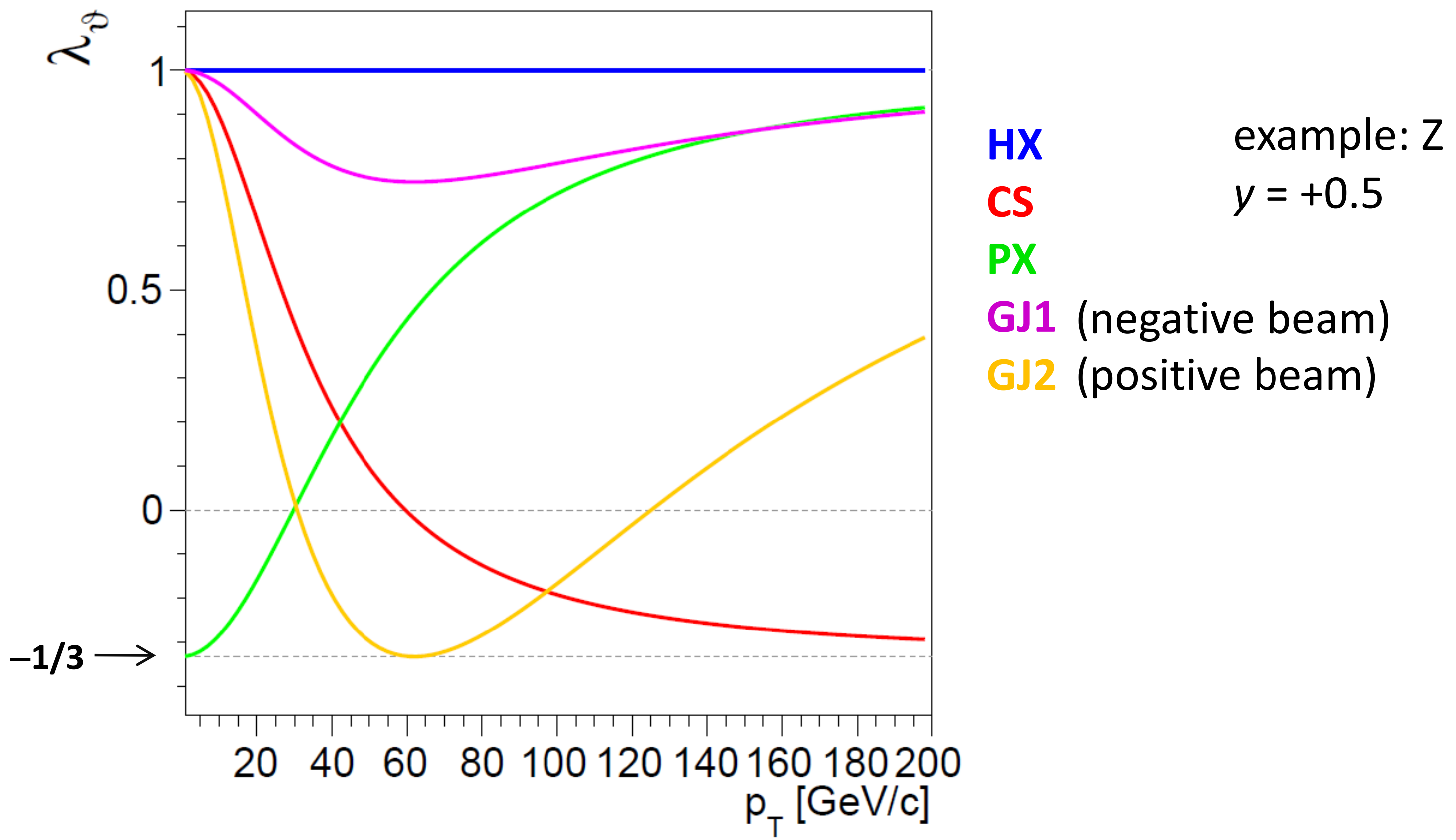
“Optimal” frames for Drell-Yan, *Z* and *W* polarizations

Different subprocesses have different “natural” quantization axes



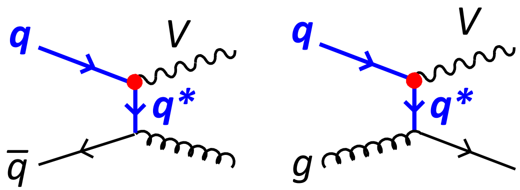
For ***s*-channel processes** the natural axis is the direction of the outgoing quark (= direction of dilepton momentum)

→ optimal frame (= maximizing polar anisotropy): **HX** (neglecting parton-parton-cms vs proton-proton-cms difference!)



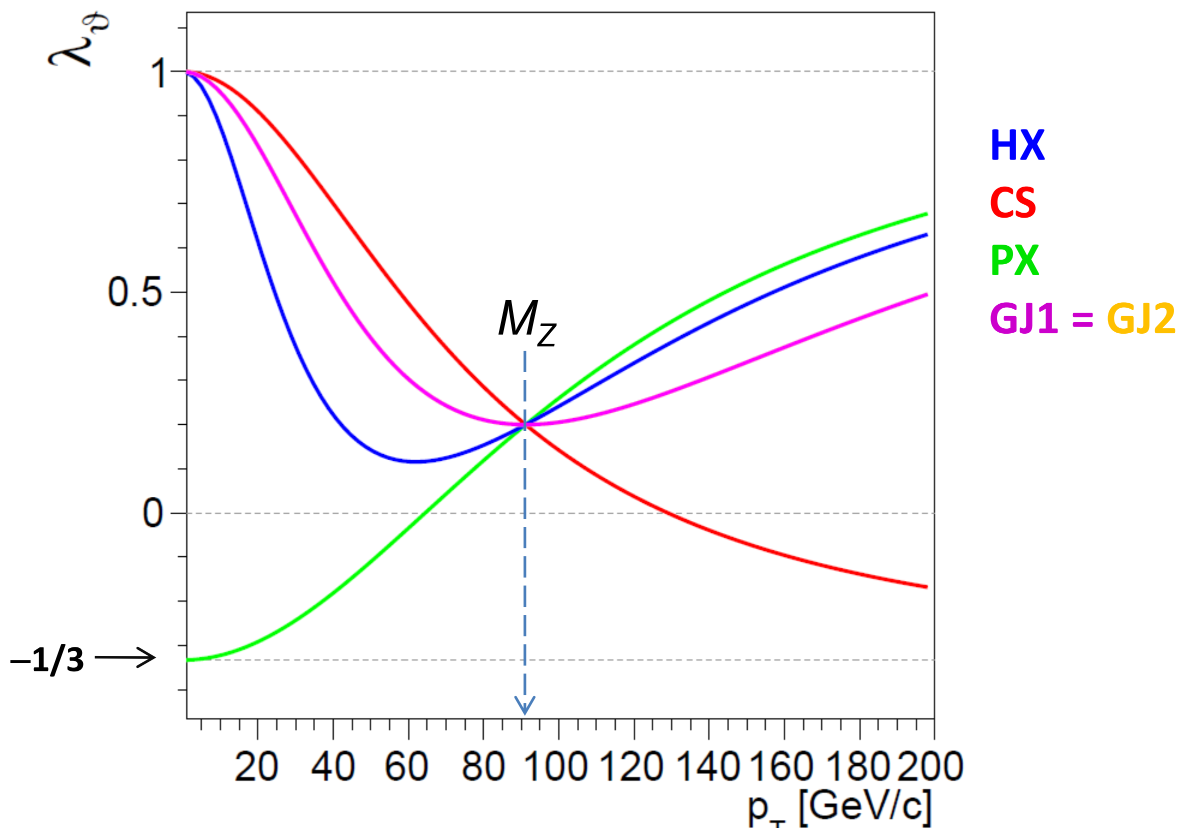
"Optimal" frames for Drell-Yan, *Z* and *W* polarizations

Different subprocesses have different "natural" quantization axes



For ***t*- and *u*-channel processes** the natural axis is the direction of either one or the other incoming parton (~ "Gottfried-Jackson" axes)

→ optimal frame: geometrical average of GJ1 and GJ2 axes = **CS ($p_T < M$)** and **PX ($p_T > M$)**



example: Z
$y = +0.5$

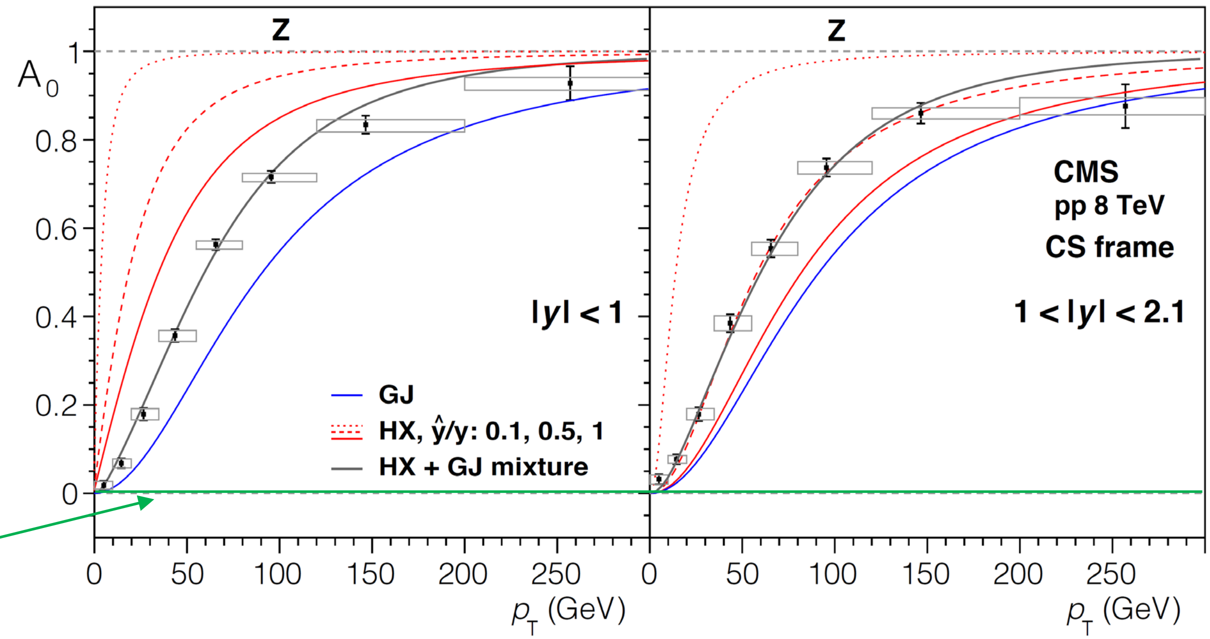
A look at Z and W data

Z by CMS

[PLB 750, 154 (2015)]

$$\frac{dN}{d\Omega} \propto 1 + \frac{1 - \frac{3}{2} A_0}{1 + \frac{1}{2} A_0} \cos^2\vartheta$$

$A_0 = 0$ ↑↓ $J_z = \pm 1$

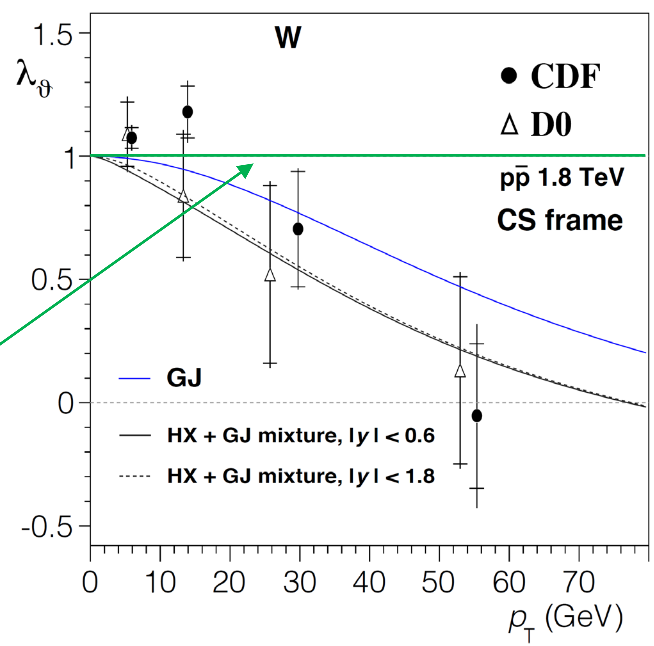


W by CDF & D0

[D0, PRD 63 (2001) 072001, CDF, PRD 70 (2004) 032004]

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\vartheta \cos^2\vartheta$$

$\lambda_\vartheta = +1 \leftrightarrow J_z = \pm 1$



The kinematic dependence of λ_ϑ (or A_0) reflects how the contributing QCD processes (s, t/u-channel) mix together as a function of p_T and y

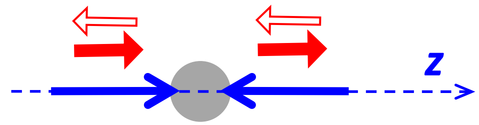
(Only) the s-channel term ($qg \to$ Z/W + quark jet) also depends on PDFs ($\hat{y}$ vs y)

→ frame-dependent parameters are certainly useful to study QCD

Rotation-invariant Drell-Yan, *Z* and *W* polarizations

- polarization is *always fully* ***transverse***...

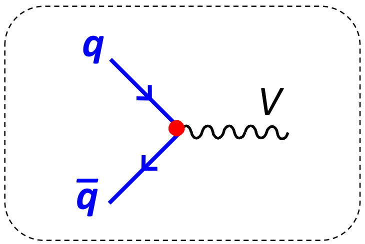
$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex,
$J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction ***z***

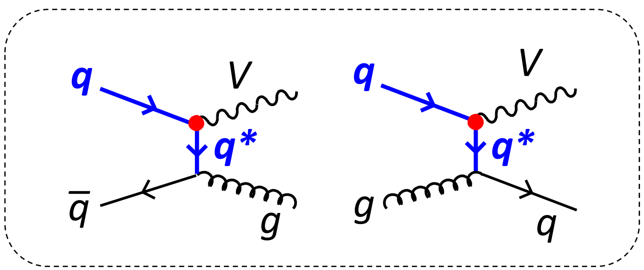
- ...but with respect to a ***subprocess-dependent quantization axis***

$O(\alpha_S^0)$

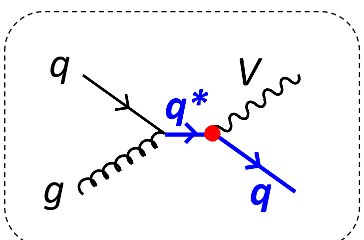


"natural" z = relative dir. of *q* and *q*bar
→ $\boldsymbol{\lambda_\vartheta}$**("CS") = +1**
wrt ***any*** axis: $\tilde{\lambda} = +1$

$O(\alpha_S^1)$
(LO) QCD corrections



z = dir. of *one* incoming quark
→ $\boldsymbol{\lambda_\vartheta}$**("GJ") = +1**
$\tilde{\lambda} = +1$



z = dir. of outgoing *q*
→ $\boldsymbol{\lambda_\vartheta}$**("HX") = +1**
$\tilde{\lambda} = +1$

Note: $\tilde{\lambda} = +1$ in **both pp-HX and qg-HX frames**!

$\tilde{\lambda} = +1$
any frame

In all these cases the *q*-*q*-*V* lines are in the production plane (***"planar" processes***)
The CS, GJ, pp-HX and qg-HX axes only differ by a rotation in the *production plane*

The Lam-Tung relation

PHYSICAL REVIEW D VOLUME 21, NUMBER 9 1 MAY 1980

Parton-model relation without quantum-chromodynamic modifications in lepton pair production

C. S. Lam
Wu-Ki Tung

...ch more to the quar... QCD modifications in LPP than jus... ...egrated Drell-Yan cross-section formula. ...lepton angular distributions are controlled by structure functions which obey parton-model relations[3,4] similar to those between F_1 and F_2 in deep-inelastic scattering (DIS). How are these relations affected by perturbative QCD corrections? The answer to this question is quite surprising: At least one of these relations—the exact counterpart of the Callan-Gross[5] relations—is not modified at all by first-order QCD corrections, although individual terms in this relation ...y be subject to large corrections. In the re... ...note, we spell out explicitly the pa... ...as the contrast betw...

...cross-section formula [essentially W^{μ}_{μ}, cf. Eq. (2)]. This appears to be a rather remarkable result; we are not aware of any other parton-model result which is not affected by QCD corrections. For this reason, we sketch in the ...endix a derivation of Eq. (5) from the dia...

...rms of helicity structure fu... ...lation takes the form $W_L = 2W_{\Delta\Delta}$, Eq. (7). ...though for LPP, the helicity structure functions depend on the choice of coordinate axes[4] (e.g., Gottfried-Jackson, Collins-Soper, etc.), *this relation remains frame independent*—i.e., if the QCD-quark-parton model is correct, the two structure functions W_L and $W_{\Delta\Delta}$ must be related by Eq. (7), for *any* choice of axes in the lepton-pair center-of-mass frame. This strong result again demonstrates the significance of this relation.

We know the angular distribution of the lepton...

$$\lambda_\vartheta + 4\lambda_\varphi = 1$$ independently of the polarization frame

Lam-Tung relation

The Lam-Tung relation after 27 years

PHYSICAL REVIEW D **76**, 074006 (2007)

Transverse momentum dependence of the angular distribution of the Drell-Yan process

Edmond L. Berger,[1,*] Jian-Wei Qiu,[1,2,†] and Ricardo A. Rodriguez-Pedraza[2,‡]

We calculate the transverse momentum $Q_\perp$ dependence of the helicity structure functions for the hadroproduction of a massive pair of leptons with pair invariant mass Q. These structure functions determine the angular distribution of the leptons in the pair rest frame. Unphysical behavior in the region $Q_\perp \to 0$ is seen in the results of calculations done at fixed order in QCD perturbation theory. We use current conservation to demonstrate that the unphysical inverse-power and $\ln(Q/Q_\perp)$ logarithmic divergences in three of the four independent helicity structure functions share the same origin as the divergent terms in fixed-order calculations of the angular-integrated cross section. We show that the resummation of these divergences to all orders in the strong coupling strength α_s can be reduced to the solved problem of the resummation of the divergences in the angular-integrated cross section, resulting in well-behaved predictions in the small $Q_\perp$ region. Among other results, we show the resummed part of the helicity structure functions preserves the Lam-Tung relation between the longitudinal and double spin-flip structure functions as a function of $Q_\perp$ to all orders in α_s.

Is this really a "QCD result" ?

The Lam-Tung relation today

A fundamental result of the theory of vector-boson polarizations (Drell-Yan, directly produced *Z* and *W*) is that, at leading order in perturbative QCD,

$$\lambda_\vartheta + 4\lambda_\varphi = 1$$ independently of the polarization frame

Lam-Tung relation

This identity was considered as a surprising result

Today we know that it is only a *special* case of general frame-independent polarization relations, corresponding to a *transverse* intrinsic polarization:

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi} = +1 \quad \Rightarrow \lambda_\vartheta + 4\lambda_\varphi = 1$$

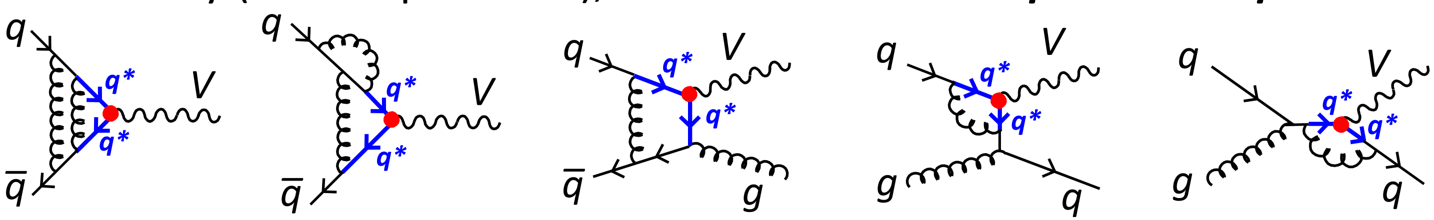
It is **not really a "QCD relation"**:
on the contrary, all "QCD details" (parton types, topology, parton PDFs) disappear from it!
It is simply a consequence of

1) rotational invariance

2) properties of the E.W. **quark-photon/Z/W couplings** (helicity conservation)

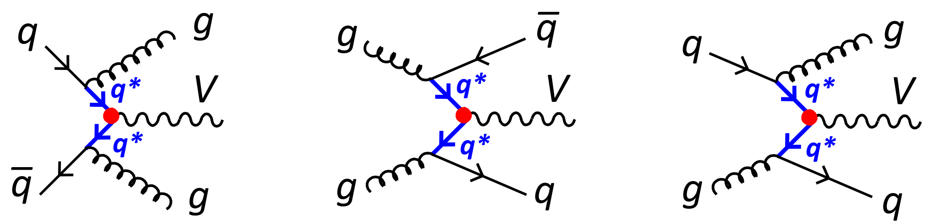
What about higher-order QCD corrections?

Any correction with "internal" gluons is harmless to the Lam-Tung relation:
helicity conservation still holds at the ***q*-q*-V*** vertex;
the **natural polarization axis** may change direction and become "inapproximable" experimentally (virtual quark line), but it ***remains in the production plane***



all *planar* topologies

Processes leading to 3 (or more) final states are different:



***non-planar* topologies**

Still, because of helicity conservation, the *V* polarization is **transverse along some axis;** but now such axis can ***"exit" the production plane***

The tilt induces a **minimal** or **negligible** modification on $\boldsymbol{\lambda_\vartheta}$ (the tilt of a "vector" produces the same reduced projection independently of the direction of the tilt)

Instead, $\boldsymbol{\lambda_\varphi}$ (measured wrt the *production* plane, which no longer *contains* the event) **is rotationally smeared** and does not compensate for the reduced λ_ϑ

$\rightarrow$ $\mathcal{F}$ **< 1/2**, $\tilde{\lambda}$ **< +1**: the **Lam-Tung relation is violated** (but the invariants remain invariant!)

In practice, **the Lam-Tung relation characterizes 2-to-1 and 2-to-2 processes**

A quantitative discriminant of physics cases

Even when the Lam-Tung relation is violated,
$\tilde{\lambda}$ can always be defined and is *always frame-independent*
→ any violation, $\tilde{\lambda} - 1 \neq 0$, is quantitatively frame-independent

$$\mathcal{F} = \frac{1 + \tilde{\lambda}}{3 + \tilde{\lambda}}$$

$\tilde{\lambda} = +1$ ($\mathcal{F} = 1/2$) → Lam-Tung. New interpretation: only ***vector boson – quark – quark*** couplings (in planar, 2-to-1 and 2-to-2 processes) → automatically verified in DY at QED & LO QCD levels and in several higher-order QCD contributions

$\tilde{\lambda} = +1 - \mathcal{O}(0.1)$ ($\mathcal{F} = 1/2 - \mathcal{O}(0.1)$)
with $\tilde{\lambda} \to +1$ for $p_T \to 0$ ($\mathcal{F} \to 1/2$)
→ same, "ordinary" vector-boson – quark – quark couplings, but in ***non-planar* 2-to-3+ processes (e.g. Z + n jets, with n>1)**

OR

smearing due to intrinsic **parton k_T**

$-1 < \tilde{\lambda} << +1$ ($0 < \mathcal{F} << 1/2$)

$+1 < \tilde{\lambda} < +\infty$ ($1/2 < \mathcal{F} < 1$)

→ contribution of ***different/new couplings or processes***
(e.g.: *Z* from Higgs, *W* from top, triple *ZZγ* coupling, higher-twist effects in DY production, etc.)

$\tilde{\lambda} < -1$ ($\mathcal{F} < 0$ or $\mathcal{F} > 1$) → experimental mistake