

Training deep neural density estimators to identify mechanistic models in science

Pedro J. Gonçalves

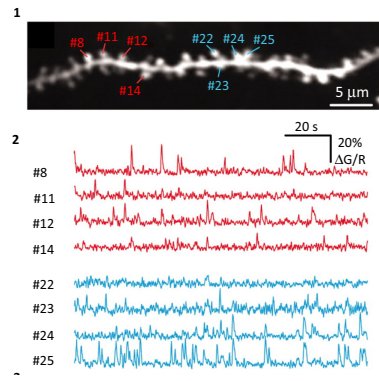
Joint work with Jan-Matthis Lueckmann, Michael Deistler, Marcel Nonnenmacher, Giacomo Bassetto, Kaan Öcal, David Greenberg, Jakob H. Macke

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graph LR; A[Cellular mechanisms] --> B[Network dynamics]; B --> C[Behaviour]
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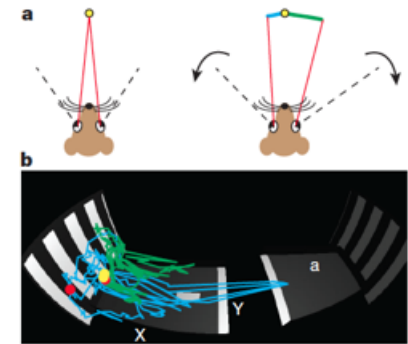
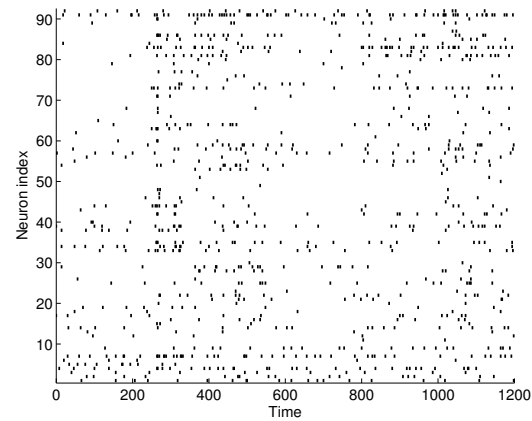
Cellular mechanisms

Network dynamics

Behaviour



Takahashi et al 2012

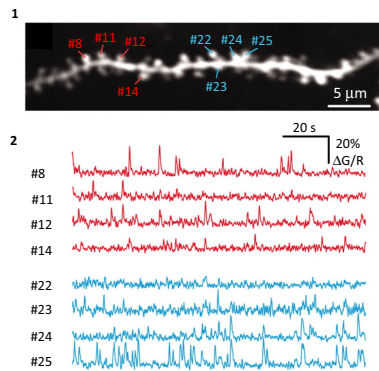


Wallace, Greenberg, Sawinsinki et al 2013

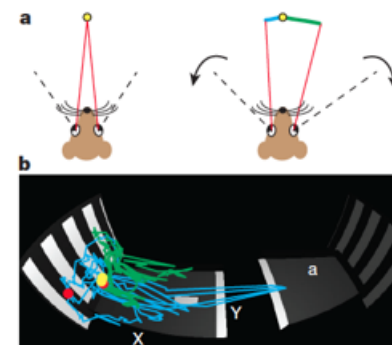
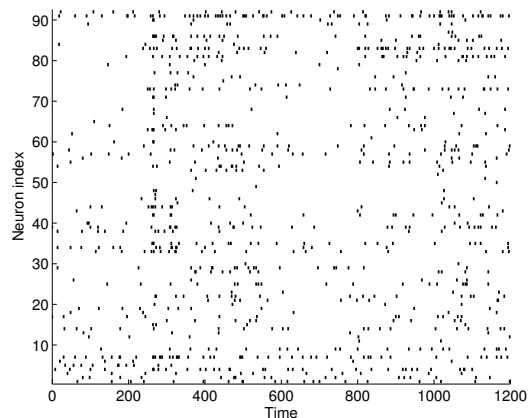
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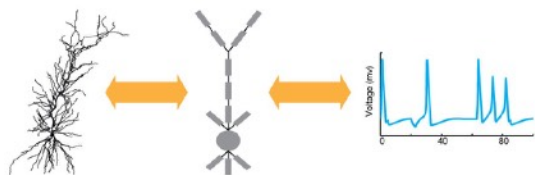
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Cellular mechanisms

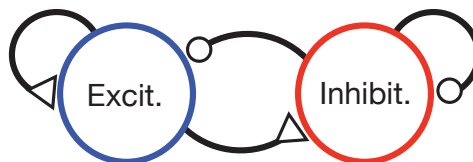
Network dynamics

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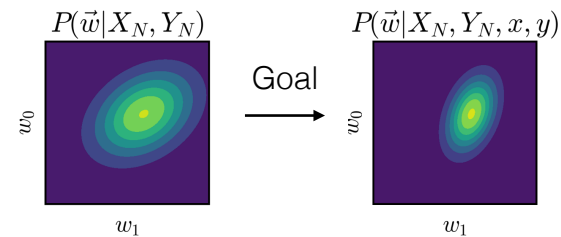
$$C \frac{\dot{V}(t)}{dt} = \sum_c \bar{g}_c g_c(t) [E_c - V(t)] + I(t)$$



$$\tau_m \frac{dV_i}{dt} = V_{rest} - V_i + R I_i(t)$$



$$\arg \max_{\boldsymbol{\theta}} H[\boldsymbol{\theta}|\mathcal{D}] - \mathbb{E}_{y \sim p(y|\mathbf{x}\mathcal{D})} [H[\boldsymbol{\theta}|y, \mathbf{x}, \mathcal{D}]]$$



Mechanistic models

vs.

Statistical/ML models

Hodgkin-Huxley model

Multi-compartment
model

Conductance-
based LIF

Balanced
networks

Biophysical network
simulations



Mechanistic models

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Generalized linear
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RNNs

Maximum
Entropy models

Gaussian
Process Factor
Analysis

Deep Nets

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- can be directly fit to data

Deep Nets

Analysis

How can we make statistical inference tractable for mechanistic models of neural dynamics?

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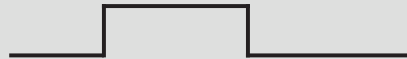
- 1) An algorithm
- 2) A couple of applications

Bayesian inference: finding parameters of a model which are consistent with data and prior knowledge

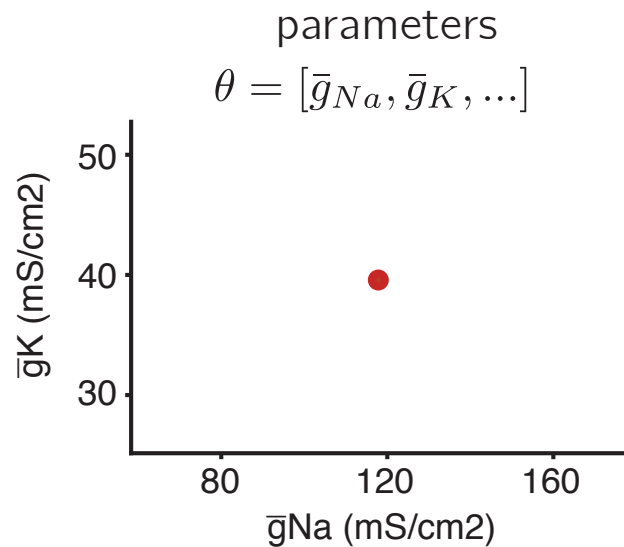
model

$$C_m \frac{dV}{dt} = \sum_c \bar{g}_c g_c [E_c - V] + I$$

input (e.g. step current)



Bayesian inference: finding parameters of a model which are consistent with data and prior knowledge

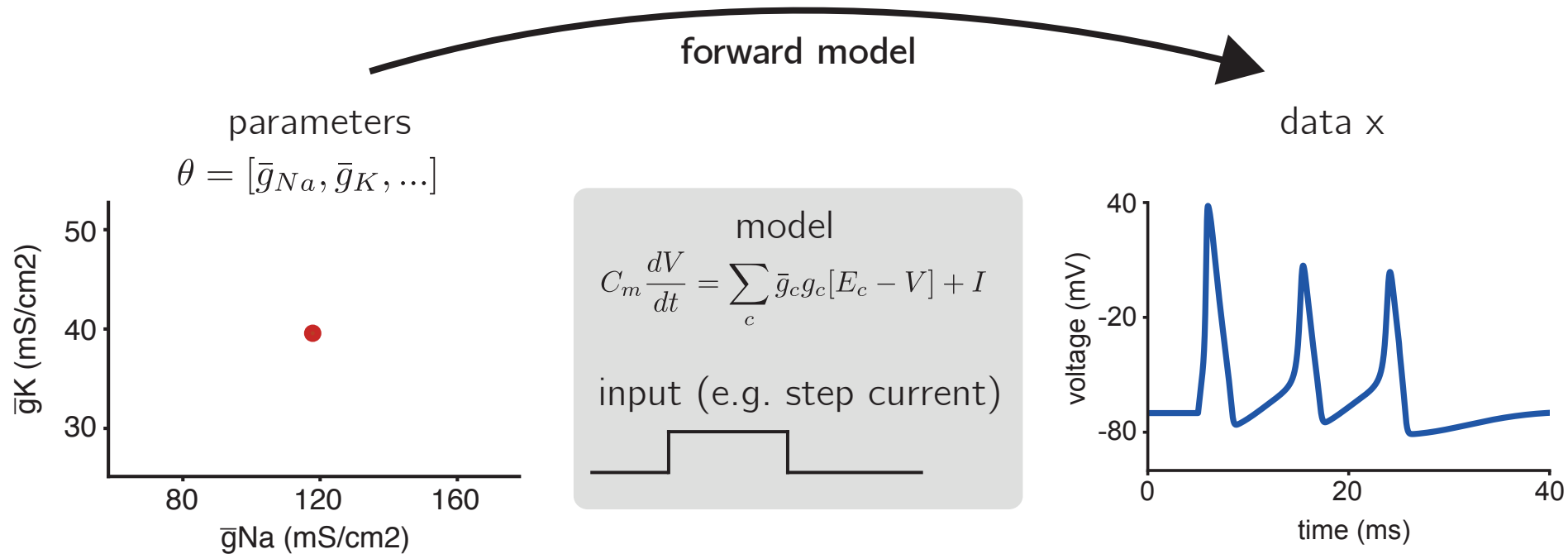


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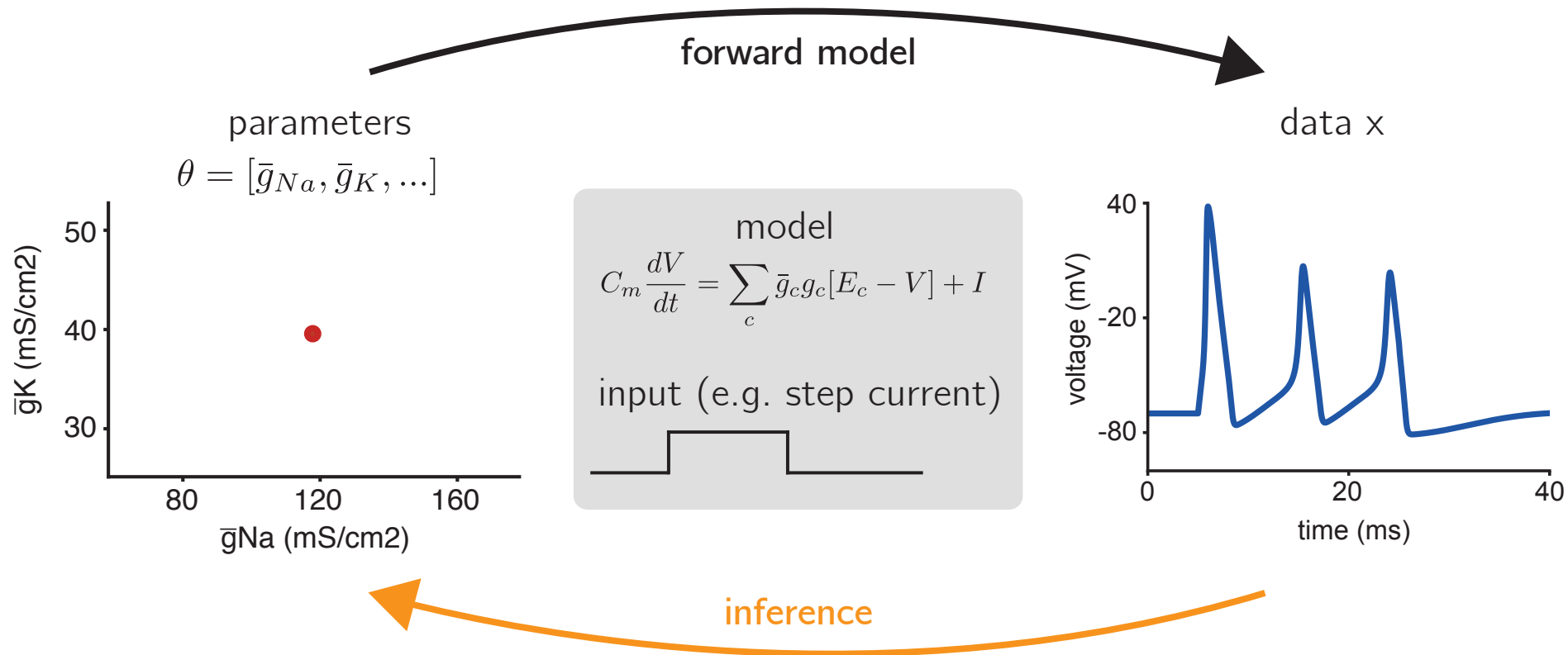
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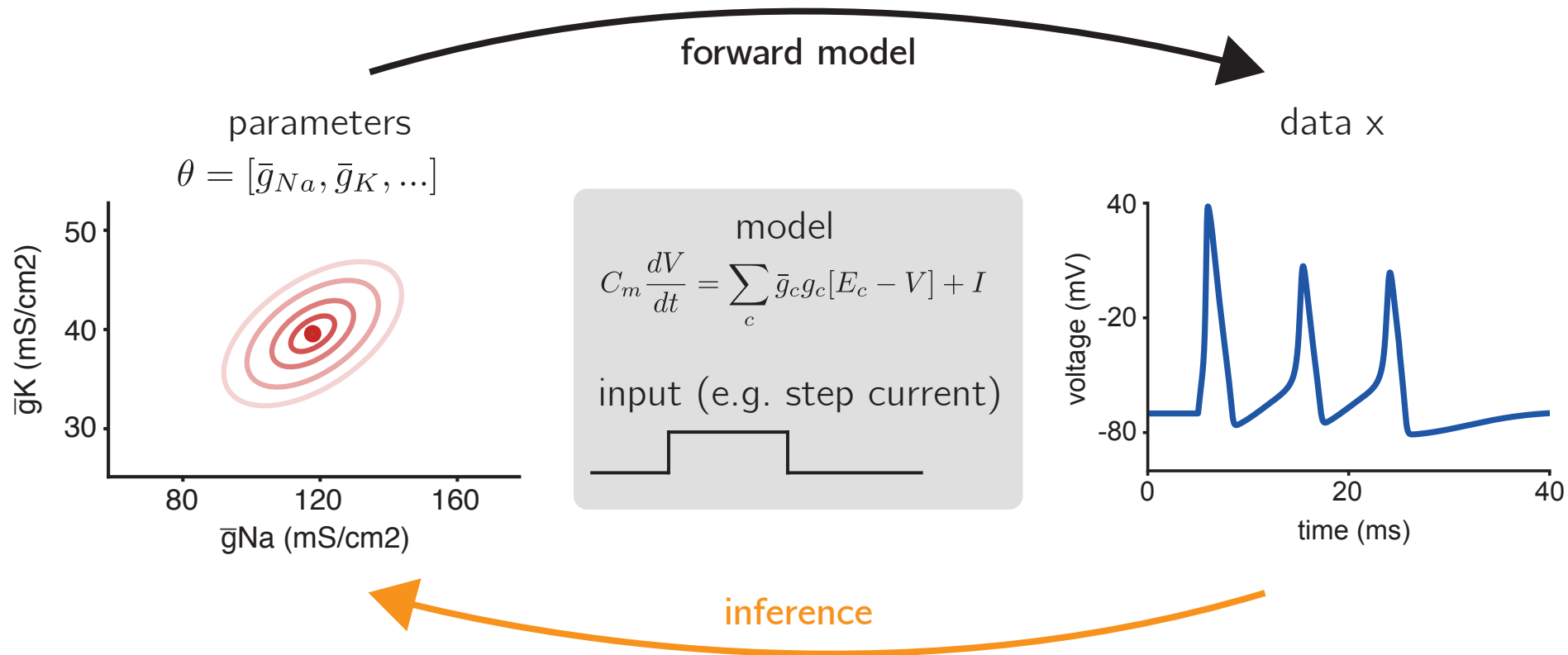
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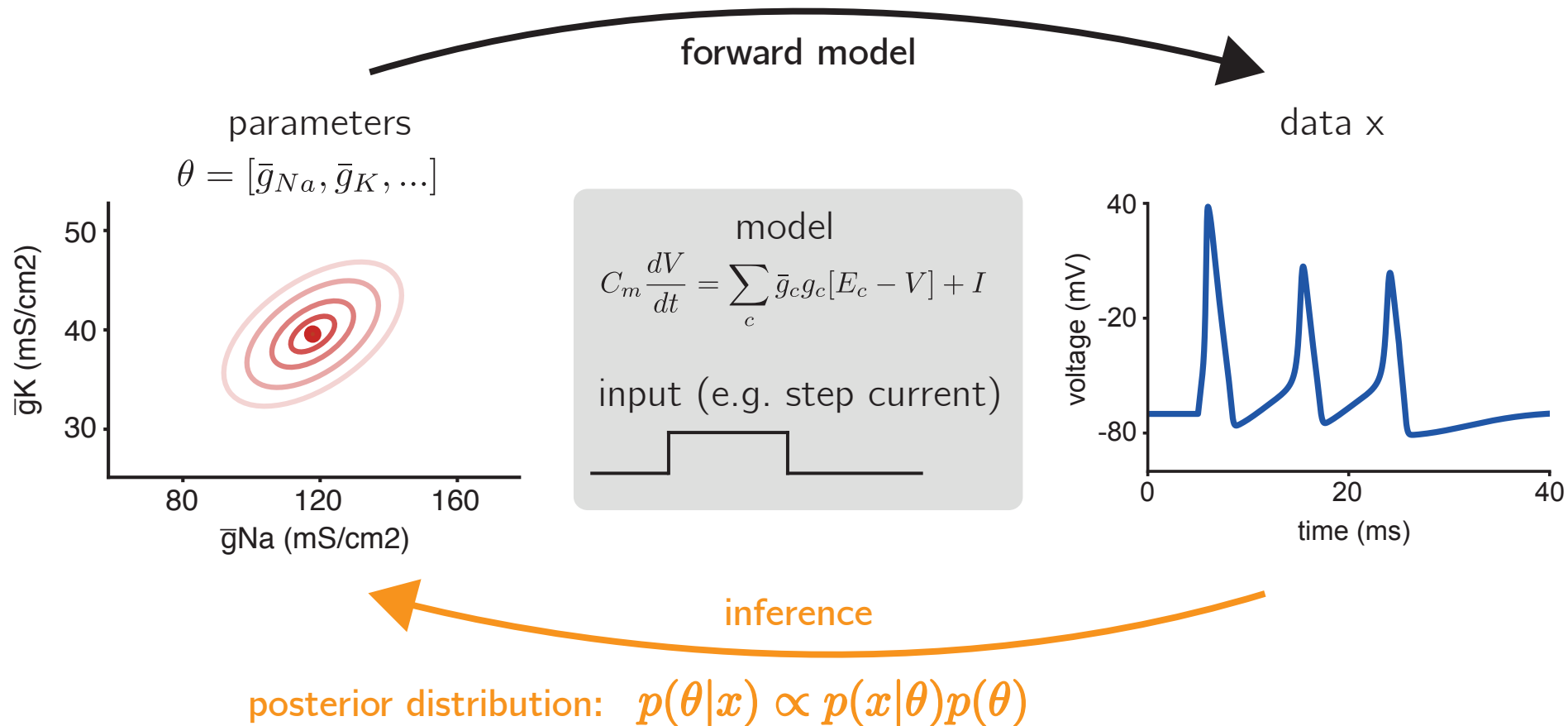
Bayesian inference: finding parameters of a model which are consistent with data and prior knowledge



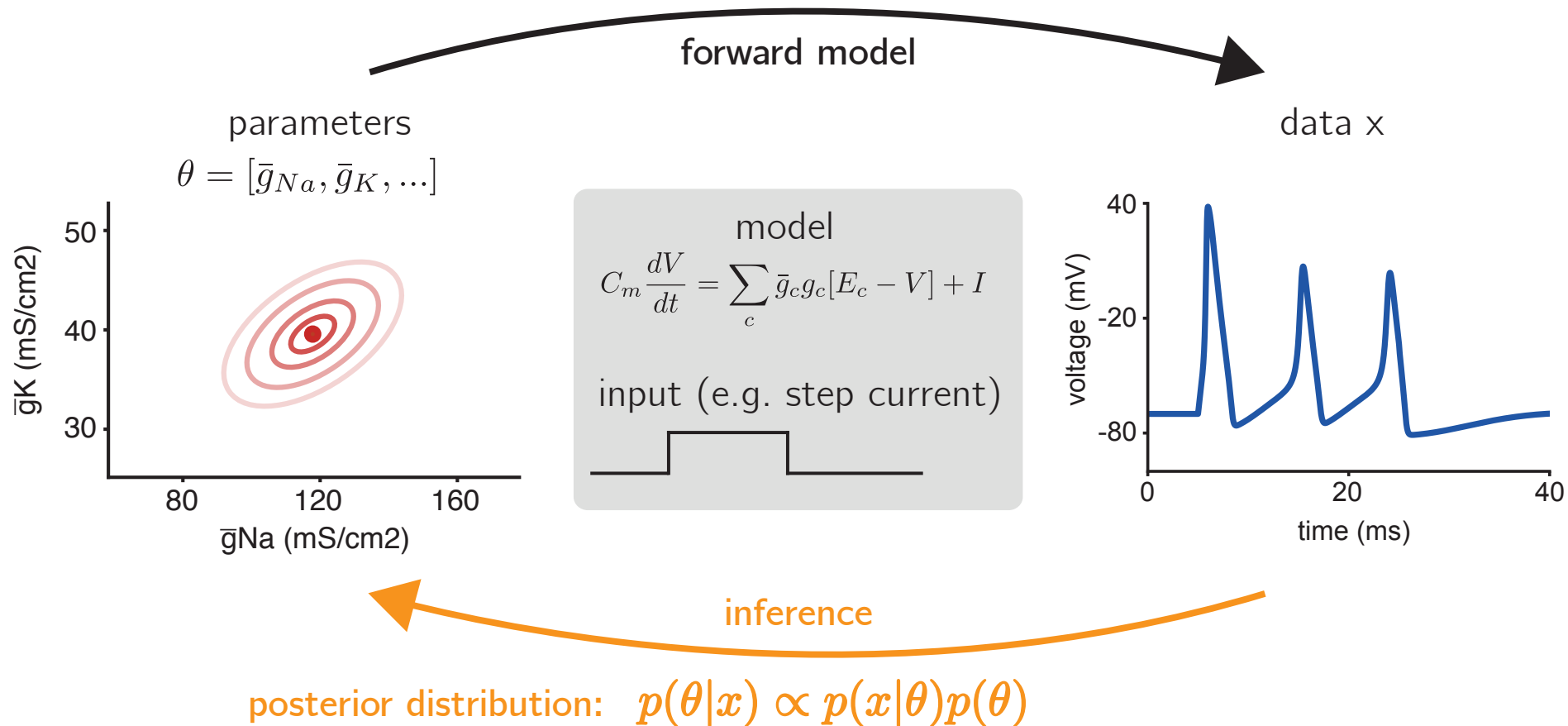
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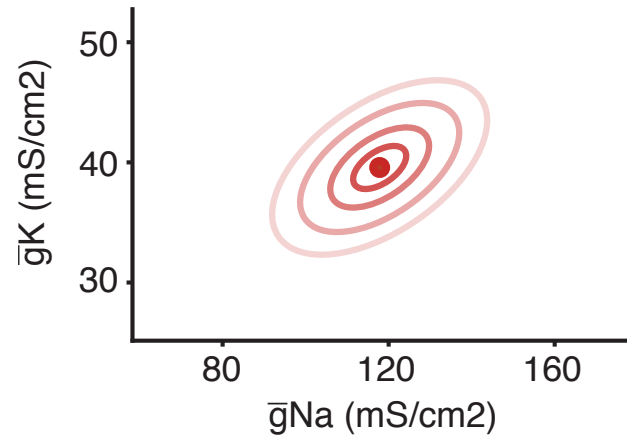


Bayesian inference: finding parameters of a model which are consistent with data and prior knowledge



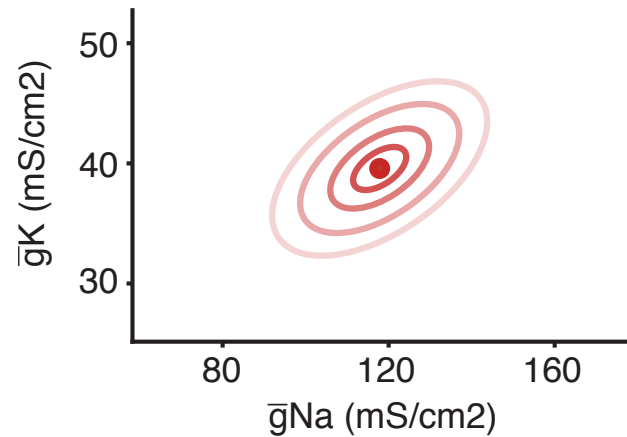
Challenging! Simulation-based inference comes to the rescue

Bayesian inference in simulation based models



$$p(\theta|x) \propto p(x|\theta)p(\theta)$$

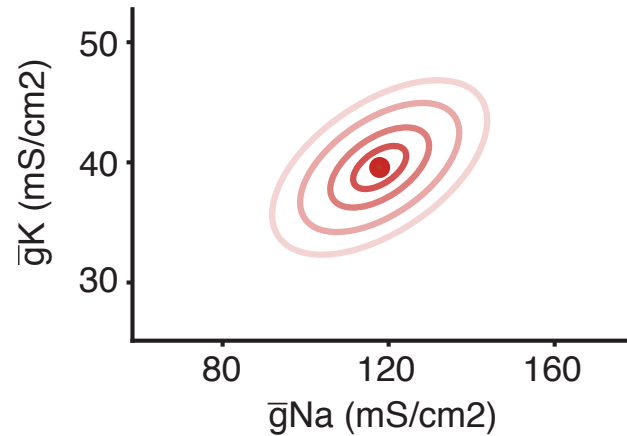
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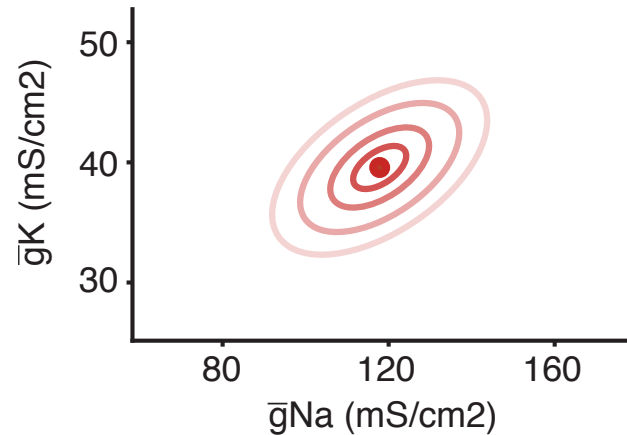


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Bayesian inference in simulation based models



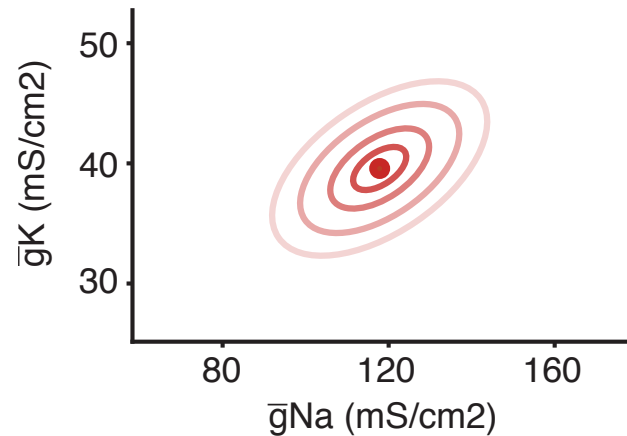
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For ‘simulation based’ models (and neuroscience models in particular), we can simulate x , but we cannot evaluate the likelihood $p(x|\theta)$.

Bayesian inference in simulation based models



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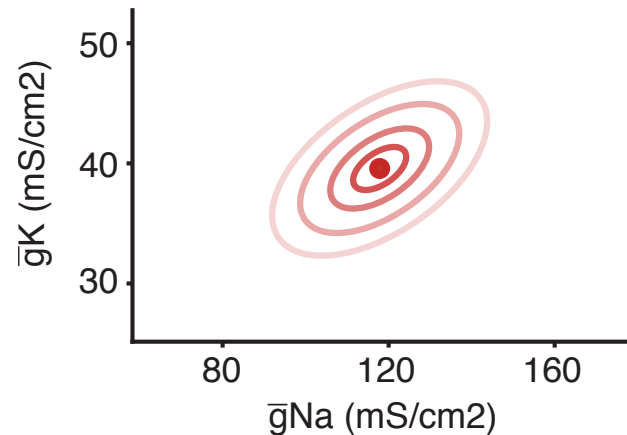
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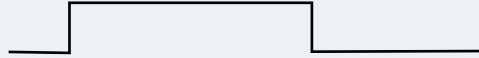
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Likelihood-free Bayesian Inference or Approximate Bayesian Computation (ABC)
Blum & Francois 2010, Papamakarios & Murray 2016

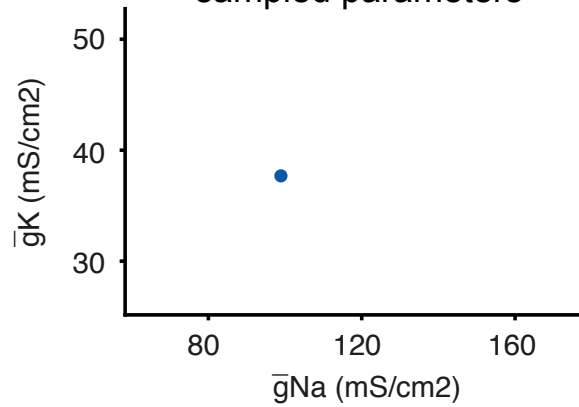
$$C\dot{V} = \sum_c \bar{g}_c g_c [E_c - V] + I$$



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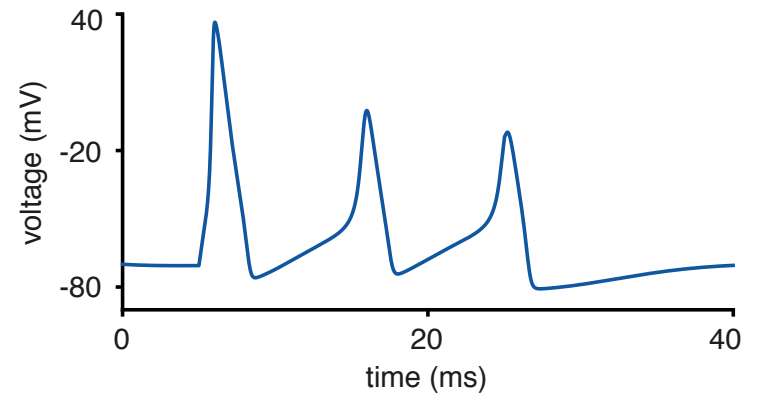
sampled parameters



simulate data



simulated data

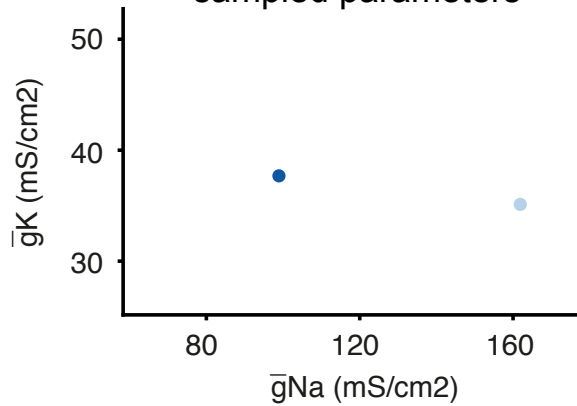


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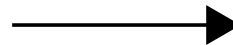


1. Sample data from multiple parameters

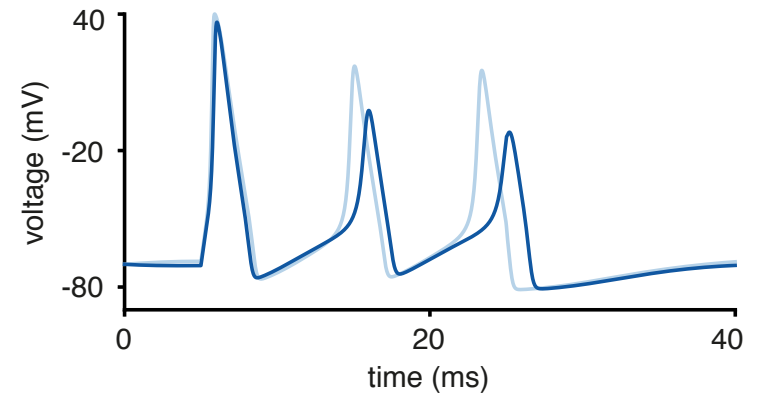
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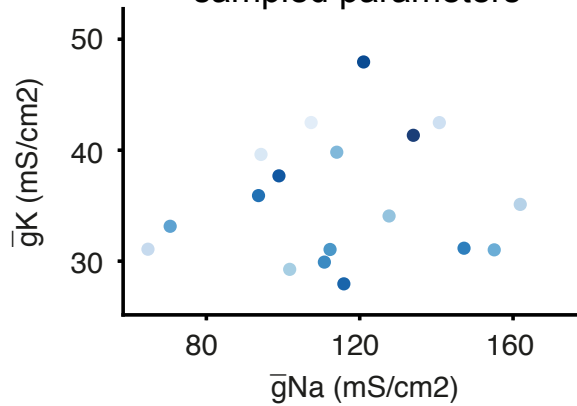


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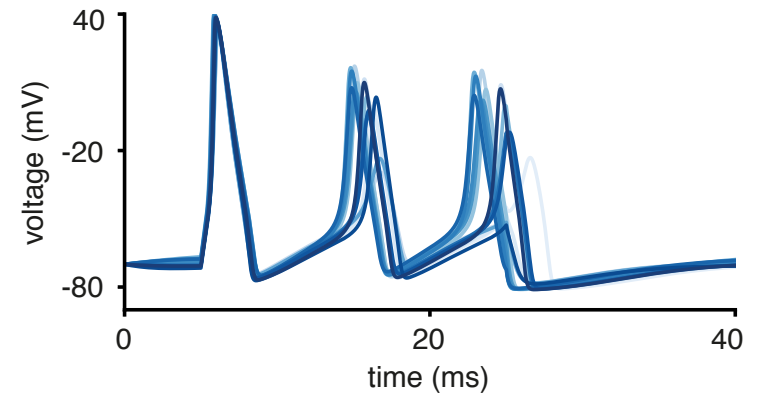
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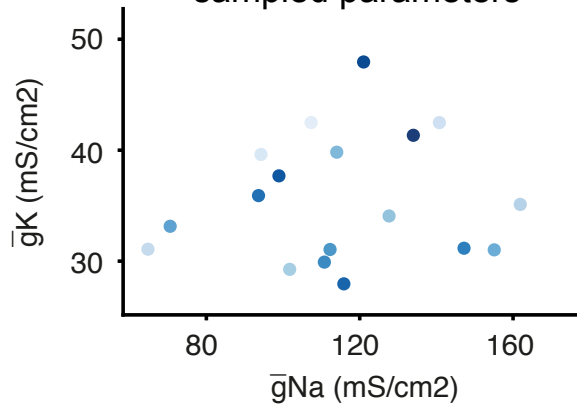
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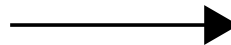
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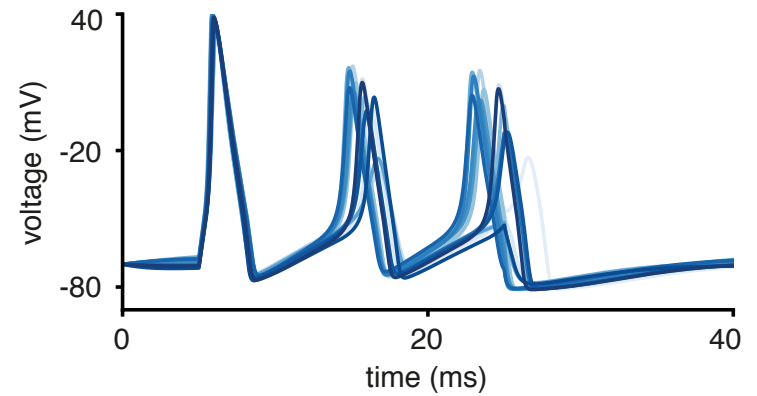
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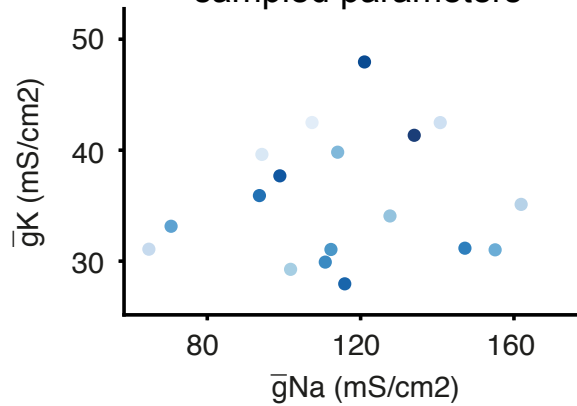


Extract summary statistics
(mean, correlations...)

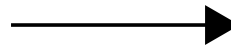
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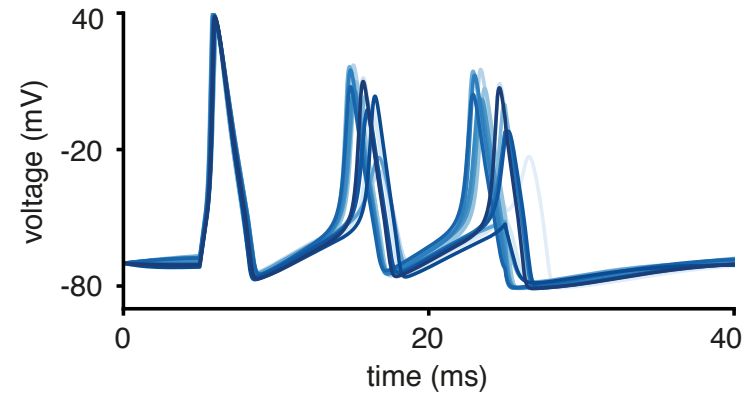
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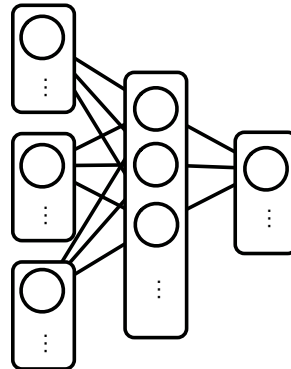
simulate data



simulated data



neural network



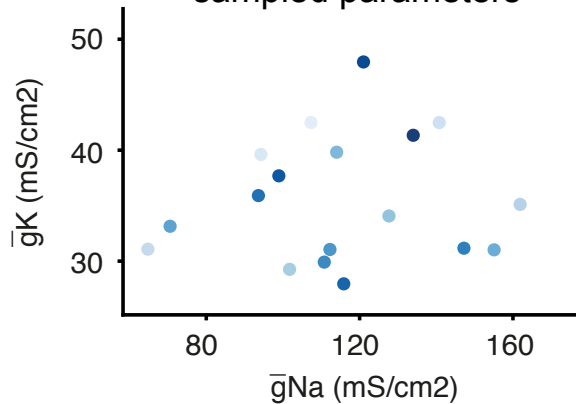
$$p(\bar{g}_{\text{Na}}, \bar{g}_{\text{K}} | s(V)) = \text{NN}(s(V))$$

2. Train Neural network (NN)
to predict parameters from
samples

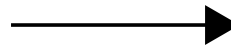
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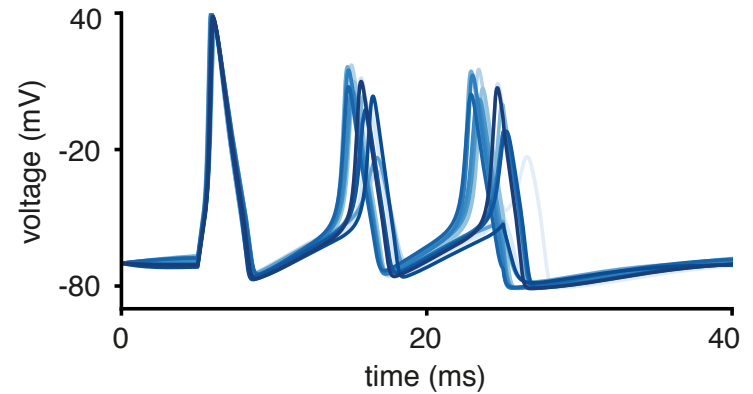
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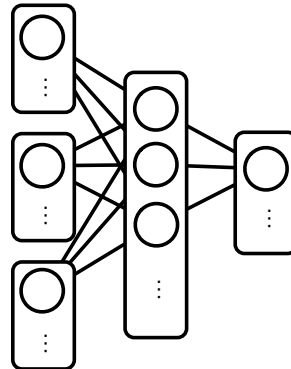
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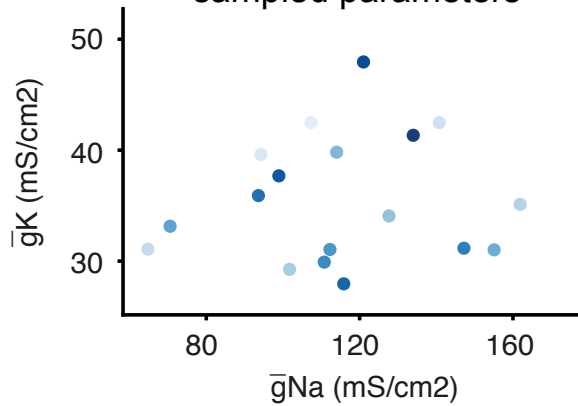
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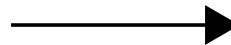
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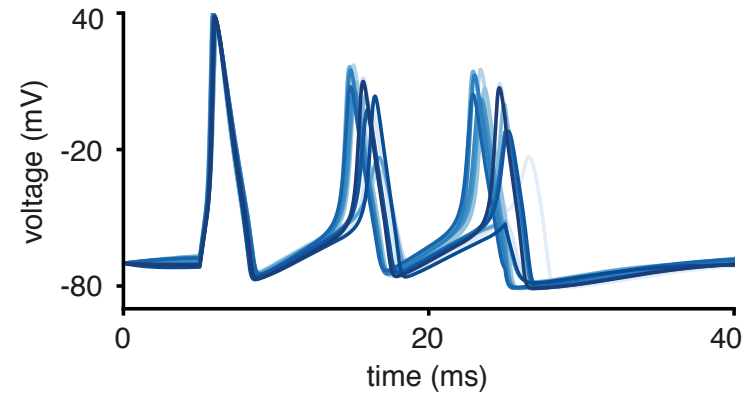
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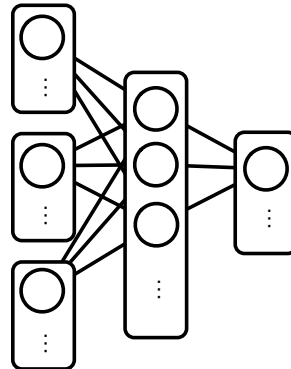
simulate data



simulated data



neural network



3. Use NN to propose new samples

train neural network

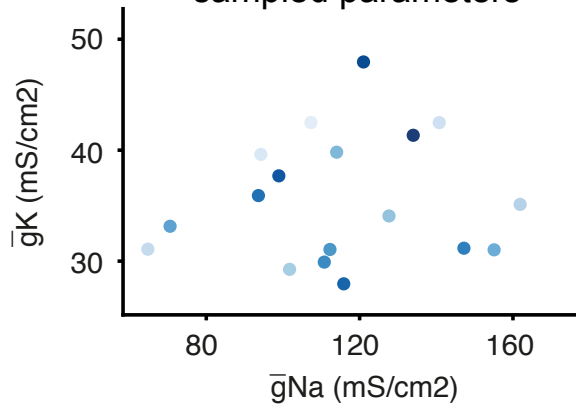
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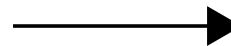
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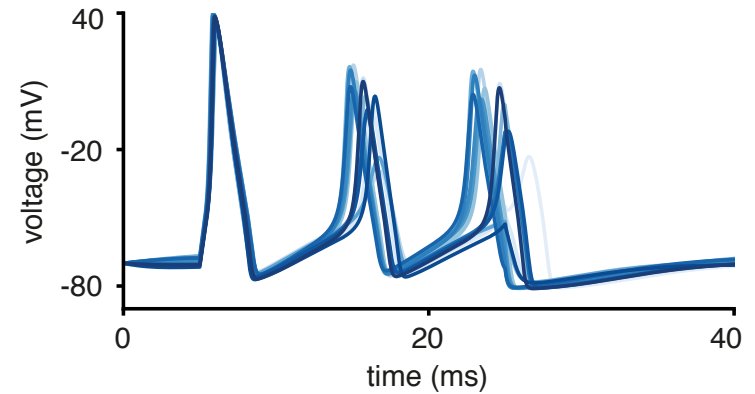
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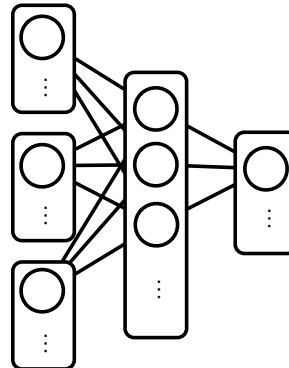
simulate data



simulated data



neural network



propose new parameters



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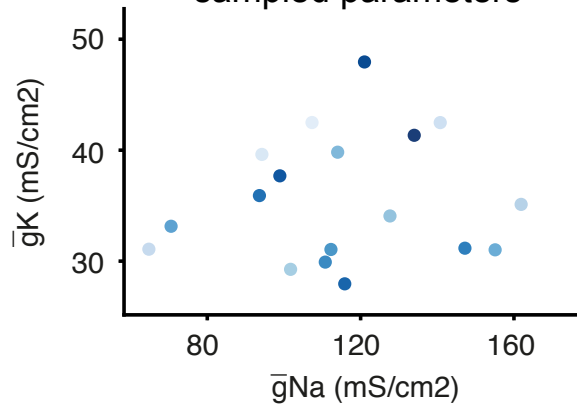
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Sequential Neural Posterior Estimation, SNPE

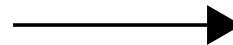
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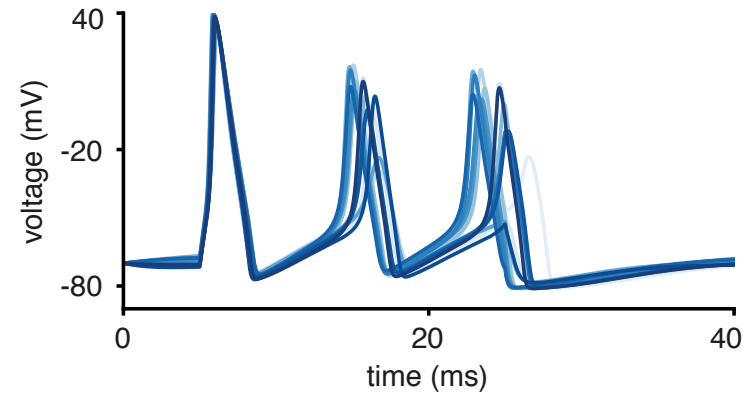
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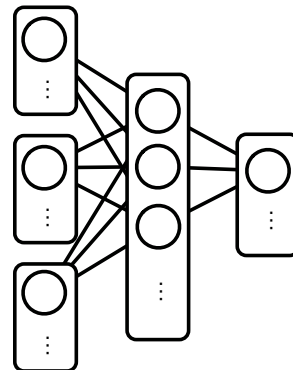
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neural network



train neural network



propose new parameters

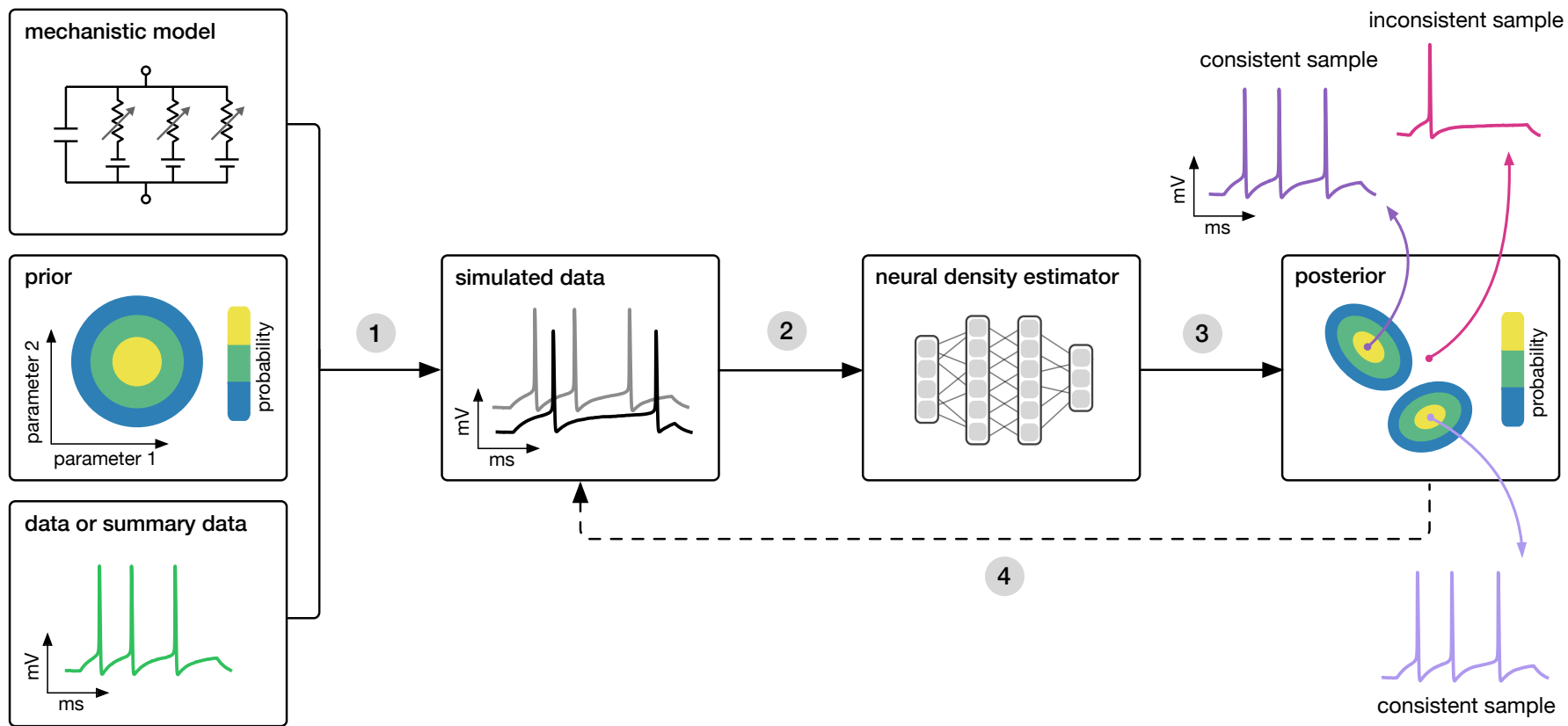


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JM Lückmann*, PJ Goncalves* et al, NeurIPS 2017

PJ Goncalves*, JM Lückmann*, M. Deistler* et al (2020) eLife

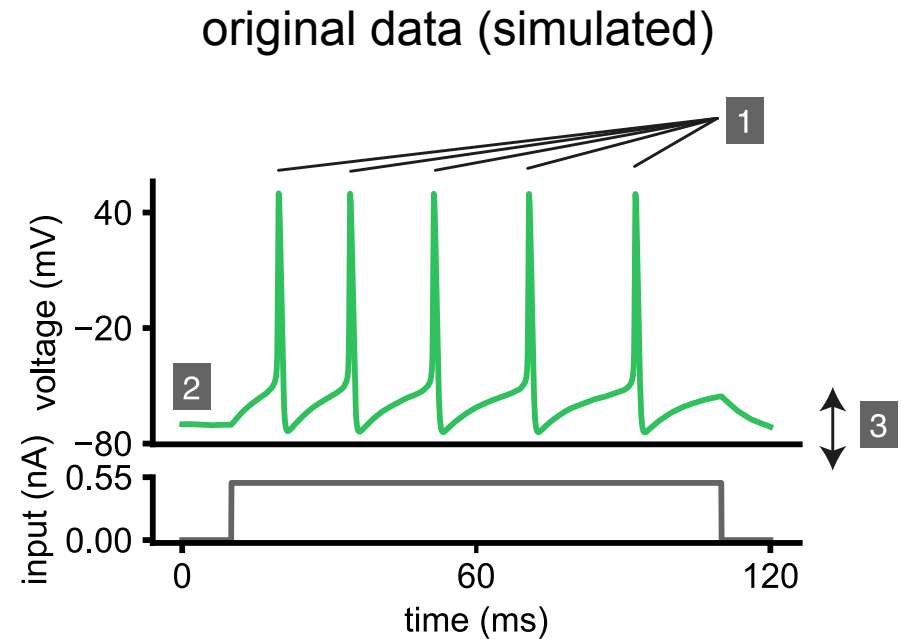
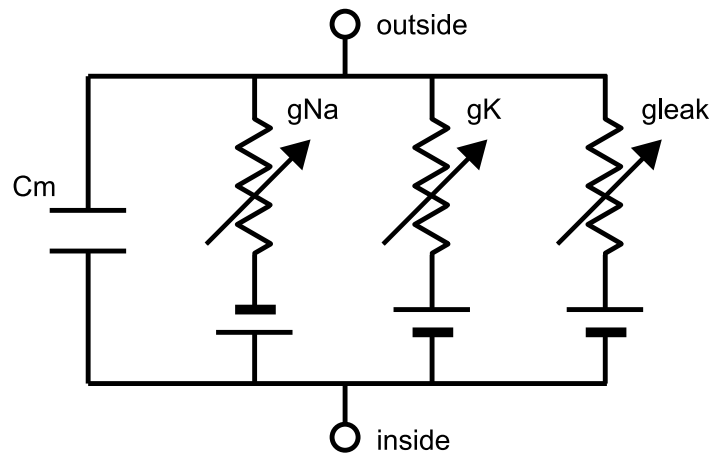
github.com/mackelab/delfi; <https://www.mackelab.org/sbi/>

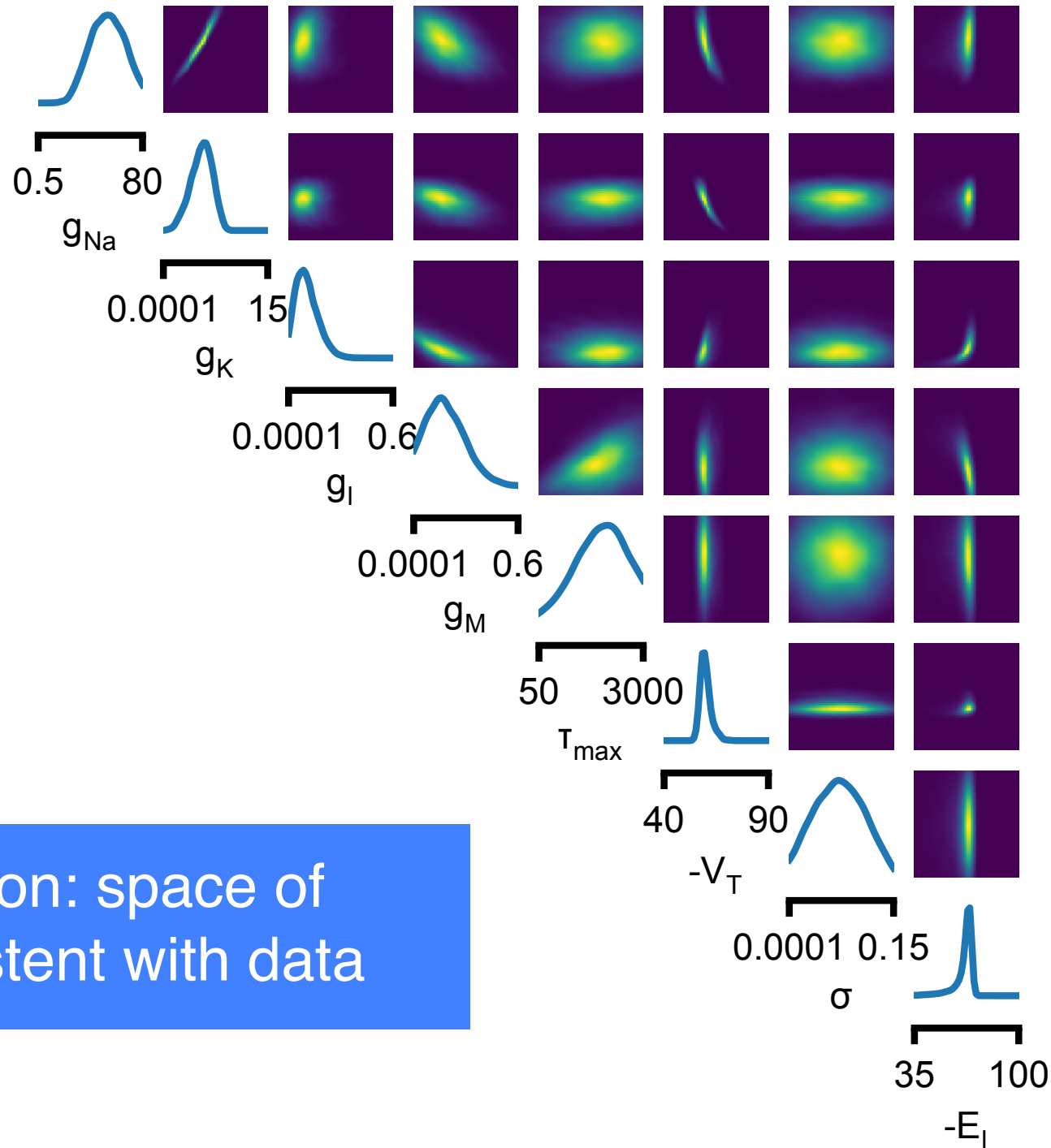


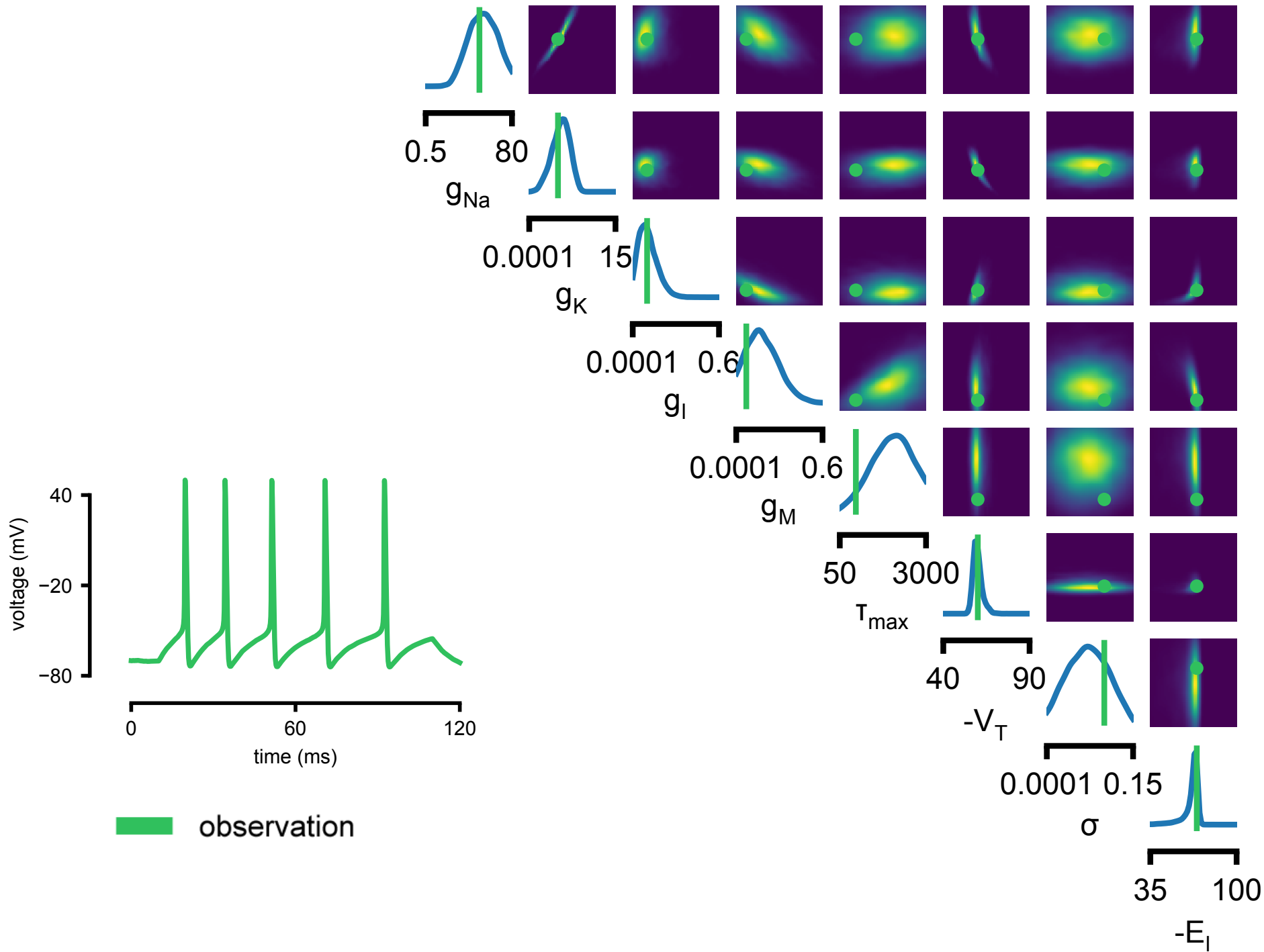
A couple of applications

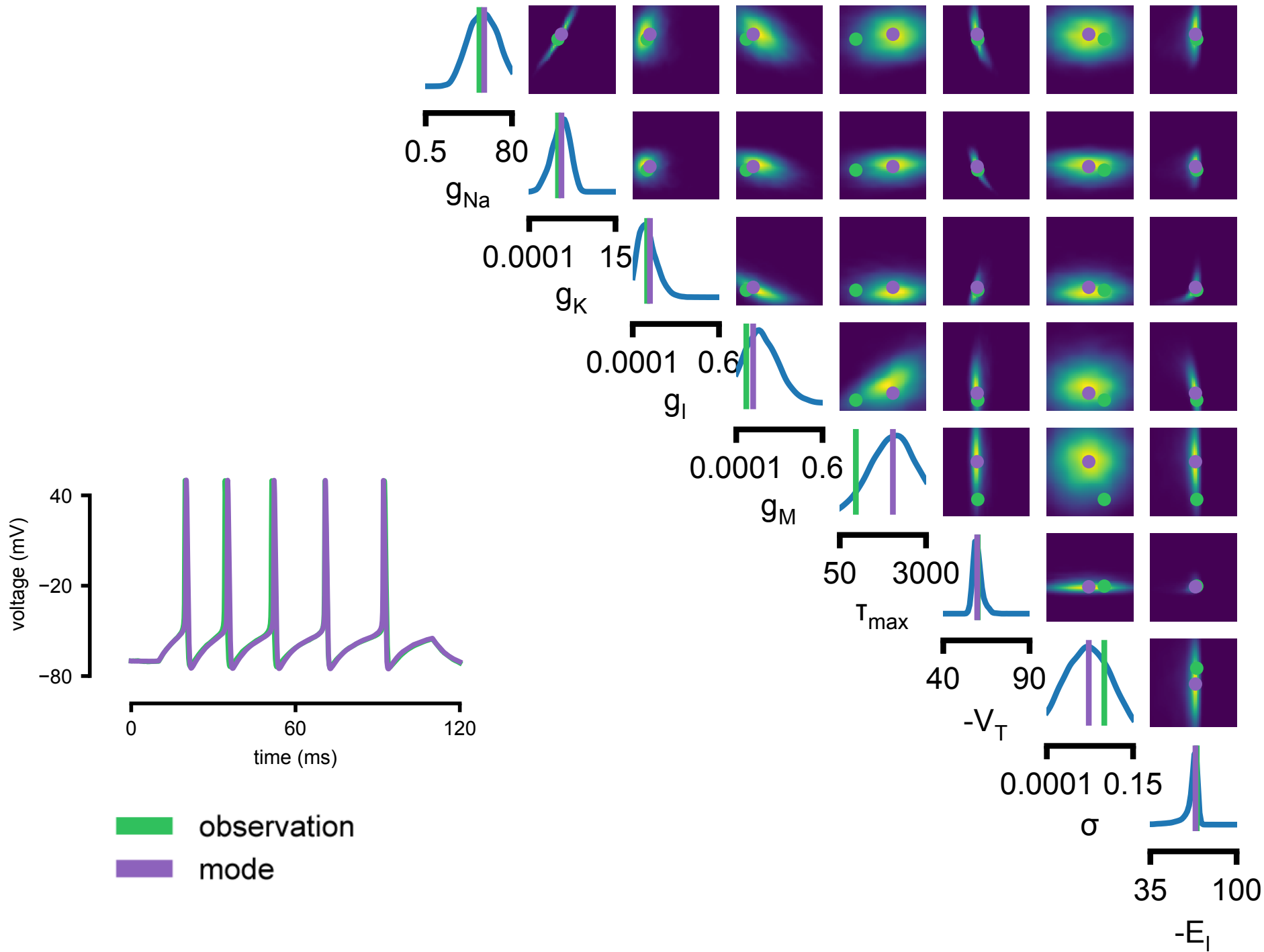
- Identification of parameters in canonical neural model
- Sensitivity to perturbations in a neural network model

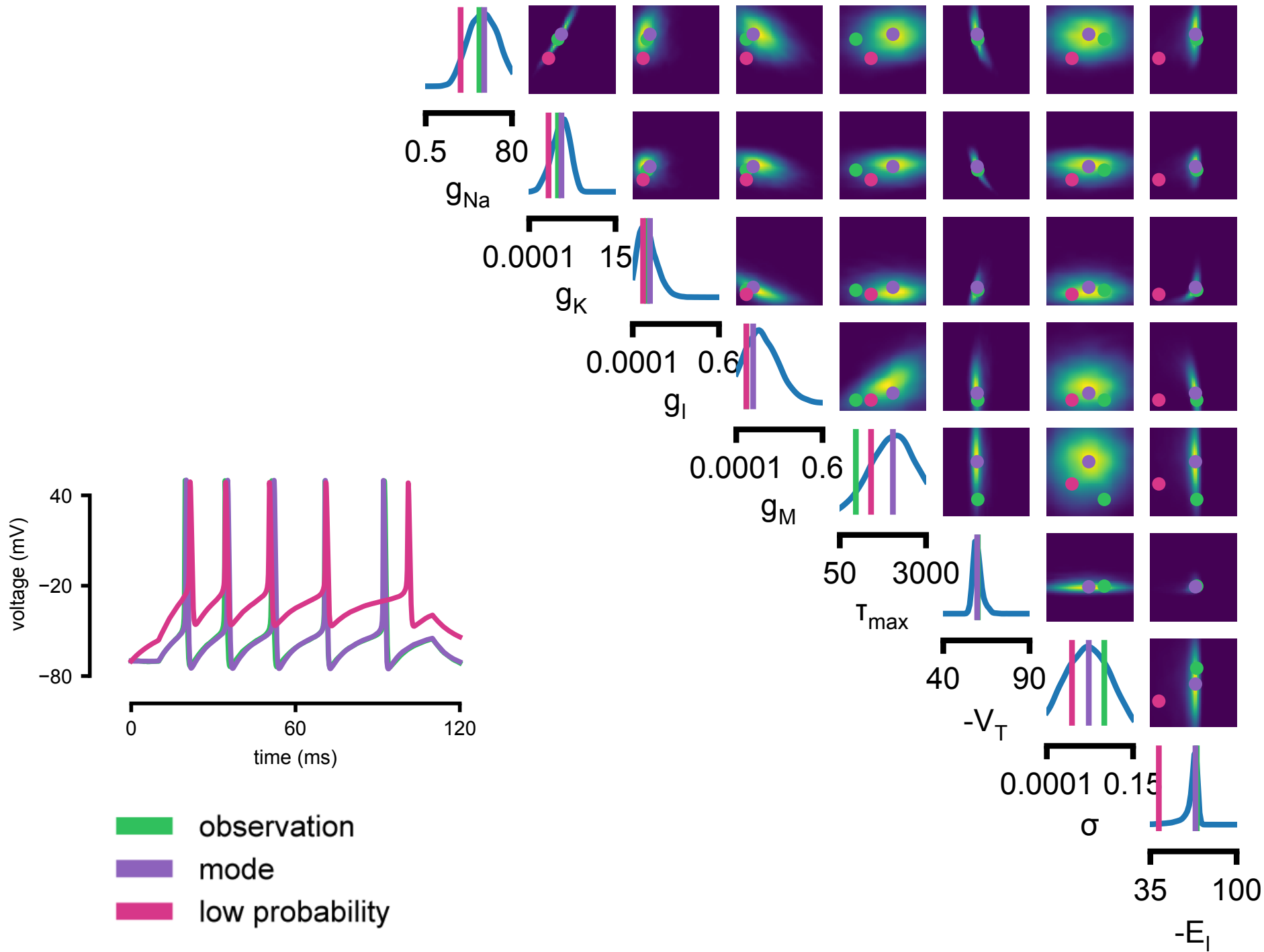
Inference of 8 biophysical parameters in Hodgkin-Huxley models



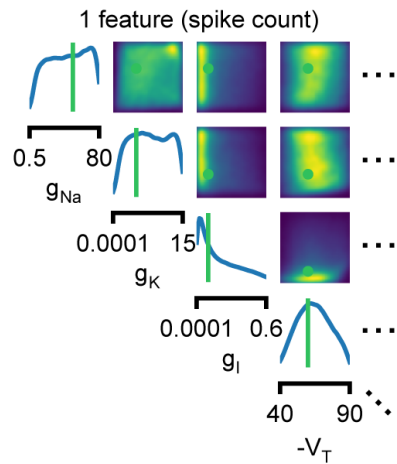




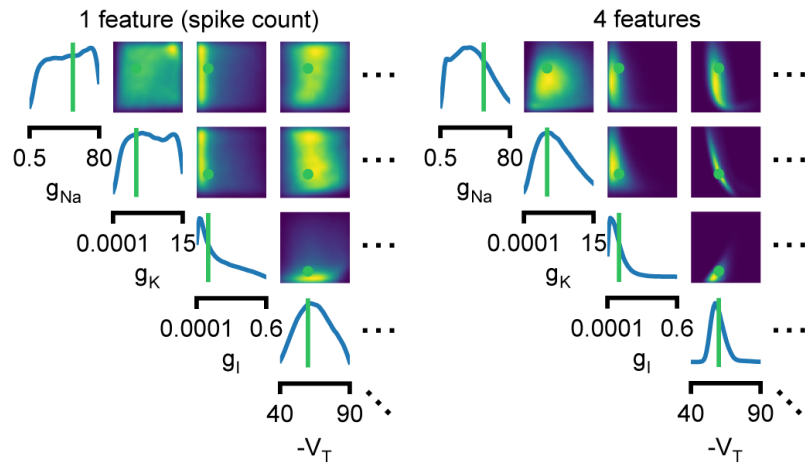




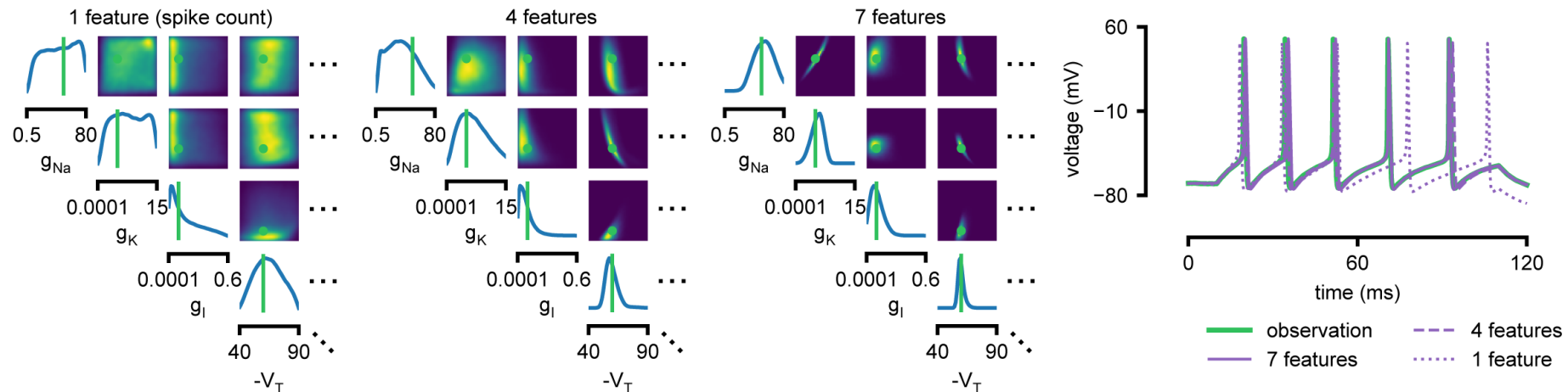
Stronger constraints from additional data features



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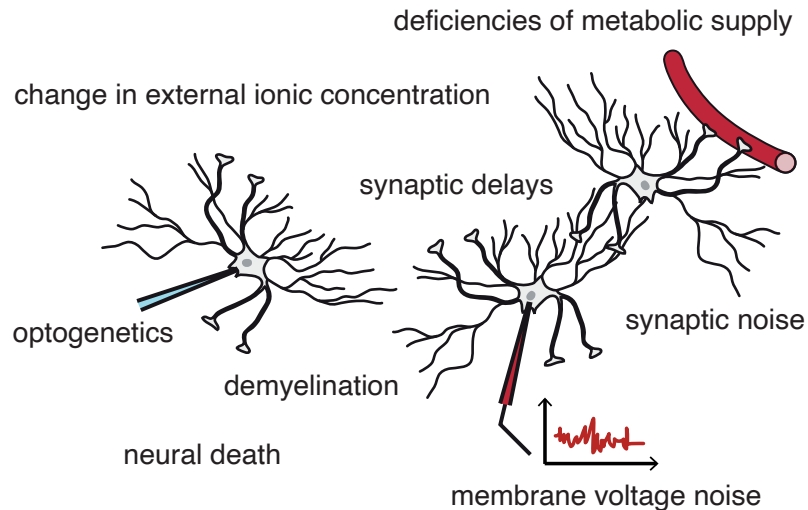


Stronger constraints from additional data features

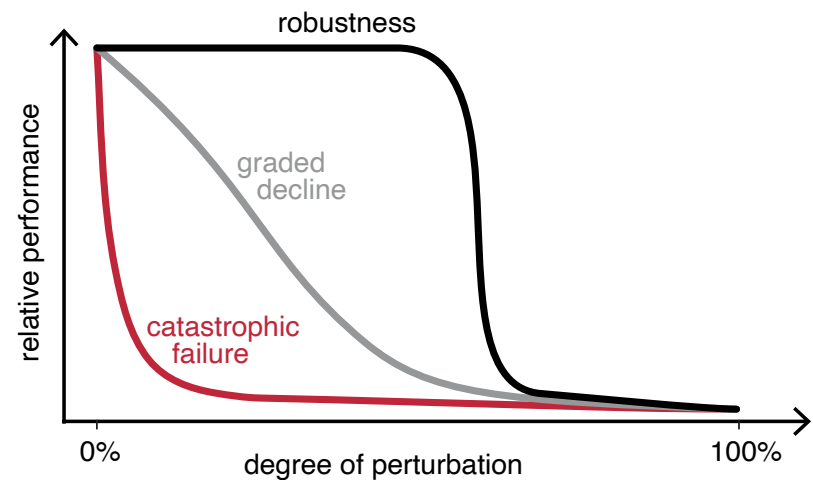
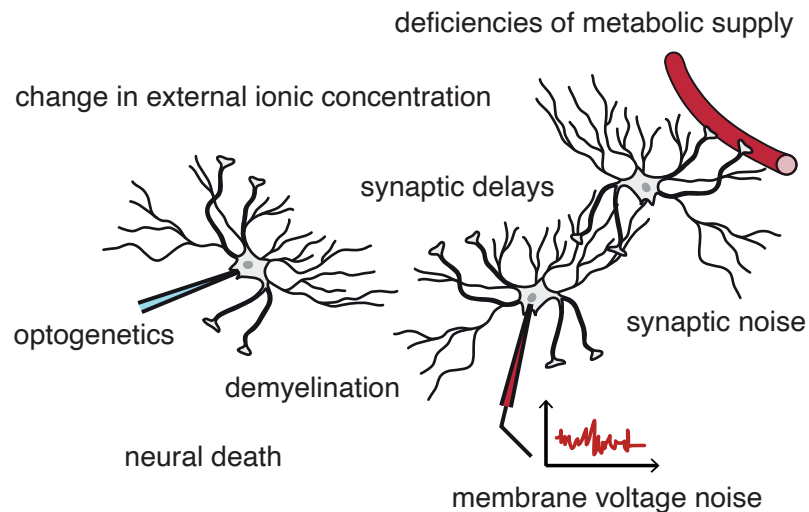


Sensitivity to perturbations in a neural network model

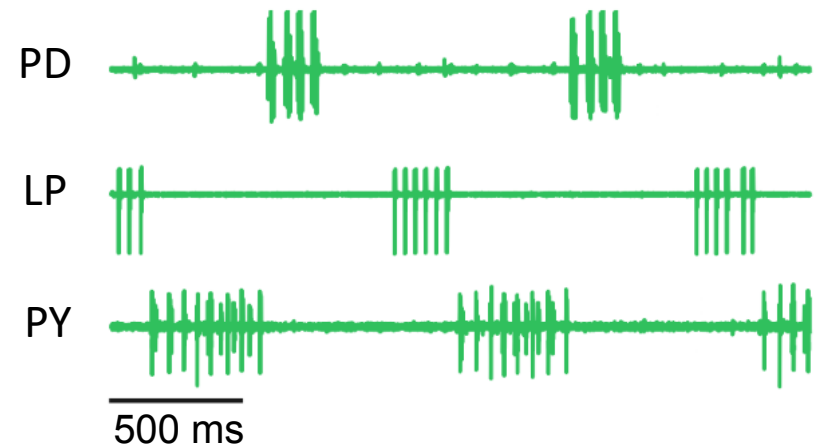
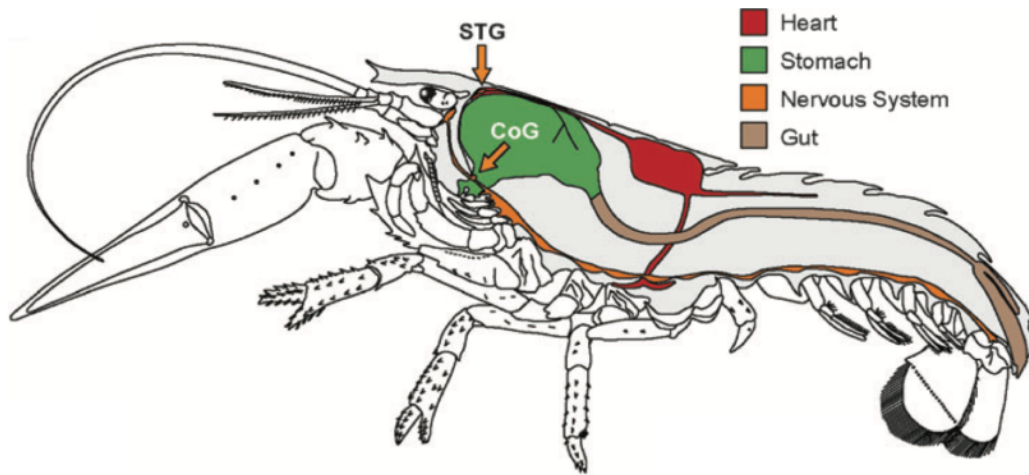
Neural systems operate under multiple perturbations



Neural systems operate under multiple perturbations

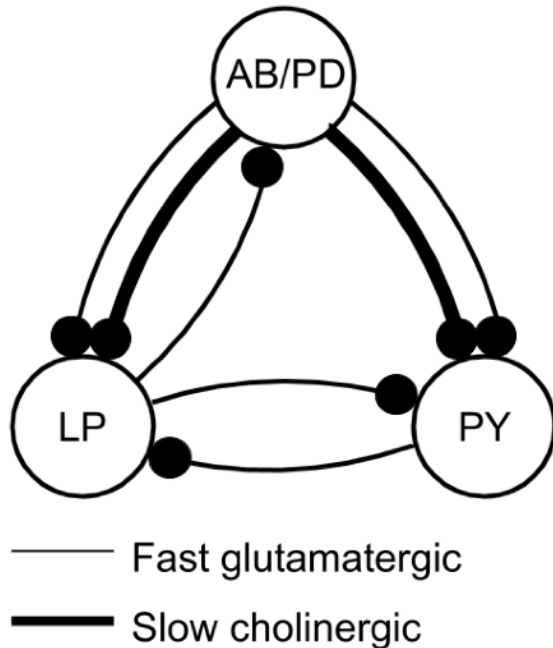


Pyloric network



Marder and Bucher 2007; Haddad and Marder 2018

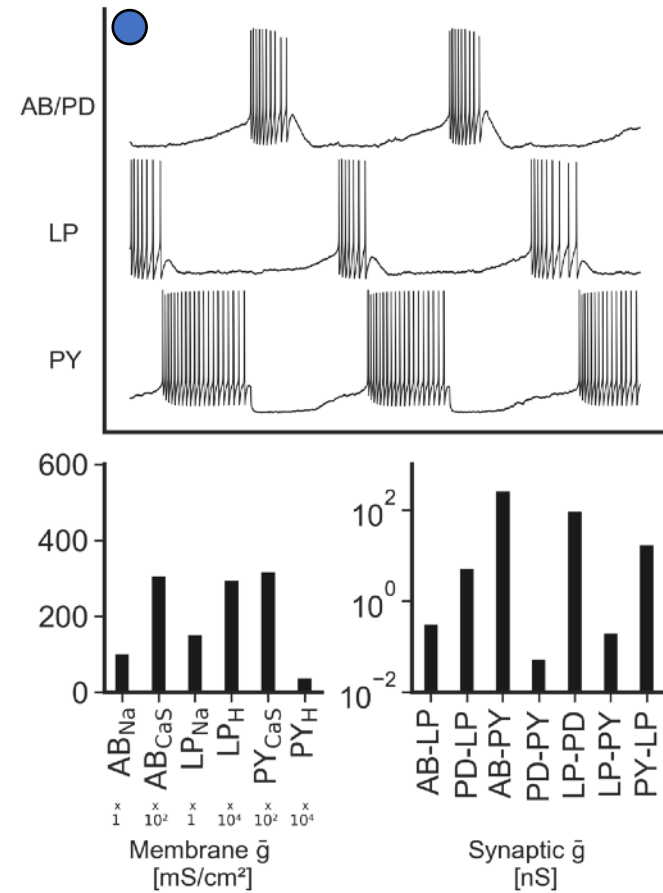
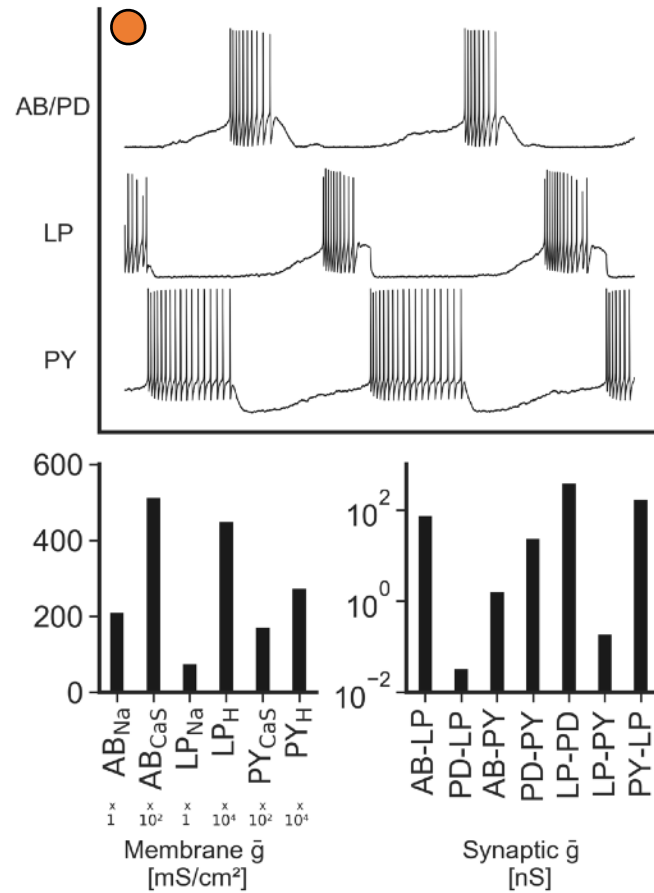
Model of the pyloric network



31 parameters:

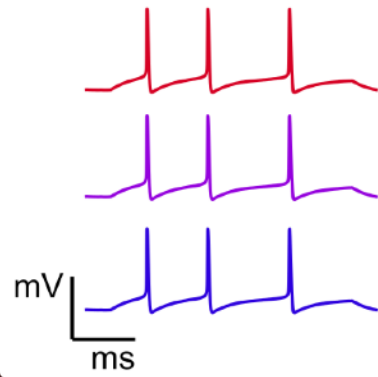
- 8 maximal membrane conductances per neuron
- 7 synaptic strengths

Model of the pyloric network

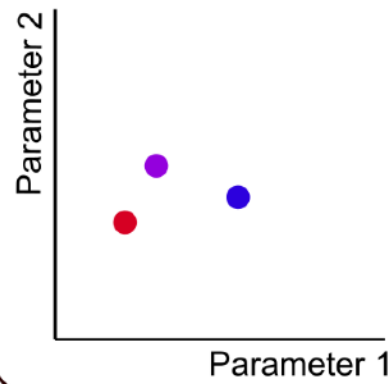


Experimental data

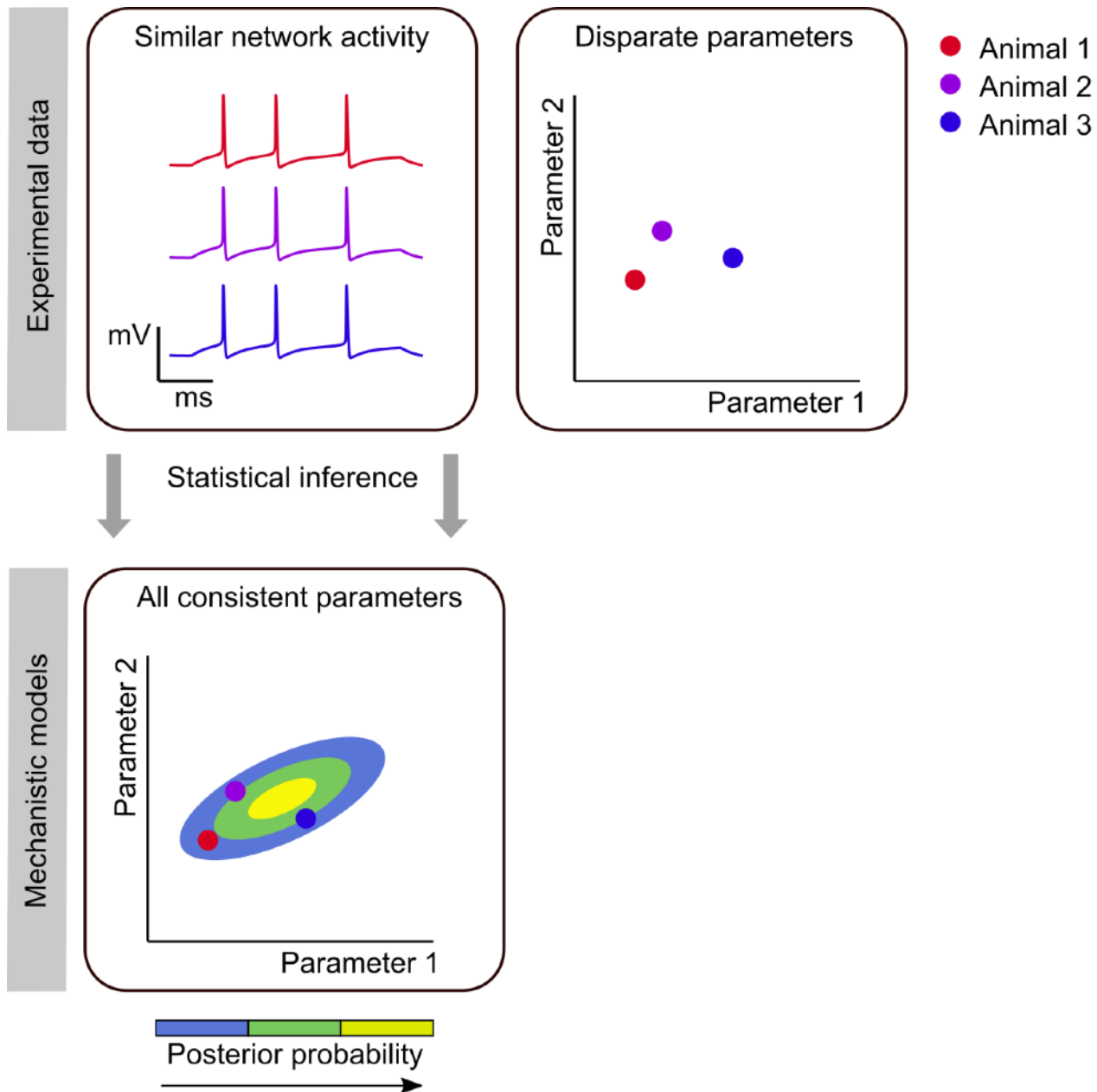
Similar network activity

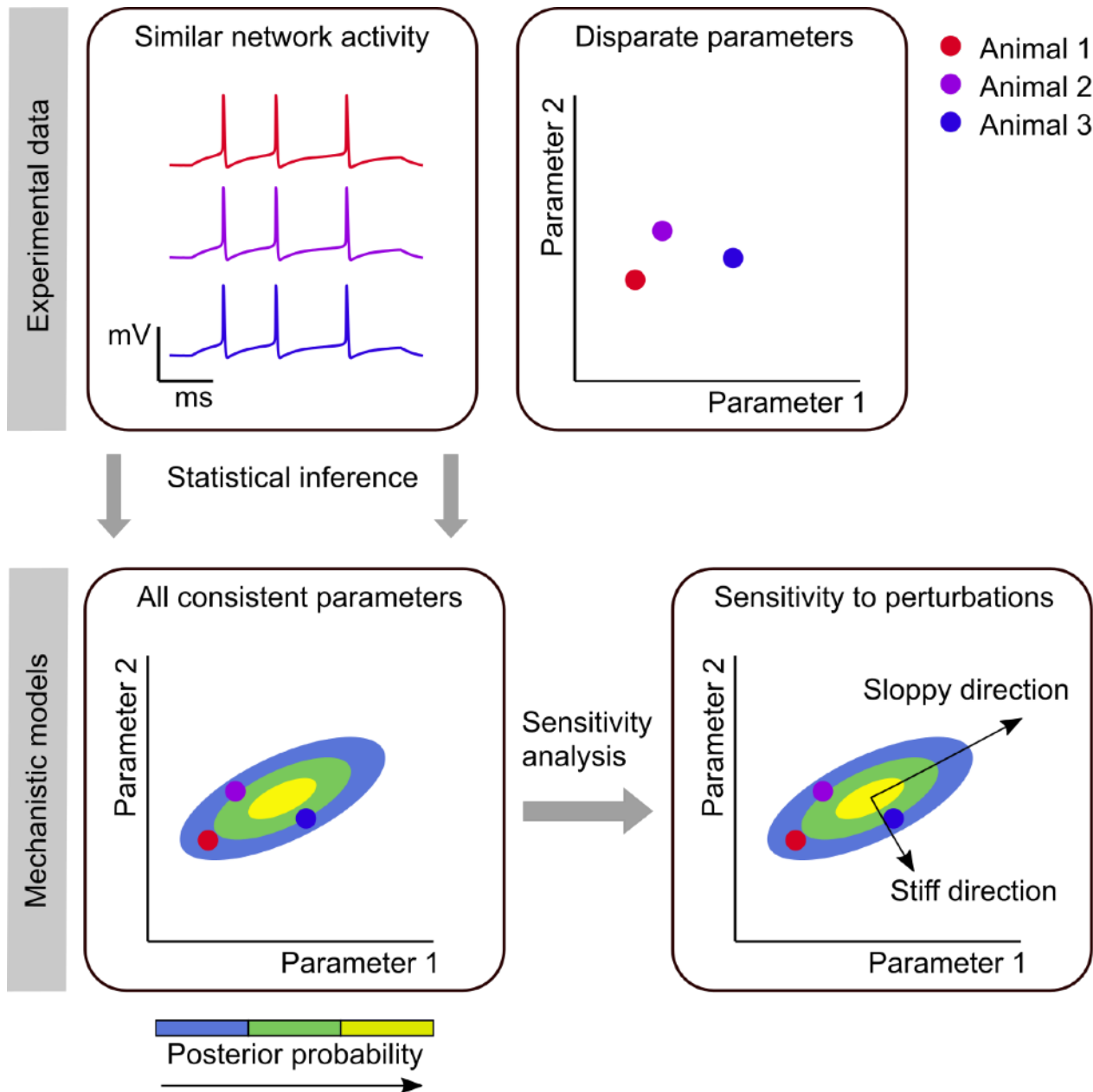


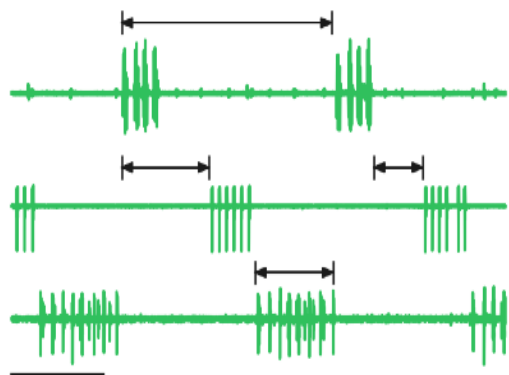
Disparate parameters



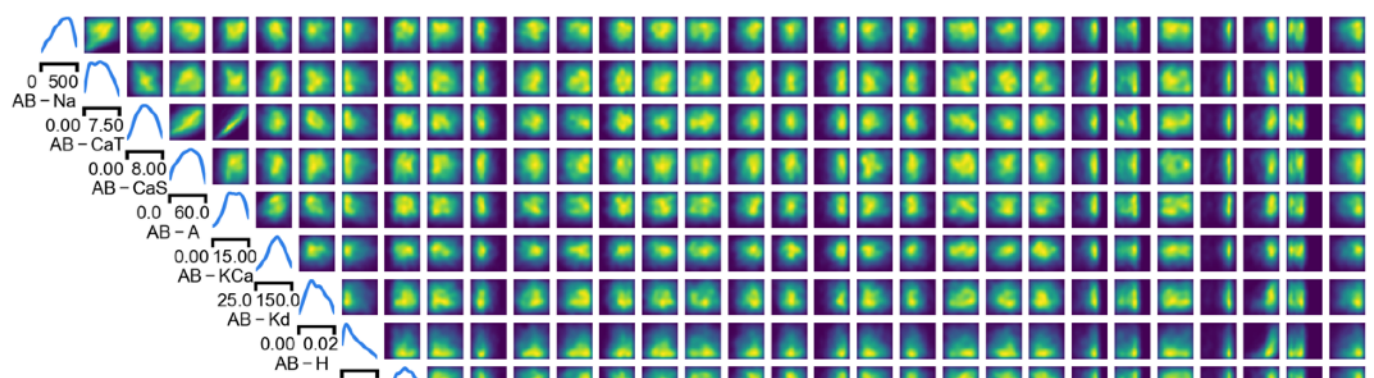
- Animal 1
- Animal 2
- Animal 3



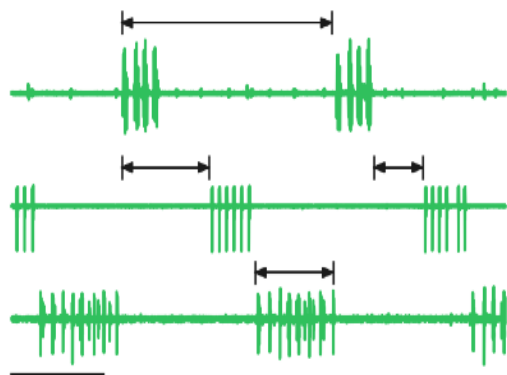




AB/PD

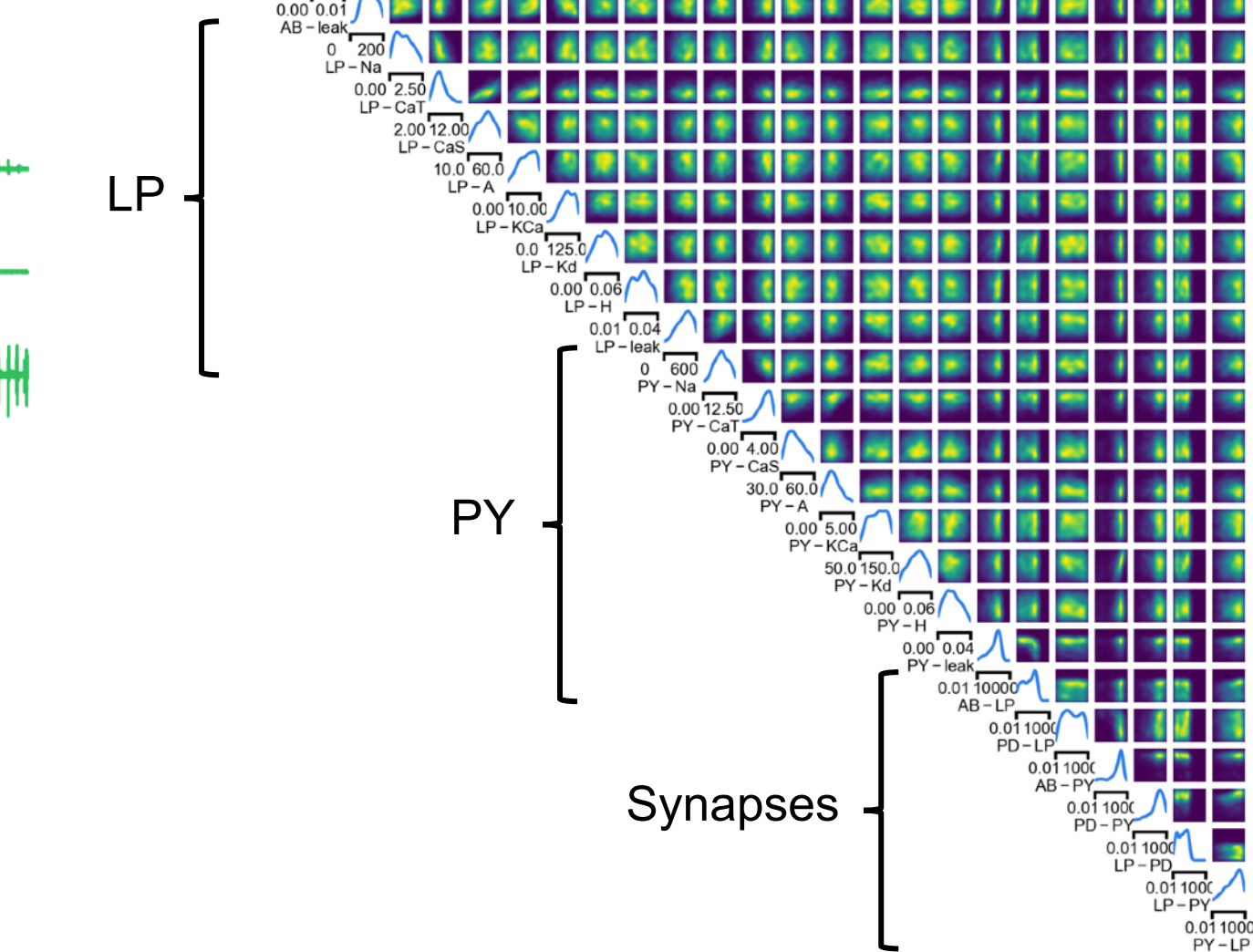


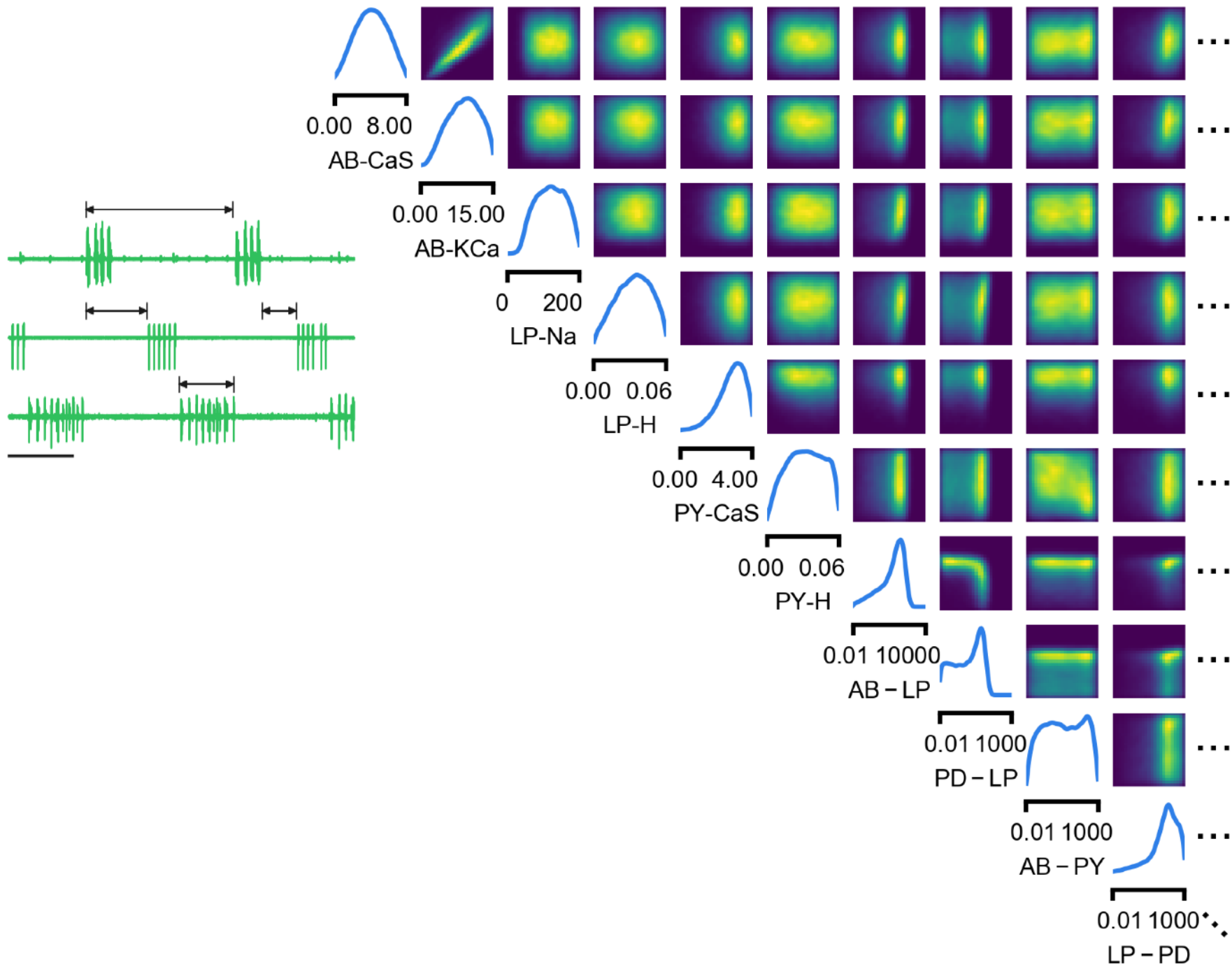
LP

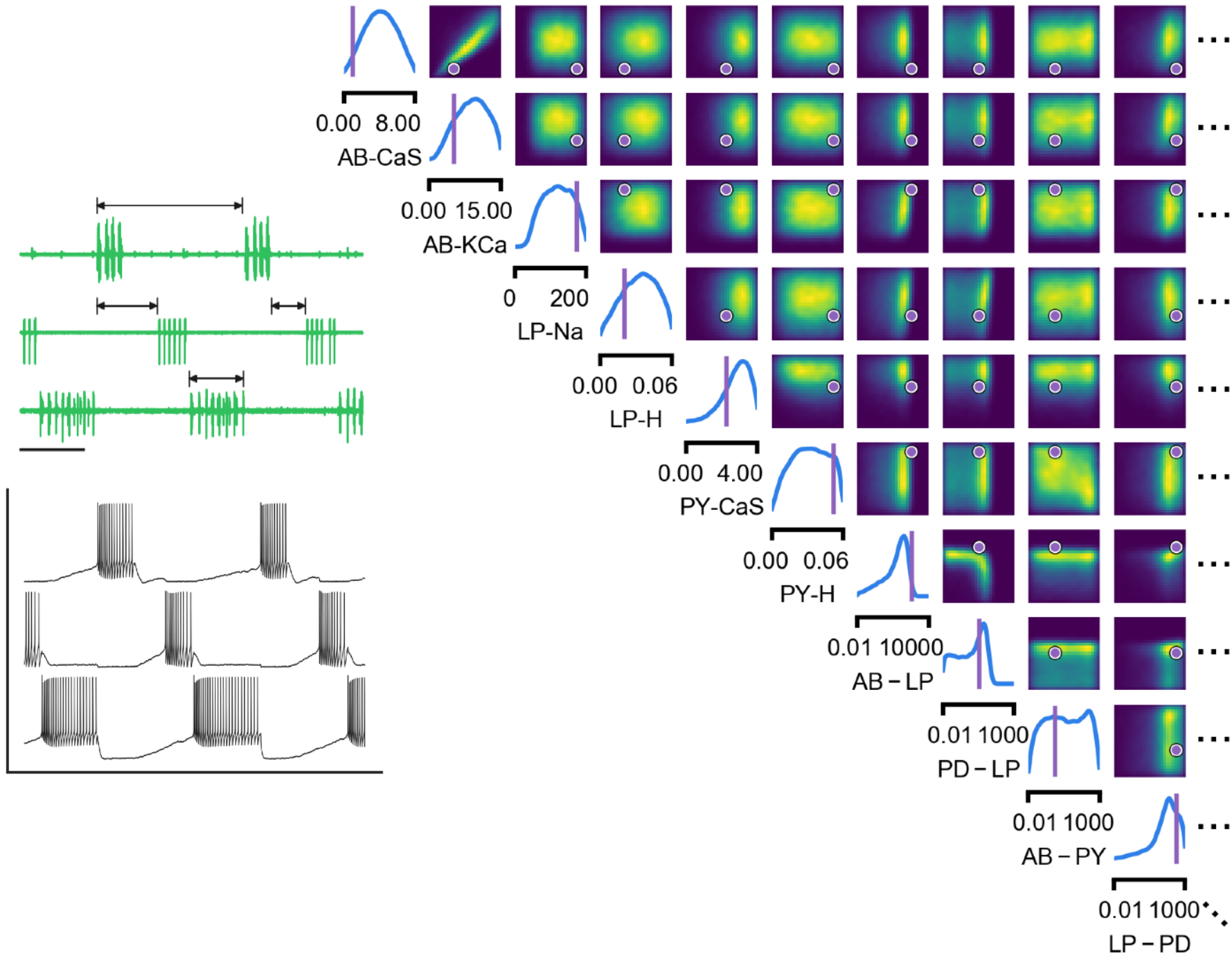


PY

Synapses

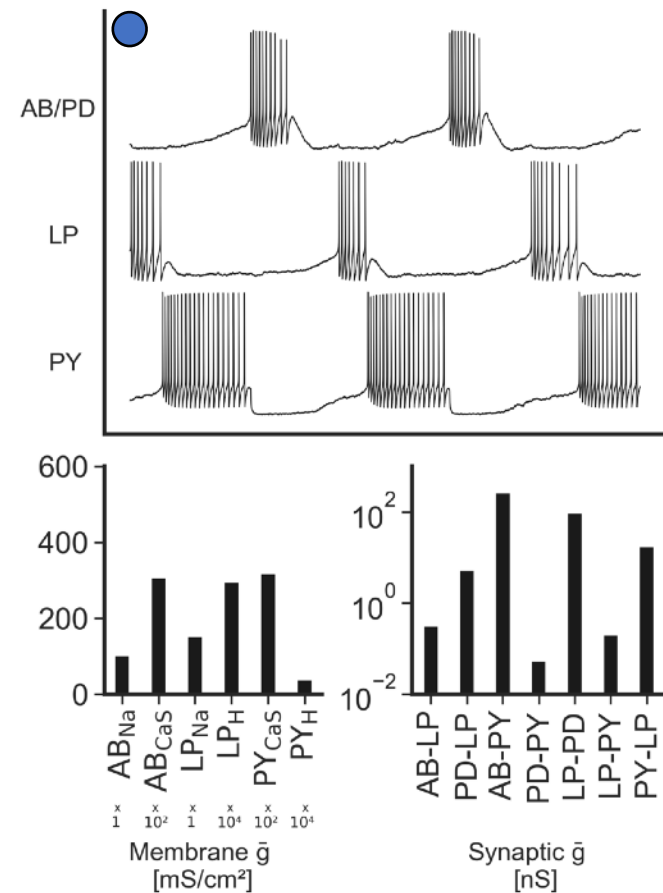
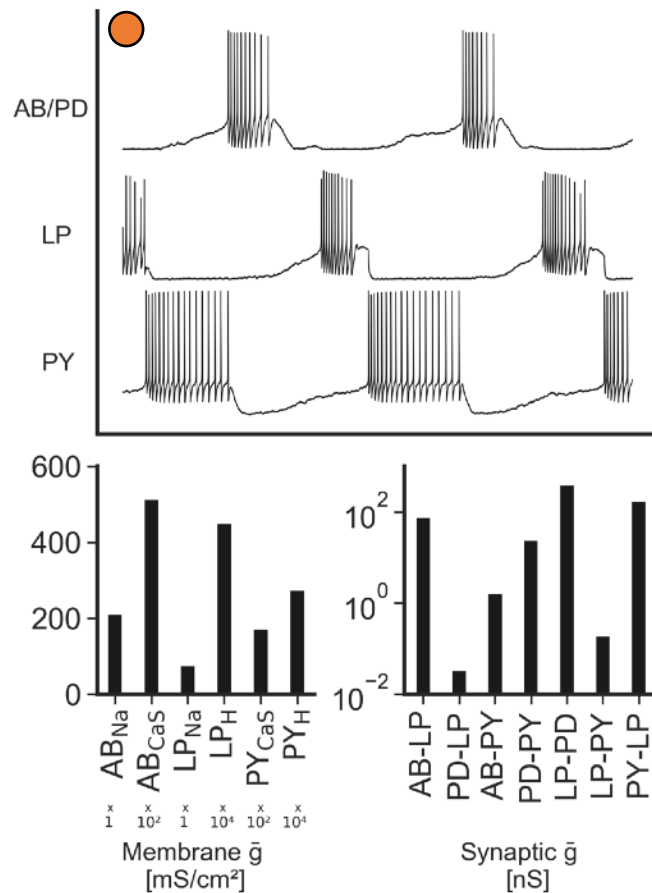




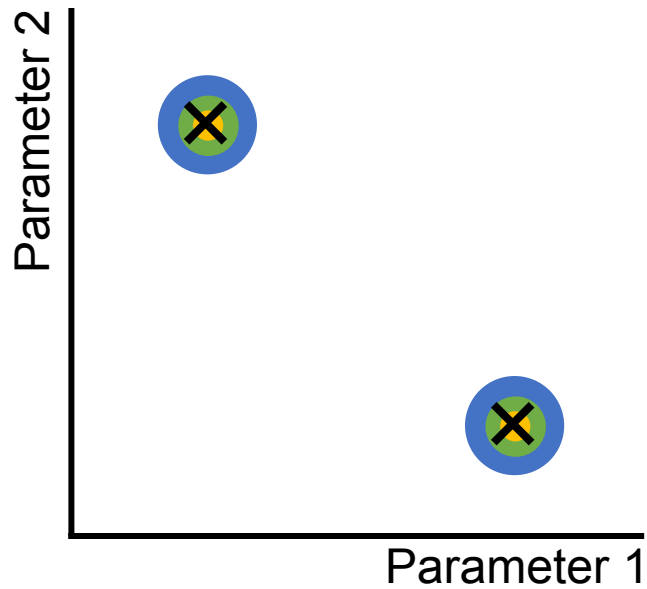


Analysing the posterior

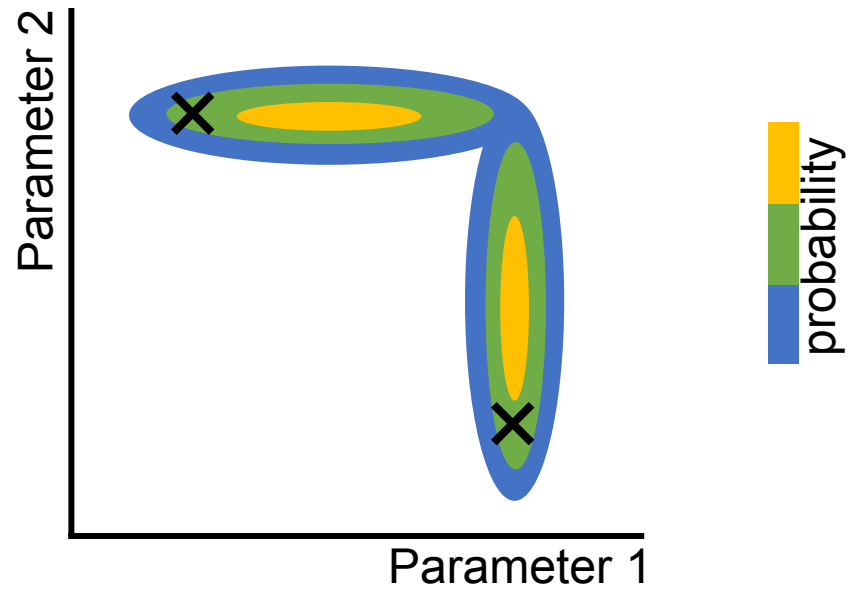
Robustness to perturbations



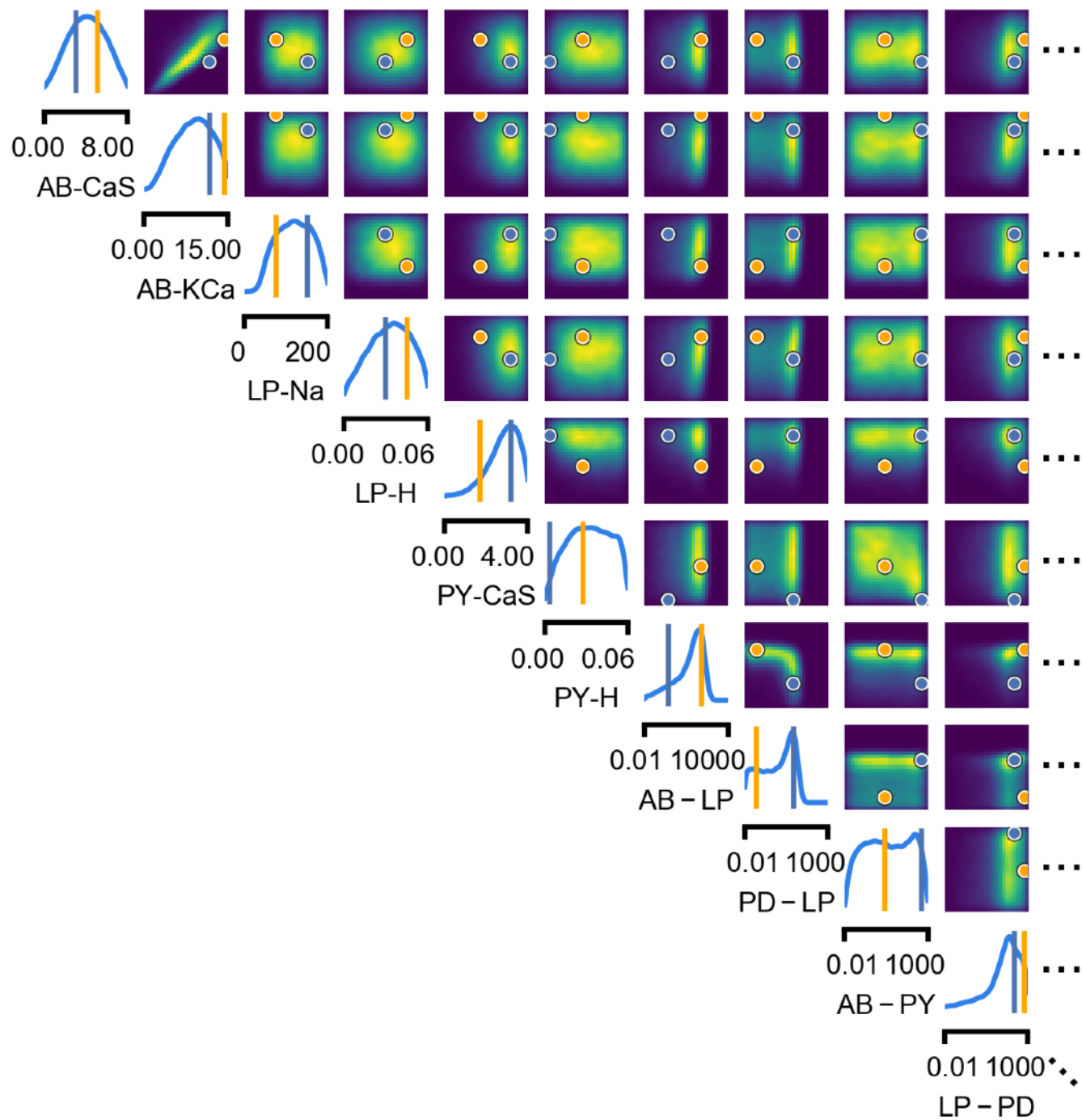
Scenario 1: parameter sets lie on separate islands

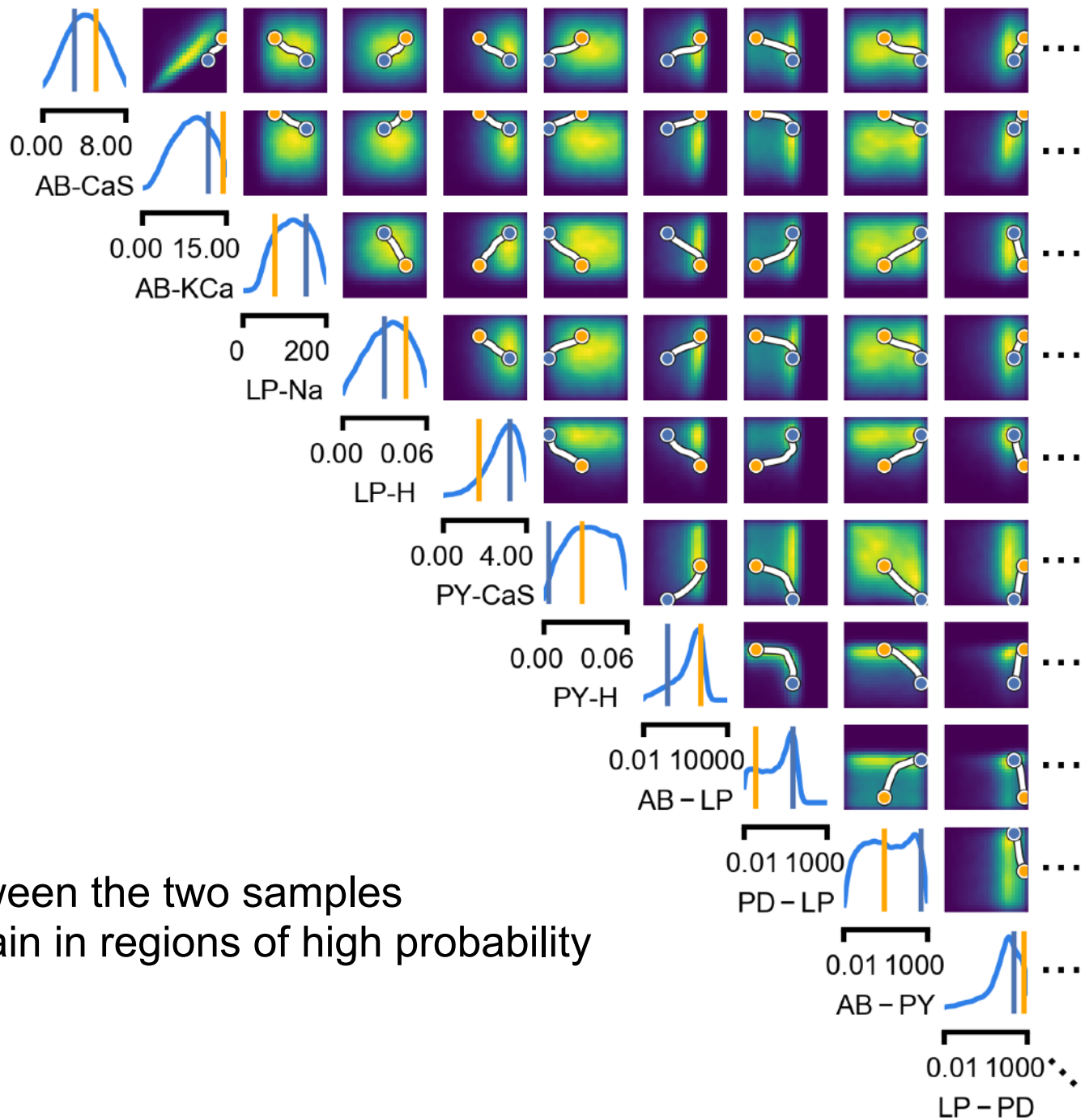


Scenario 2: parameter sets are connected

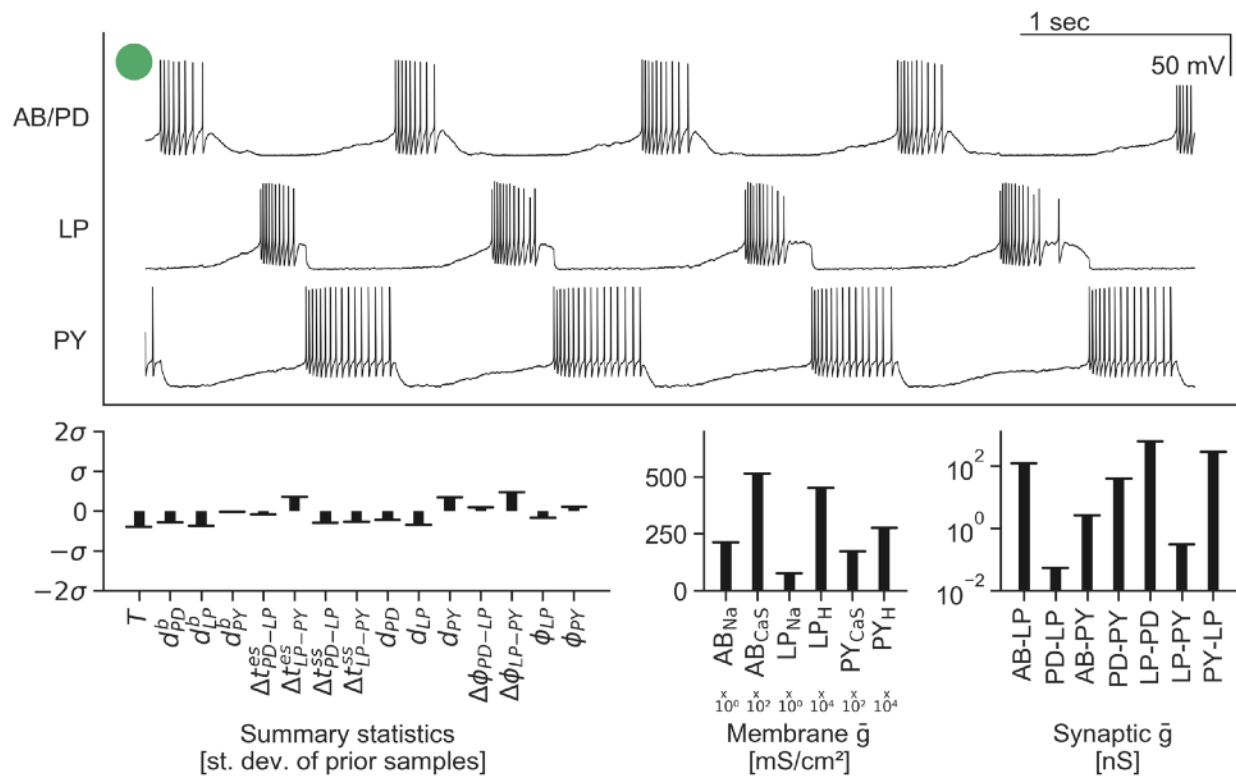
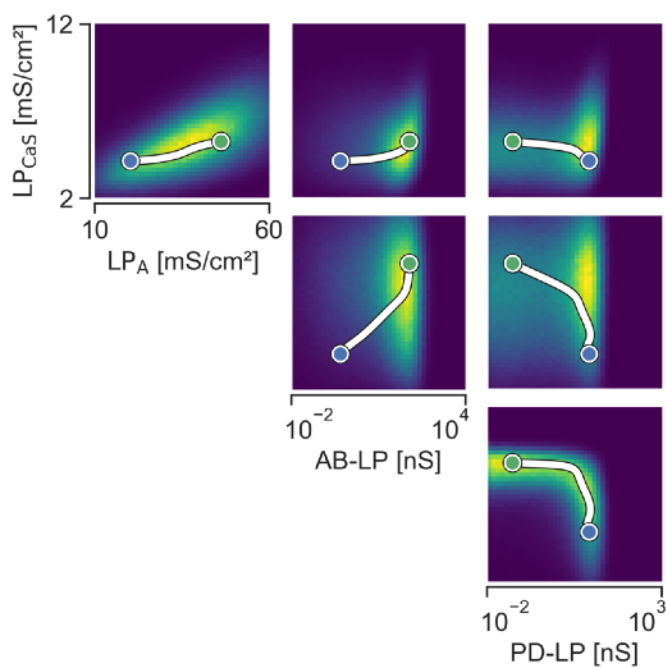


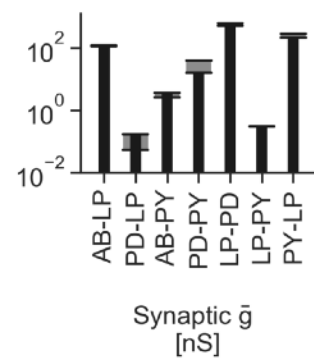
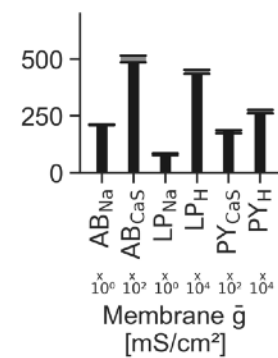
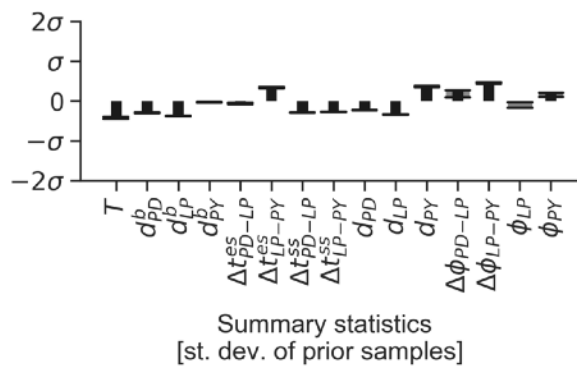
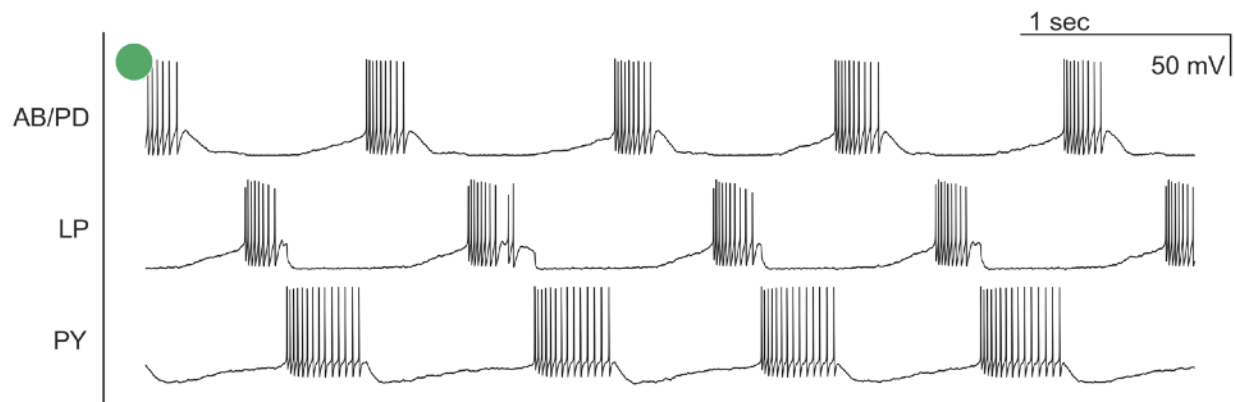
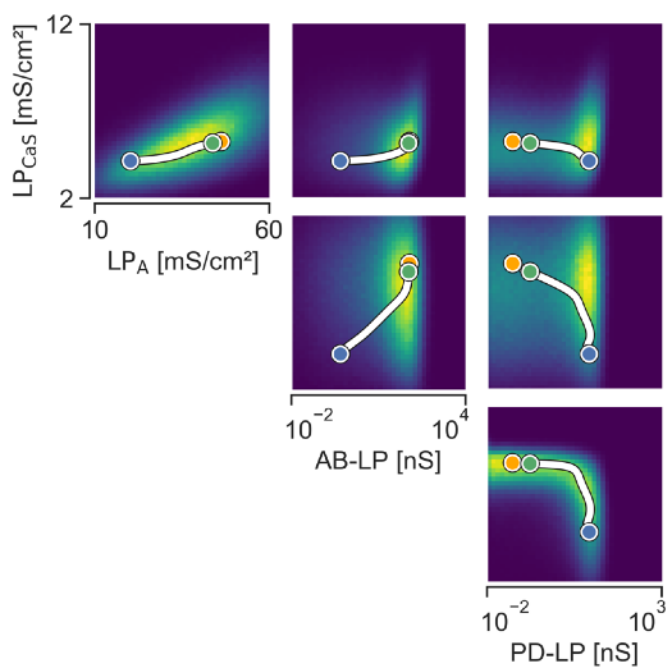
✕ Consistent parameter sets

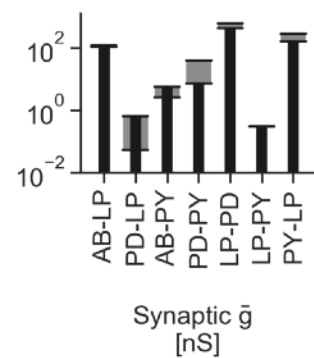
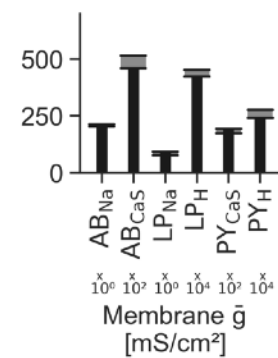
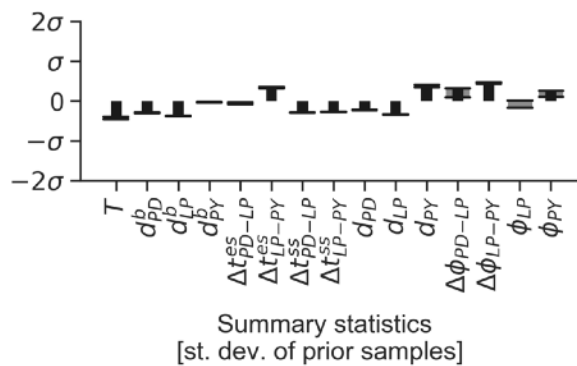
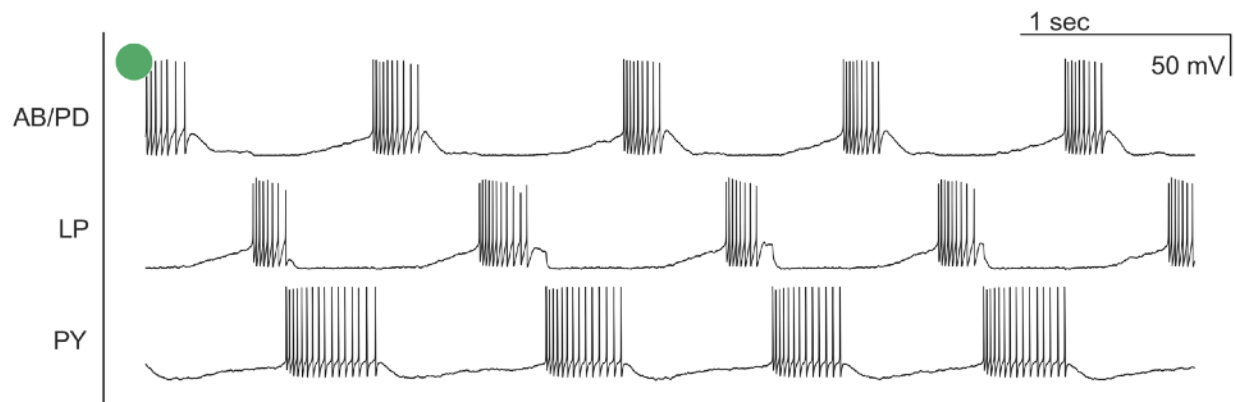
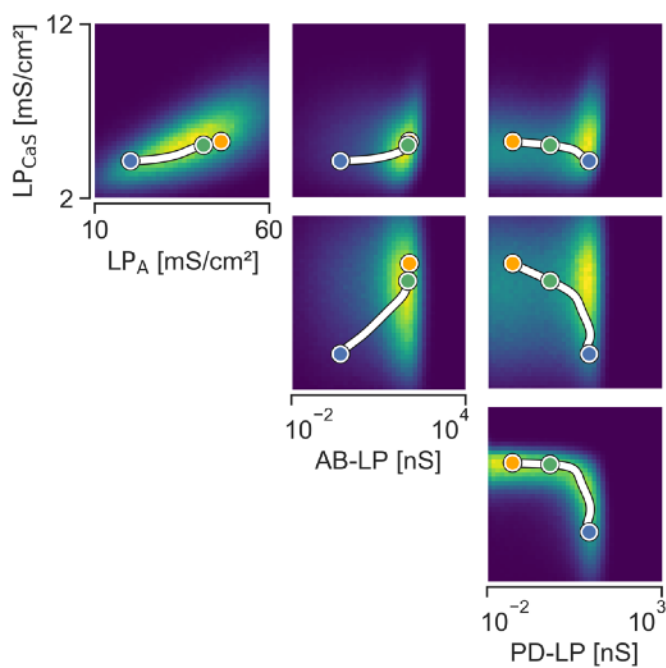


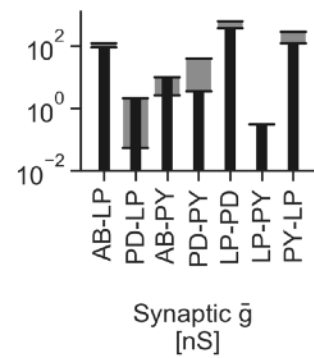
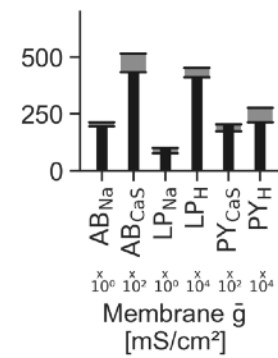
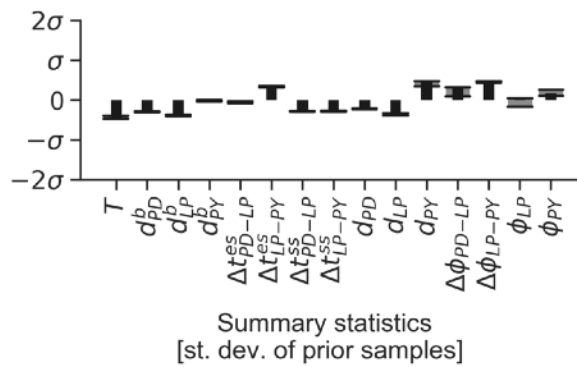
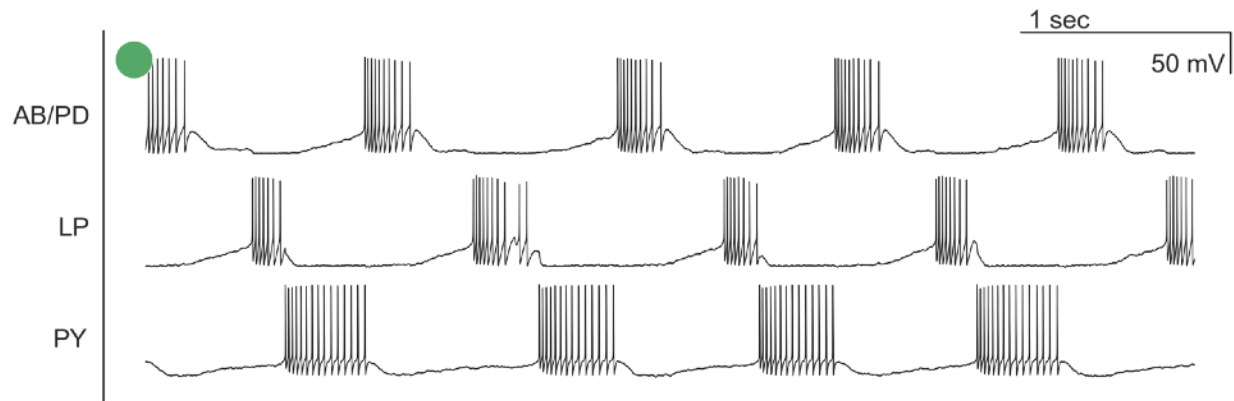
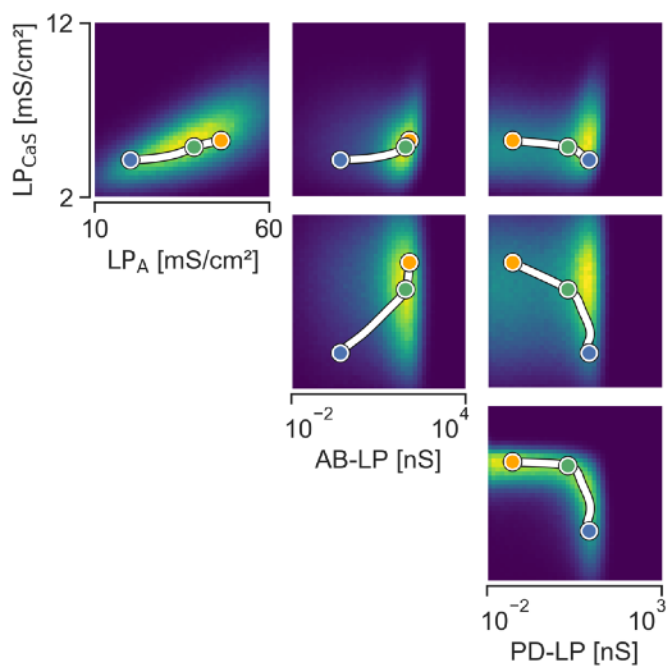


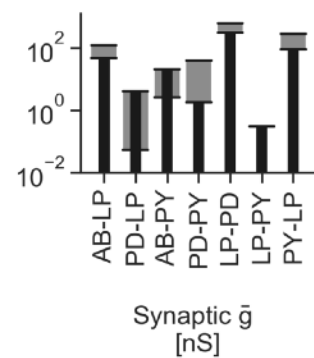
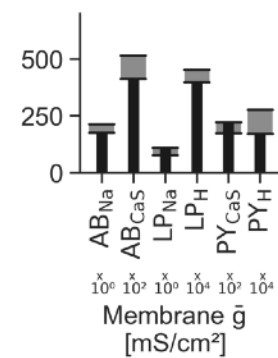
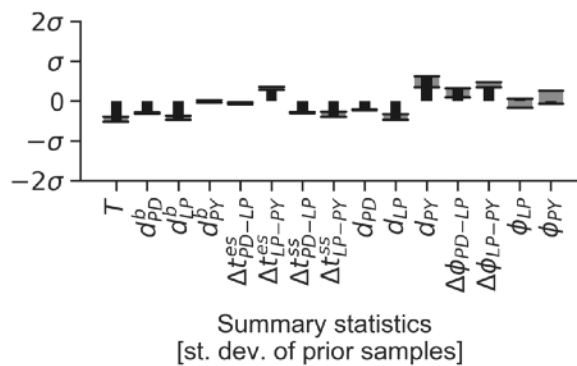
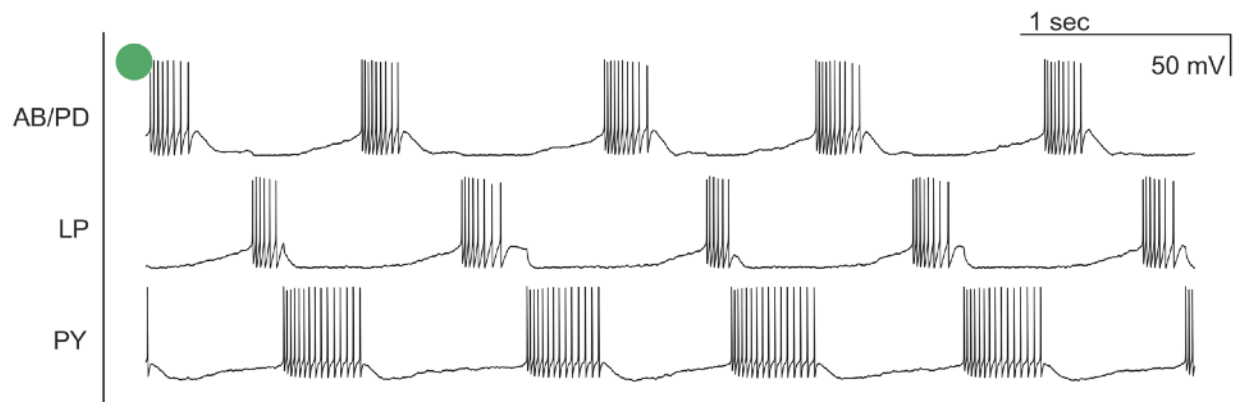
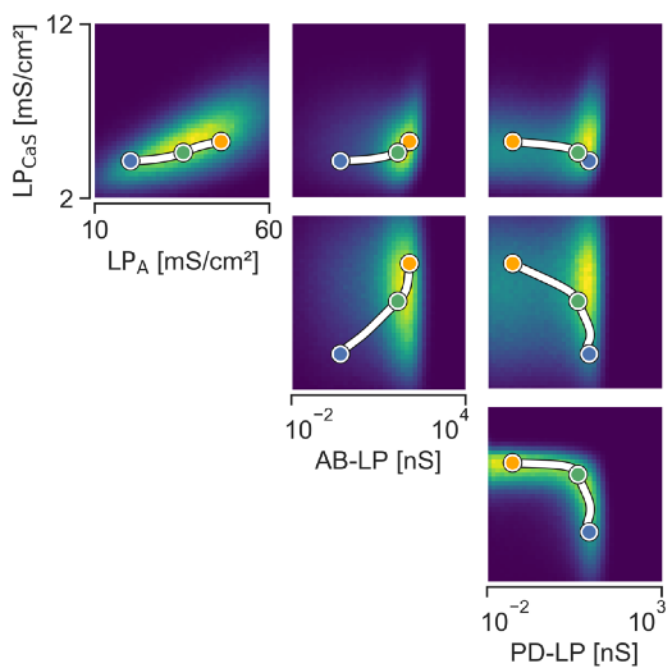
- Find a 'path' between the two samples
- Path should remain in regions of high probability

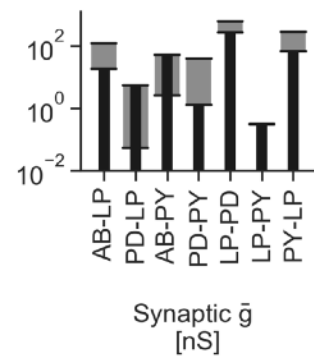
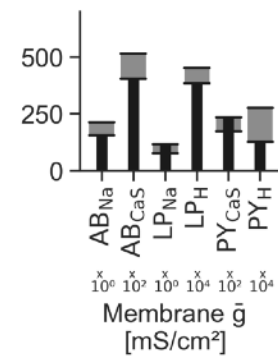
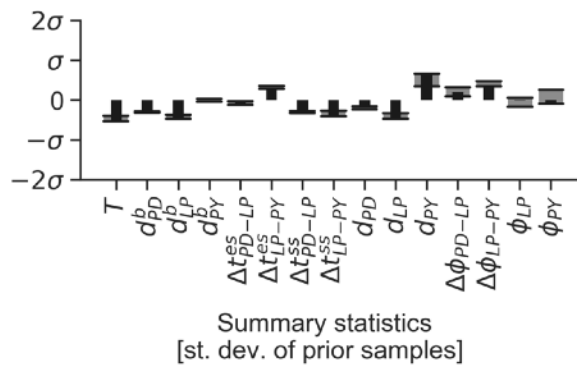
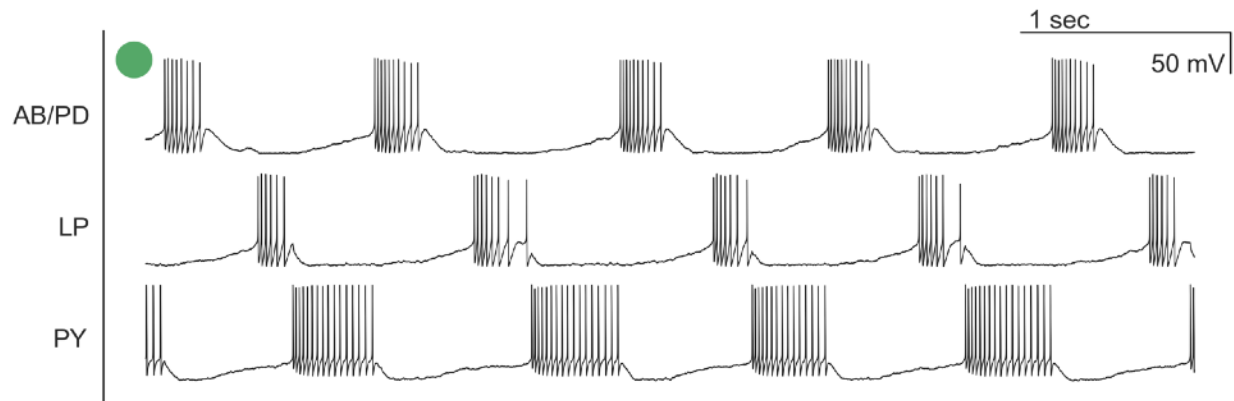
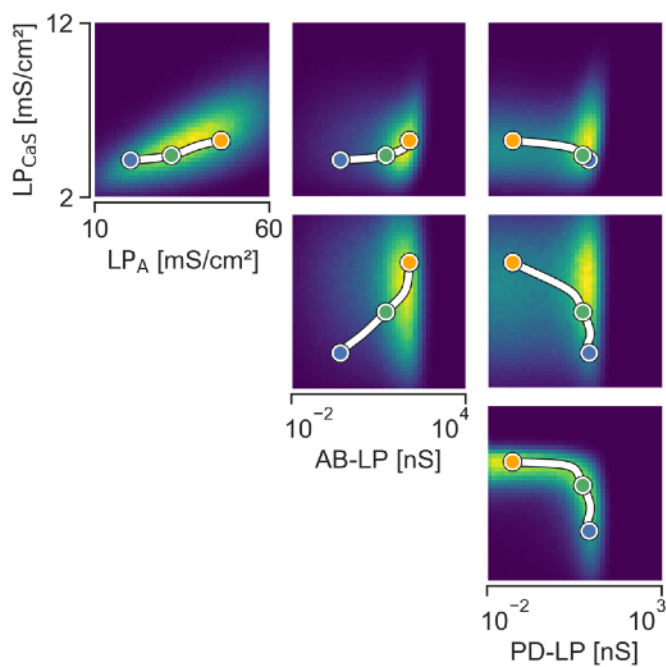


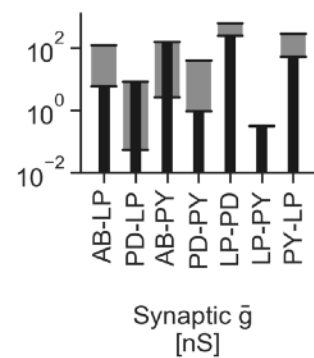
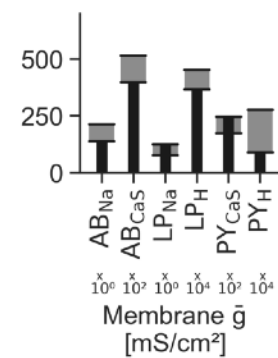
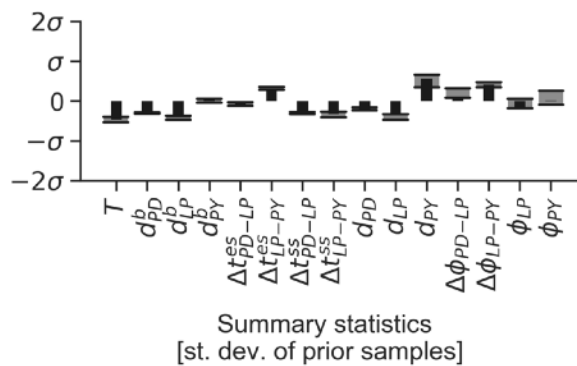
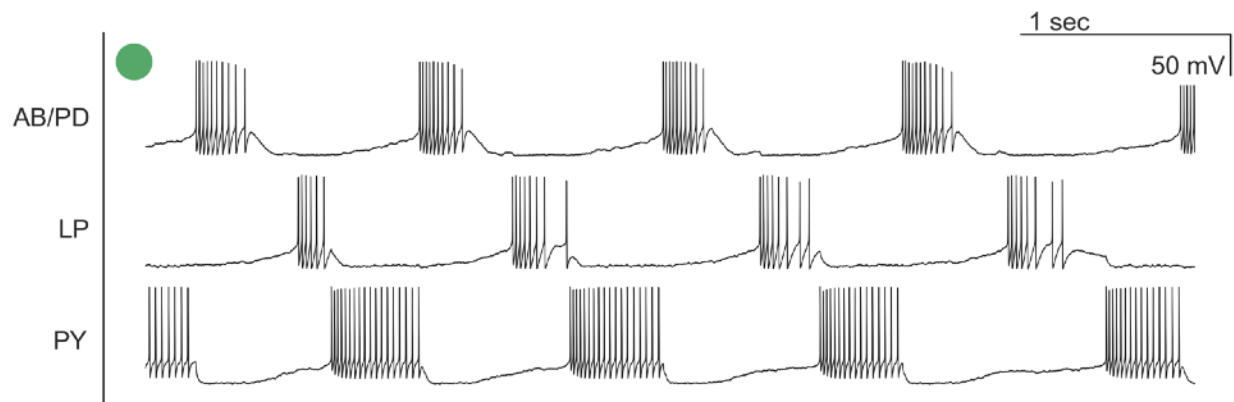
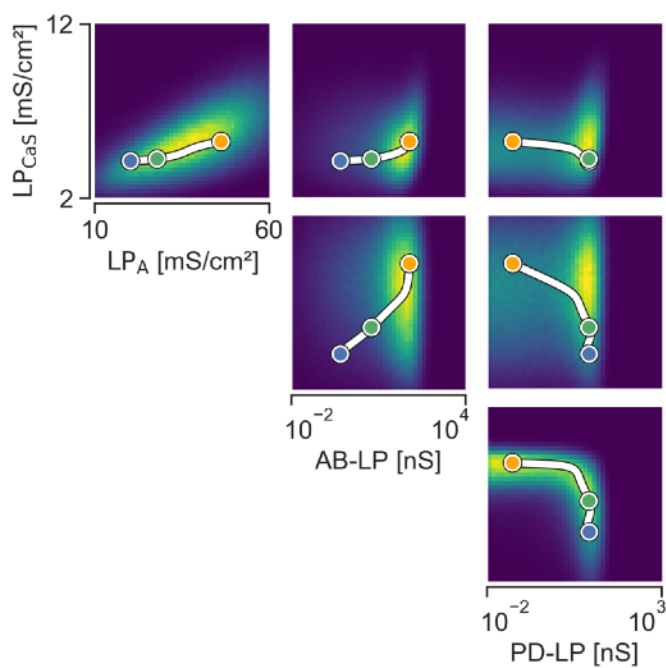


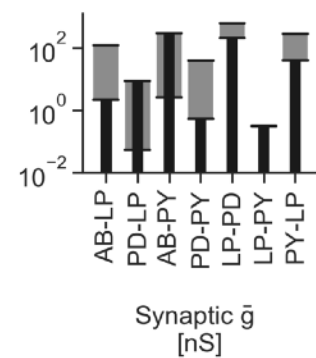
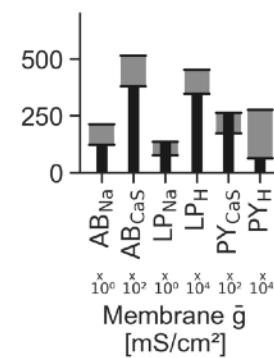
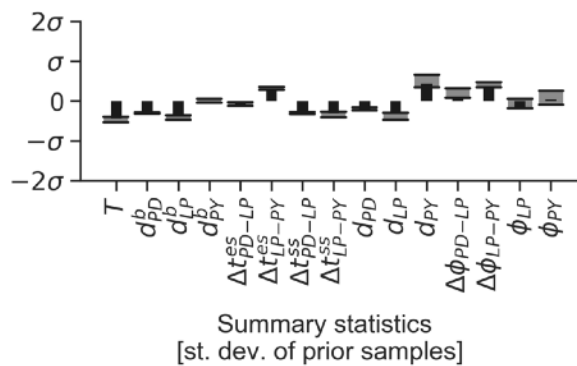
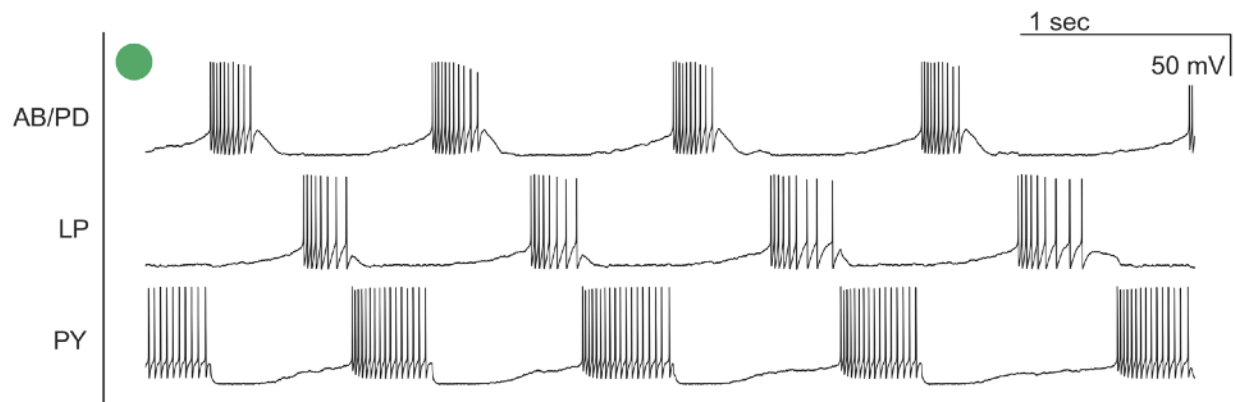
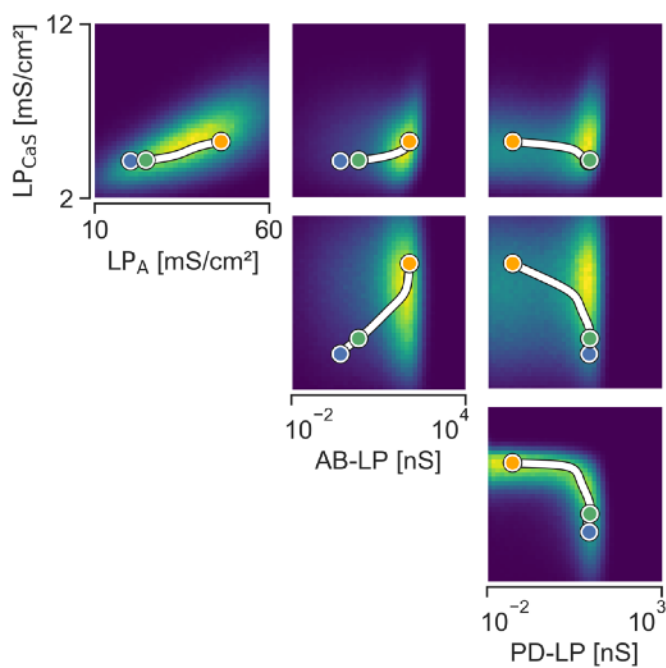


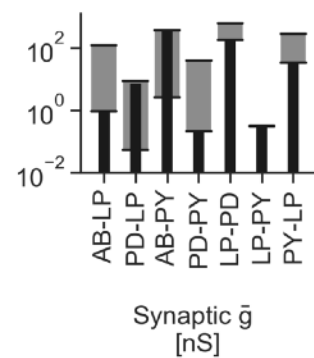
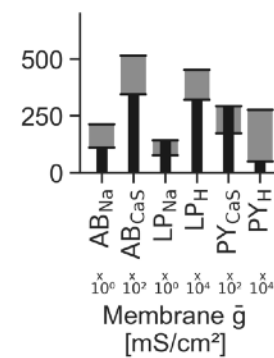
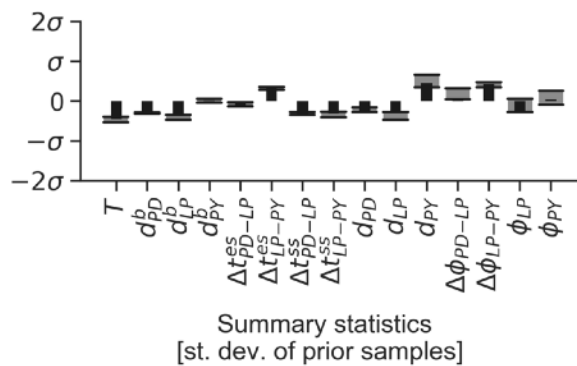
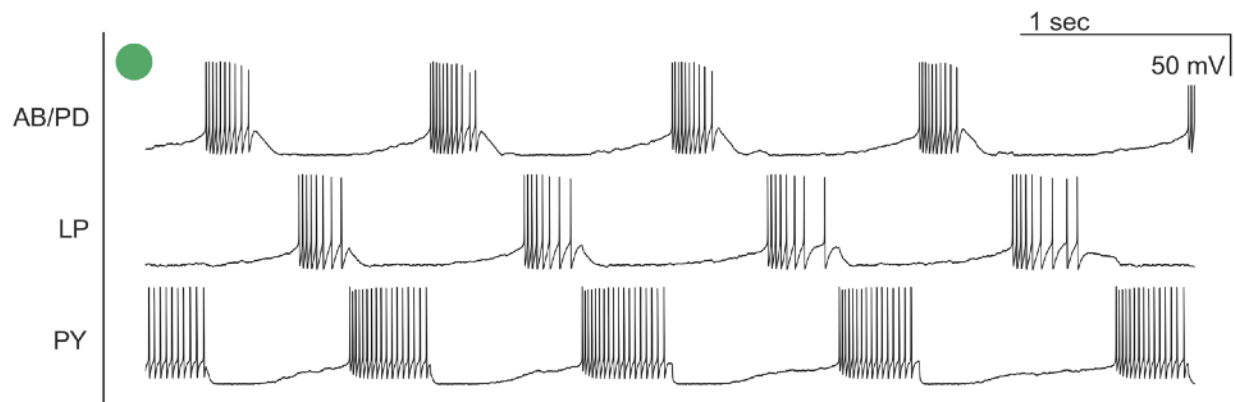
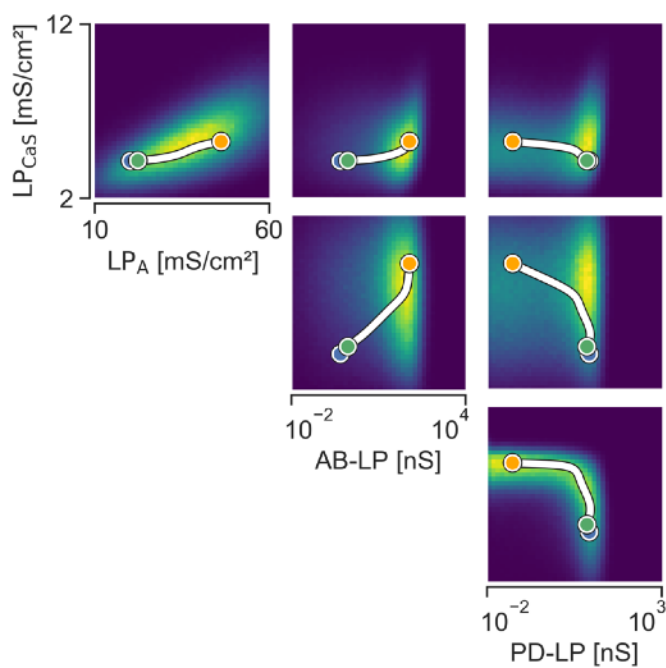




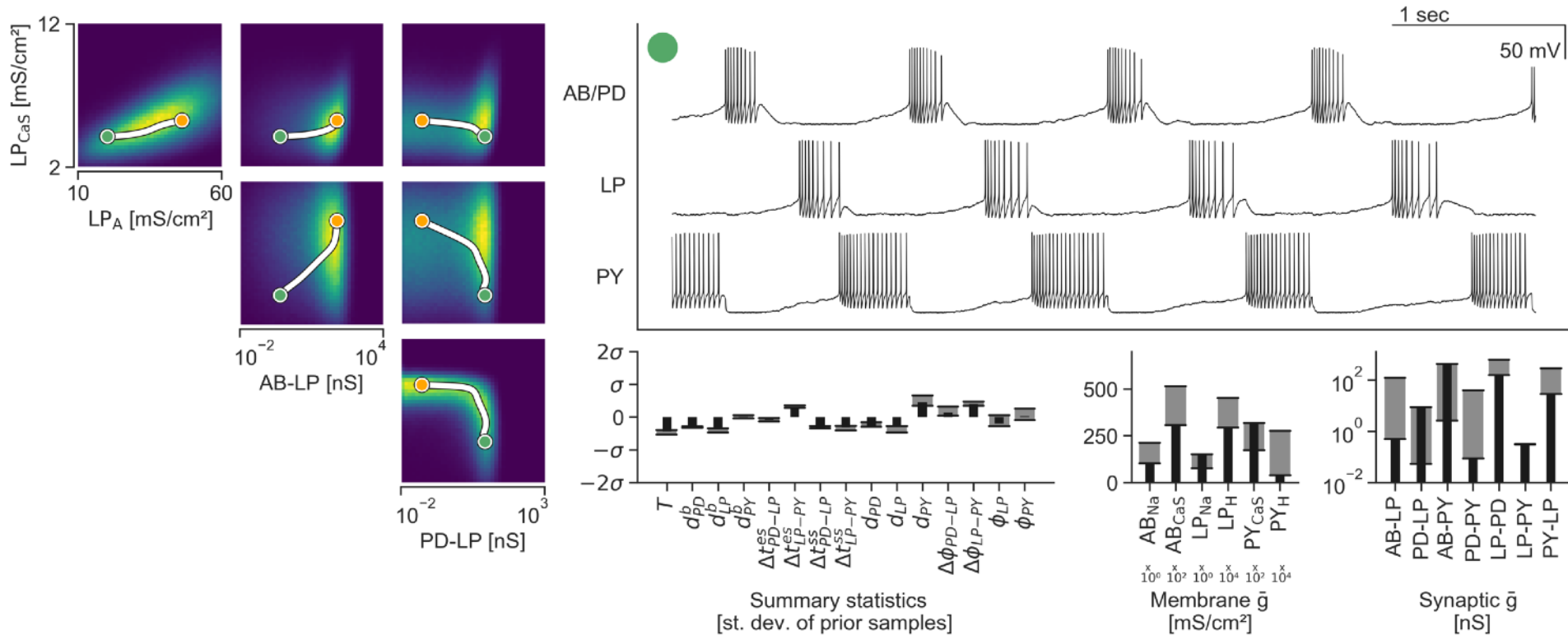




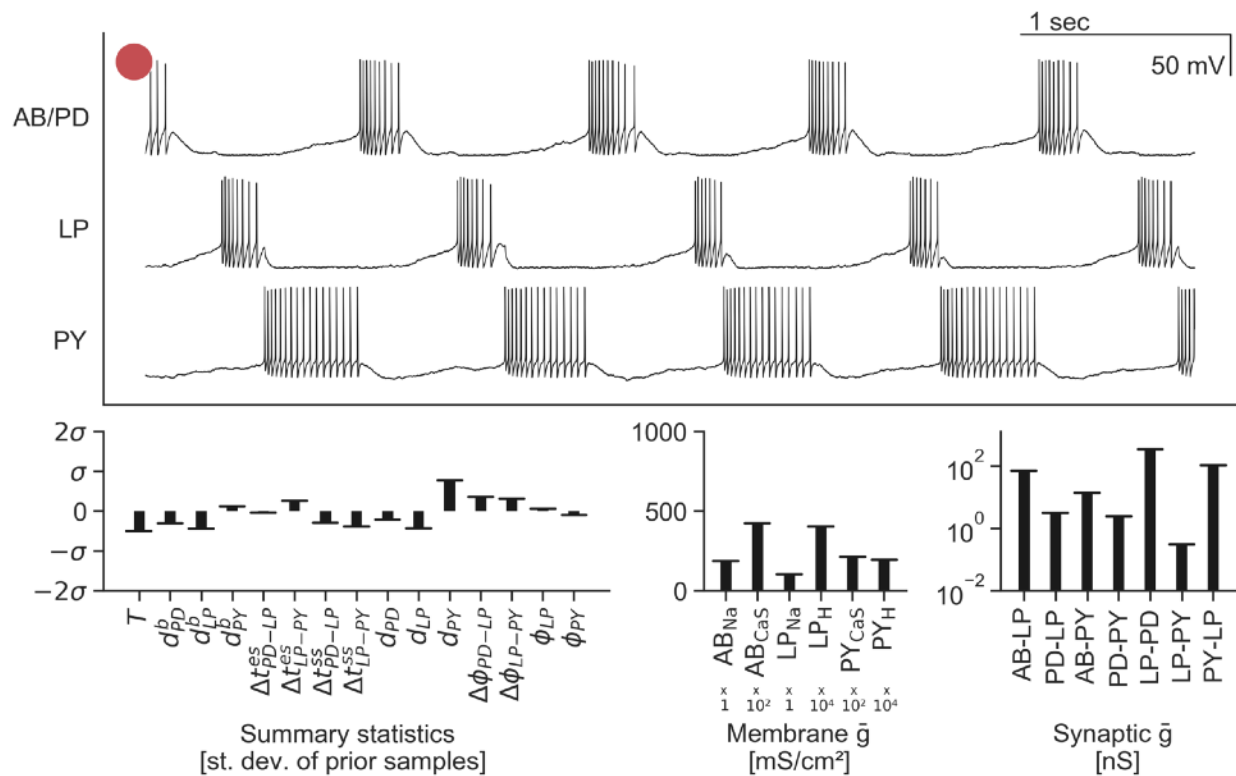


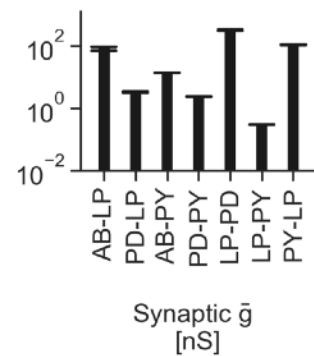
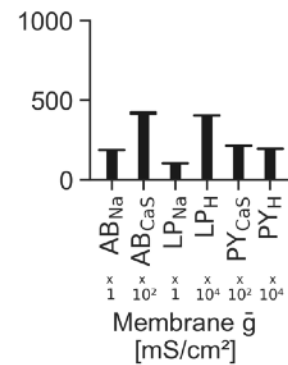
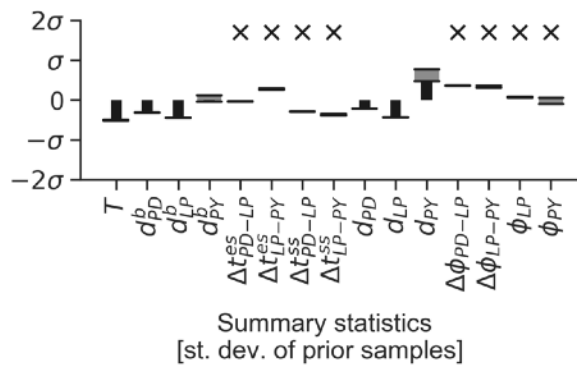
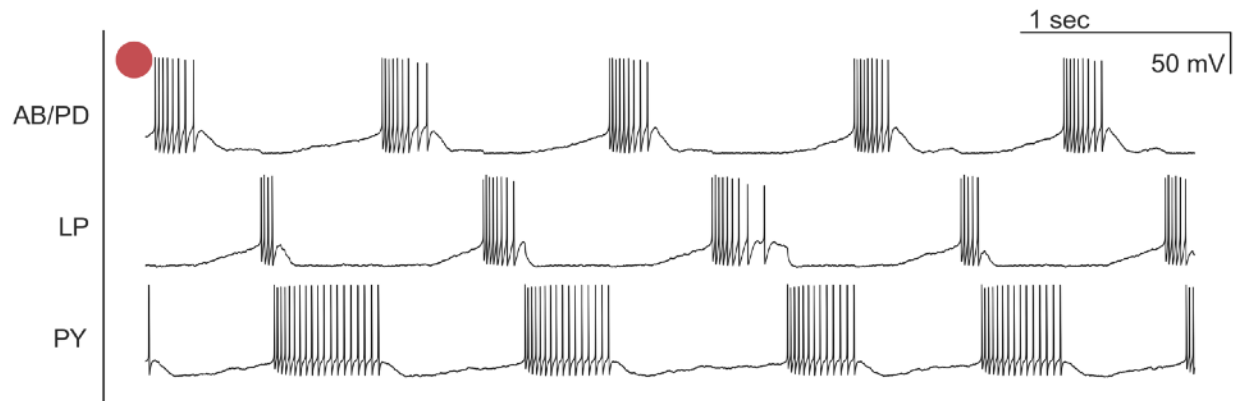
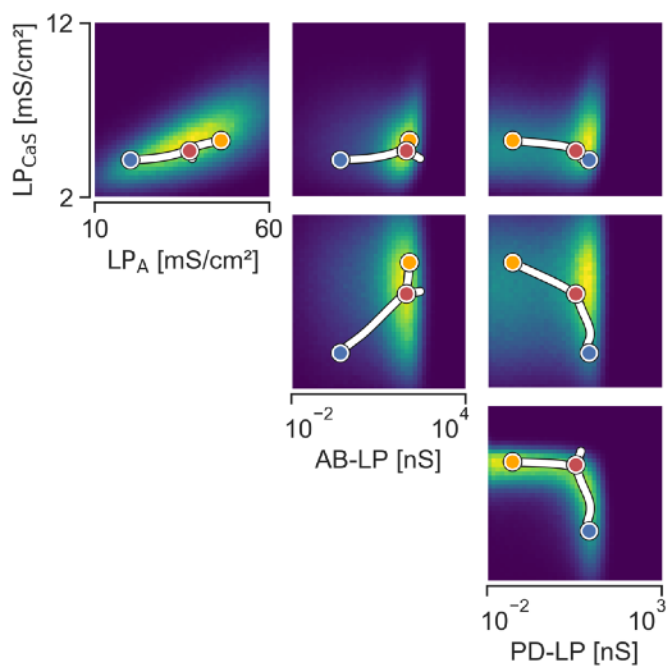


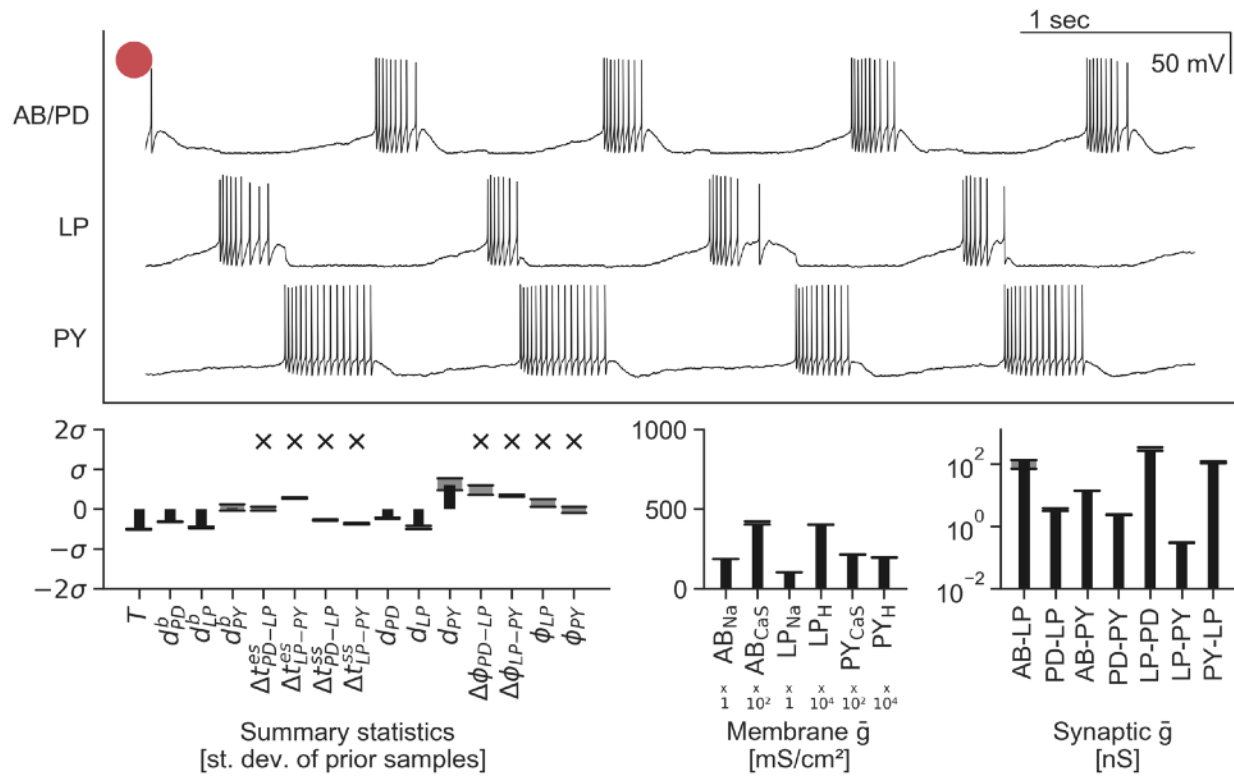
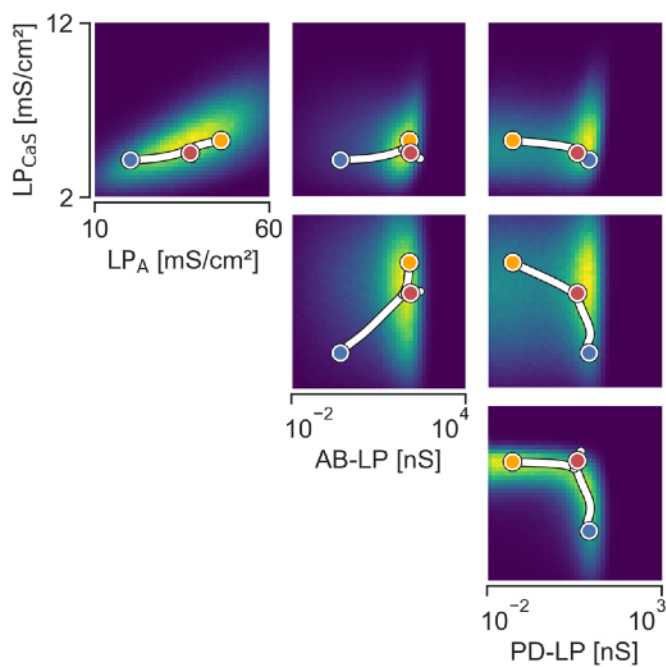
Parameter sets producing similar outputs are connected in parameter space: no 'islands'

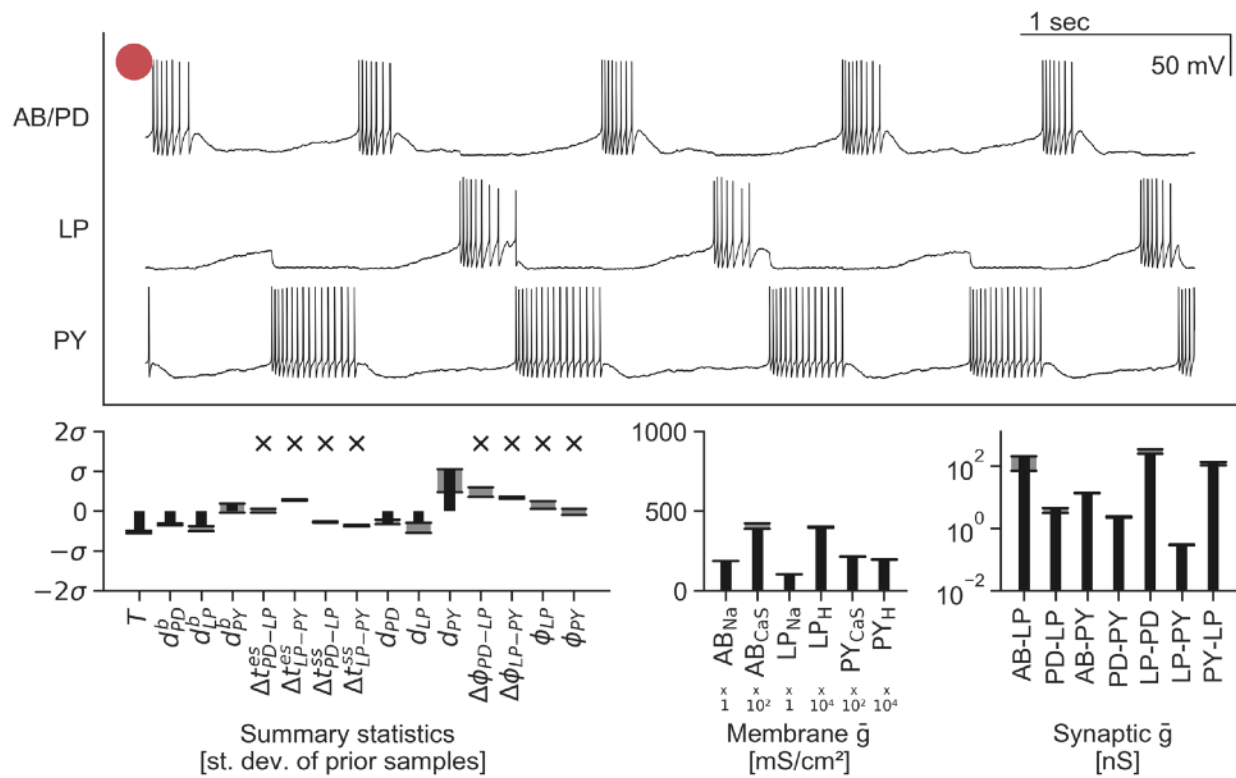


Can small perturbations lead the circuit to break down?

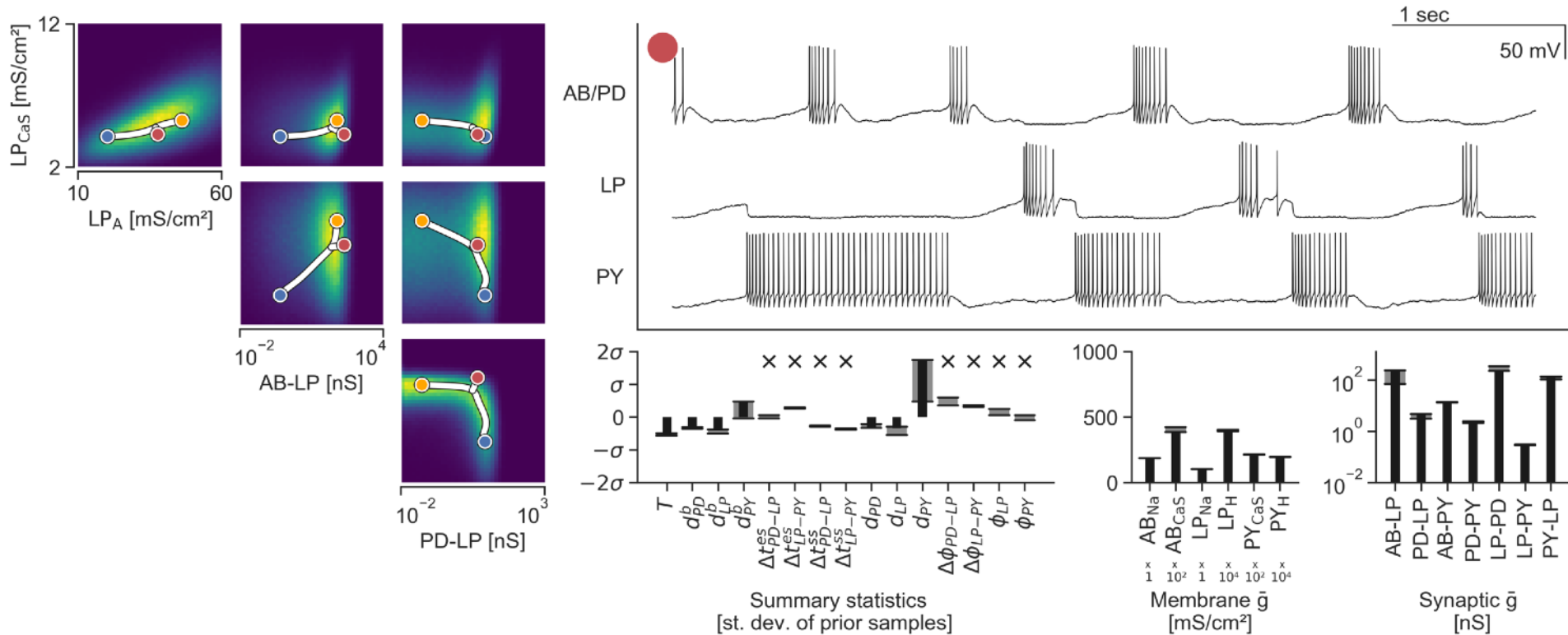






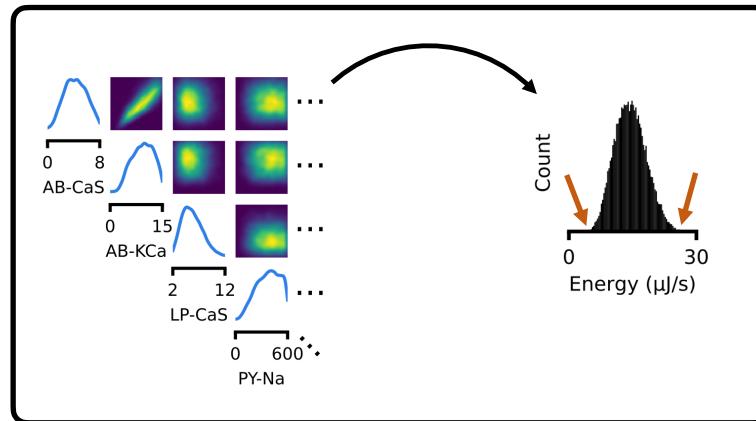


Small perturbations can lead the circuit to break down



Flexibility of simulation-based inference

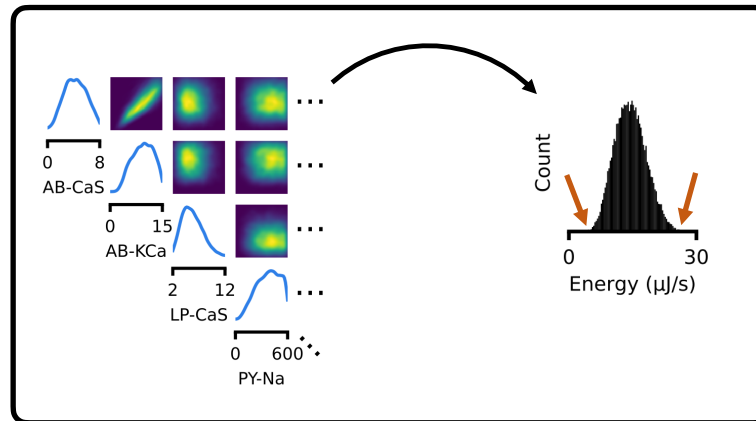
Energy efficiency and robustness



M Deistler, JH Macke*, PJ Goncalves* (2021) biorXiv

Flexibility of simulation-based inference

Energy efficiency and robustness

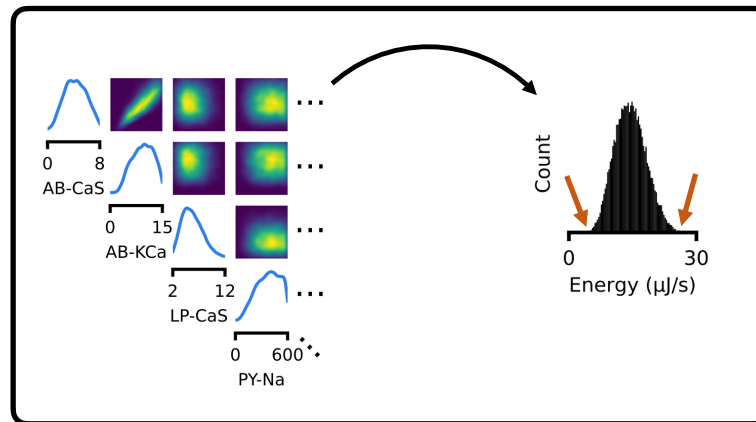


M Deistler, JH Macke*, PJ Goncalves* (2021) biorXiv

But other projects ongoing at different scales in the brain. Also, cosmology and climate sciences.

Flexibility of simulation-based inference

Energy efficiency and robustness



M Deistler, JH Macke*, PJ Goncalves* (2021) biorXiv

But other projects ongoing at different scales in the brain. Also, cosmology and climate sciences.

Workshop on simulation-based inference for scientific discovery, 09/2021, Tübingen.

Discussion

- **Flexibility:** inference approach does not depend on specifics of mechanistic model, works with simulation-based models

PJ Goncalves*, JM Lückmann*, M. Deistler* et al (2020) eLife
A Tejero-Cantero et al (2020) Journal of Open Source Software
github.com/mackelab/delfi; <https://www.mackelab.org/sbi/>

Discussion

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- Simulation-based inference allowed us to fit non-linear models, with lots of parameters and complex relationships between parameters and outputs

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- Sensitivity analysis

PJ Goncalves*, JM Lückmann*, M. Deistler* et al (2020) eLife
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Discussion

- **Flexibility:** inference approach does not depend on specifics of mechanistic model, works with simulation-based models
- Simulation-based inference allowed us to fit non-linear models, with lots of parameters and complex relationships between parameters and outputs
- Sensitivity analysis
- Lots of challenges: scaling number of parameters, model misspecification...

PJ Goncalves*, JM Lückmann*, M. Deistler* et al (2020) eLife
A Tejero-Cantero et al (2020) Journal of Open Source Software
github.com/mackelab/delfi; <https://www.mackelab.org/sbi/>

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Janne Lappalainen

Jan-Matthis Lueckmann

Kaan Oecal (former)

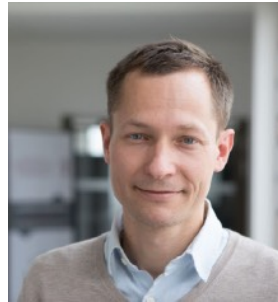
Poornima Ramesh

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Auguste Schulz

Artur Speiser

Julius Vetter



Jakob
Macke



Jan-Matthis
Lueckmann



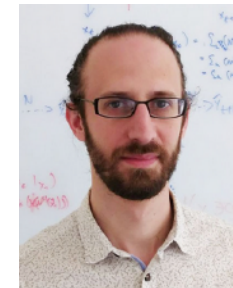
Michael
Deistler



Giacomo
Bassetto



Marcel
Nonnenmacher



David
Greenberg

