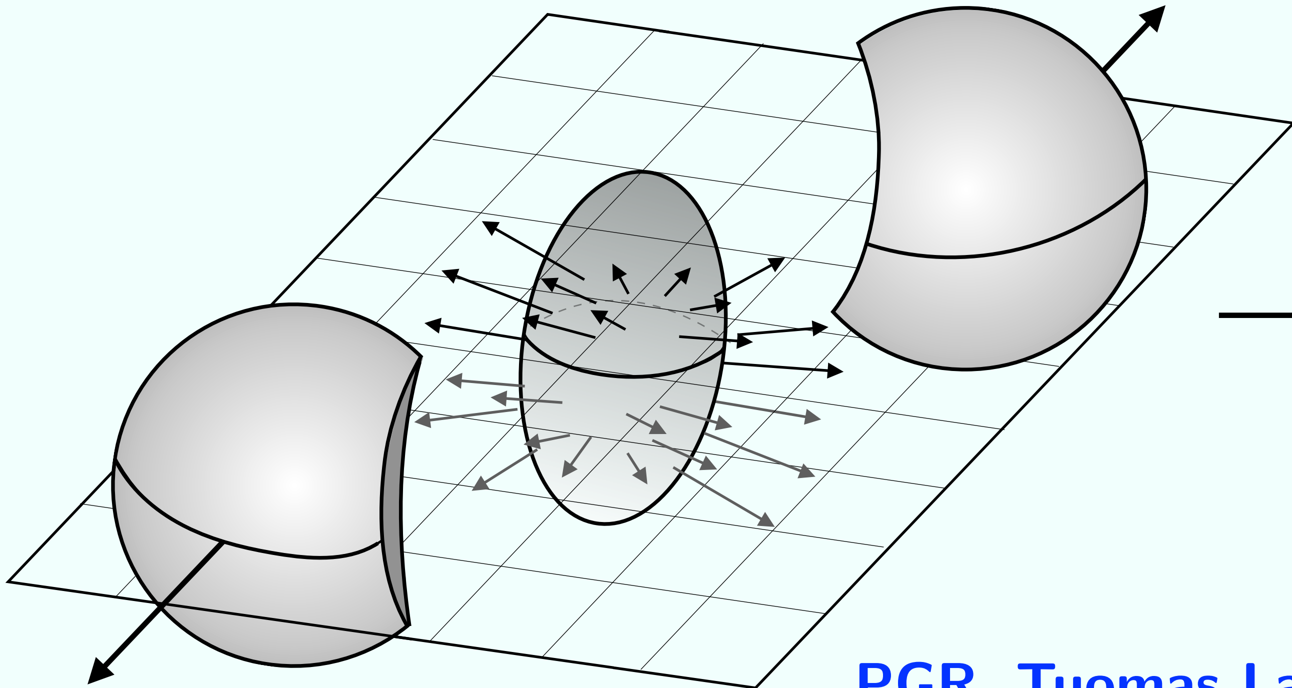
Initial Stage Fluctuations in Heavy Ion Collisions



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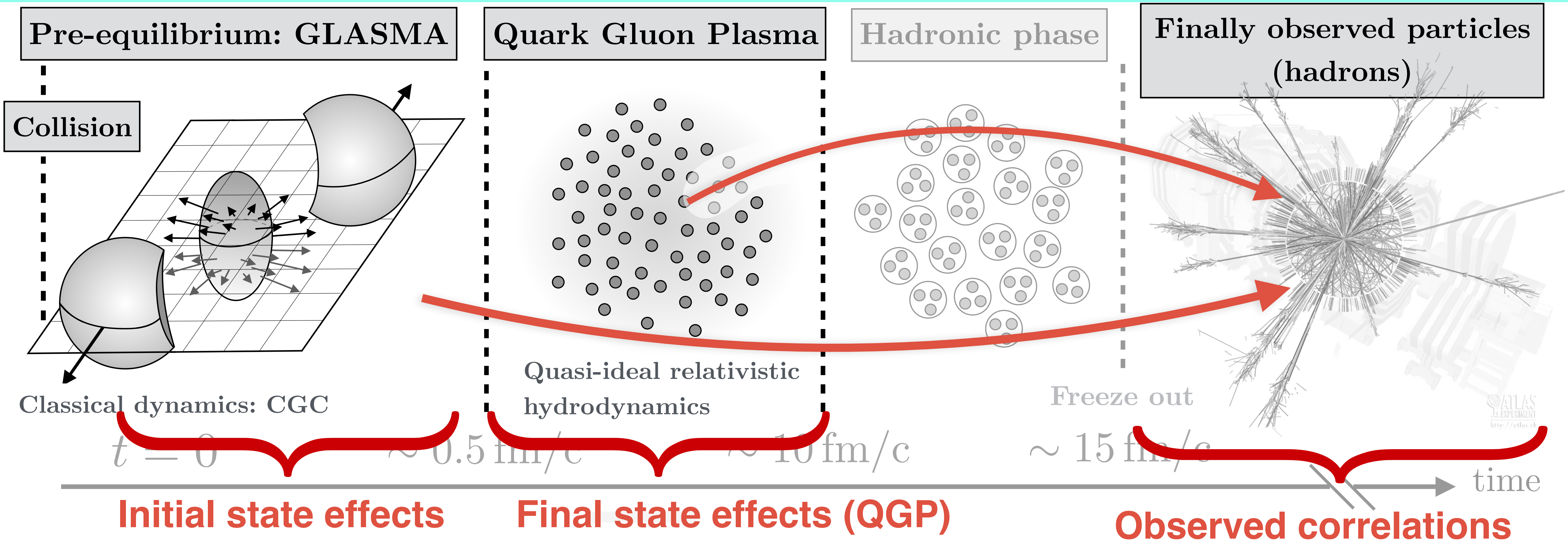
PGR, Tuomas Lappi (PRD 104, 014011 (2021) [arXiv:2102.09993])
Javier L. Albacete, PGR, Cyrille Marquet (JHEP (2019) 2019: 73 [arXiv:1808.00795])
PGR (10.1007/JHEP08(2019)026 [arXiv:1903.11602])
F.Gelis, G.Giacalone, C.Marquet, J-Y Ollitrault, PGR [arXiv:1907.10948]
Giuliano Giacalone, PGR, Matthew Luzum, Cyrille Marquet, Jean-Yves Ollitrault (PRC 100, 024905 (2019) [arXiv:1902.07168])

LABORATÓRIO DE INSTRUMENTAÇÃO
E FÍSICA EXPERIMENTAL DE PARTÍCULAS

PART I: Introduction

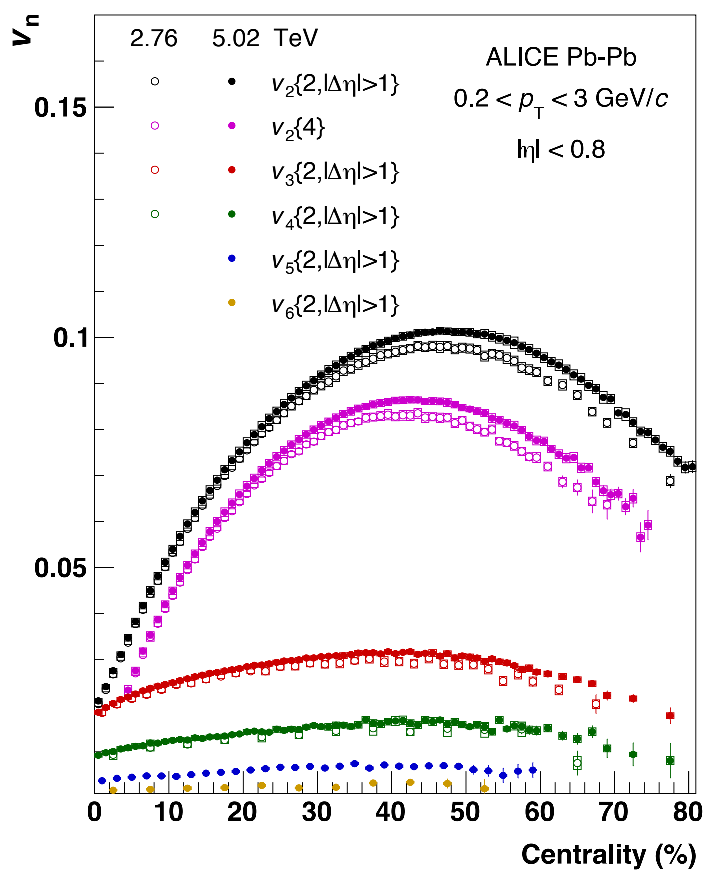
- **Standard Picture of Heavy Ion Collisions**
- **Collective Flow, Azimuthal Anisotropy**
- **Modeling the Early Stages of Heavy Ion Collisions**
- **High-energy QCD: The Color Glass Condensate**

Standard Picture of Heavy Ion Collisions



- **Initial state**: colliding ions break into a highly dense, out-of-equilibrium state known as **GLASMA**.
- The Glasma thermalizes into a **QUARK GLUON PLASMA**, an extremely hot, fluid-like state that exhibits deconfinement.
- **Hadronization**: quarks and gluons rearrange into a hadronic phase.
- **Final state**: the hadrons, along with other particles, fly off through the detectors.
- **QGP** can be studied through the **non-trivial correlations** between the measured particles
- **BUT: we need to have initial stage under control**

Collective Flow, Azimuthal Anisotropy

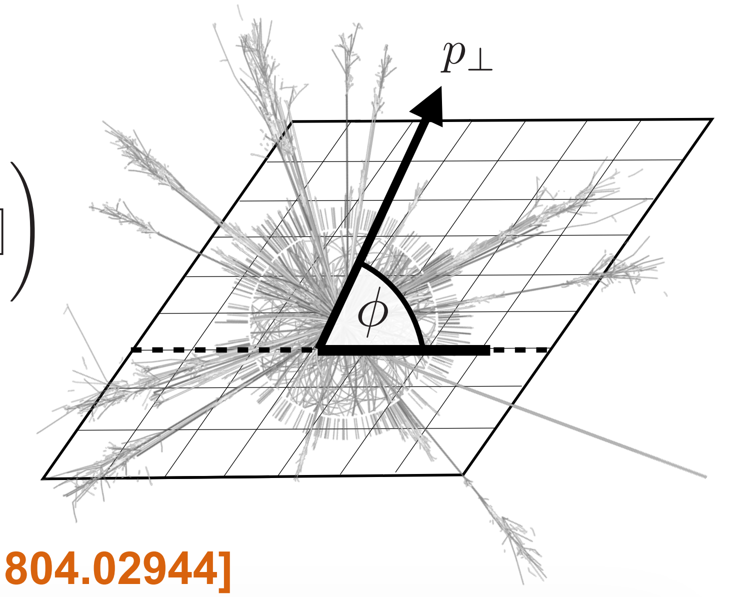


- Associated observable: **azimuthal anisotropy**, characterized by the Fourier coefficients of:

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_\perp dp_\perp dy}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos\left[n(\phi - \Psi_{RP})\right]\right)$$

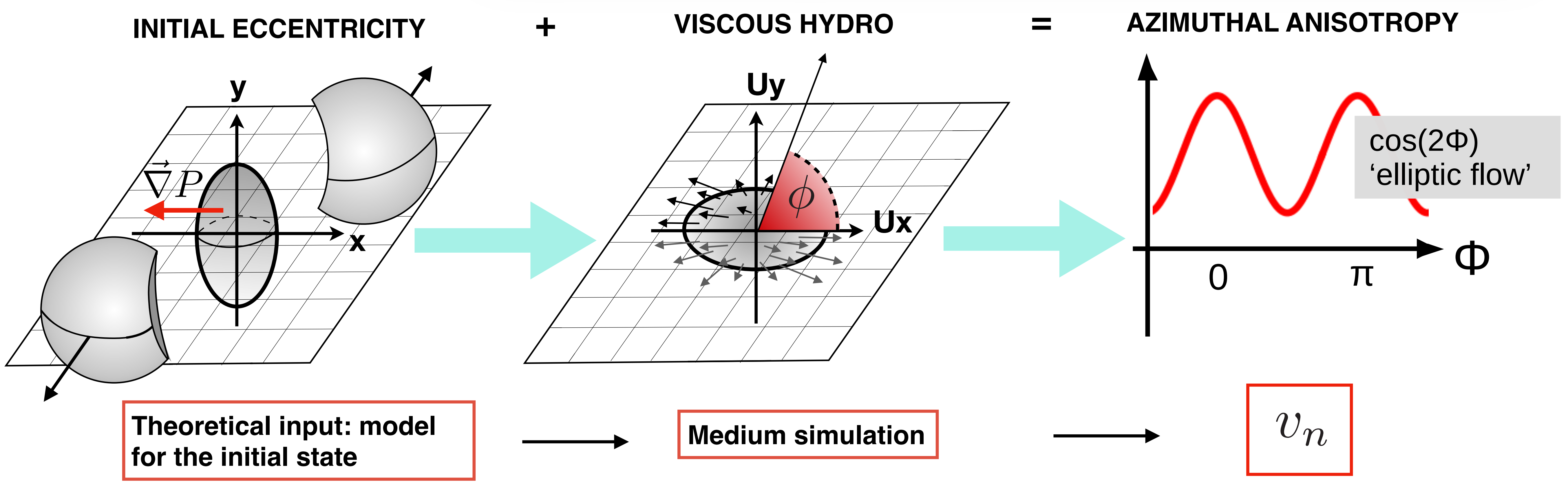
- **Harmonic flow coefficients**:

$$v_n = \langle \cos\left[n(\phi - \Psi_{RP})\right]\rangle$$



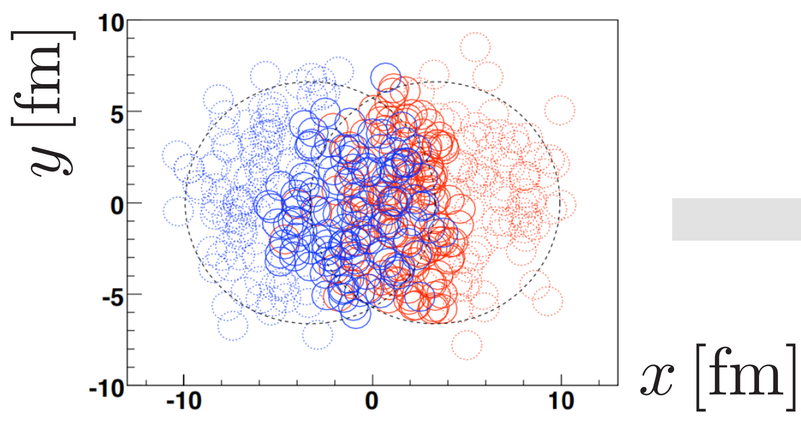
← ALICE Collaboration, JHEP 1807 (2018) 103 [1804.02944]

Collective flow has a natural explanation in a **fluid paradigm**

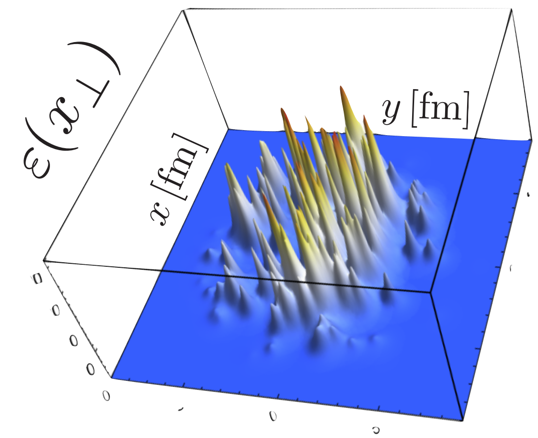


Initial stage modelization

- Traditional modelization of the initial stage: **Glauber Montecarlo Ansatz**



Random sampling of nucleon positions



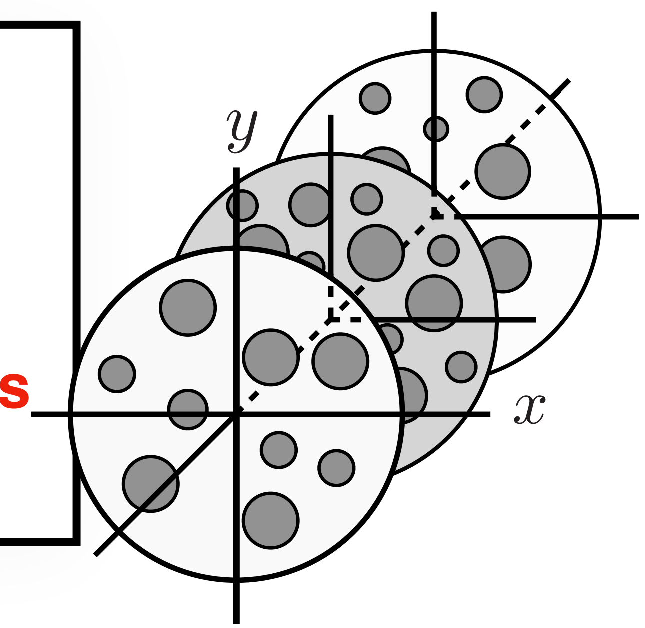
Mapping of nucleons/partons into energy density profile.

$$\epsilon_n = \frac{\int_{\mathbf{s}} \mathbf{s}^n \varepsilon(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}|^n \varepsilon(\mathbf{s})}$$

Initial state deformation characterized by **eccentricities**

- Sources of **fluctuations**:
  - → Random positions of nucleons
  - → Subnucleonic structure (e.g. Hot Spots)
  - → **Quantum fluctuations of the wave functions (i.e. primordial fluctuations)**

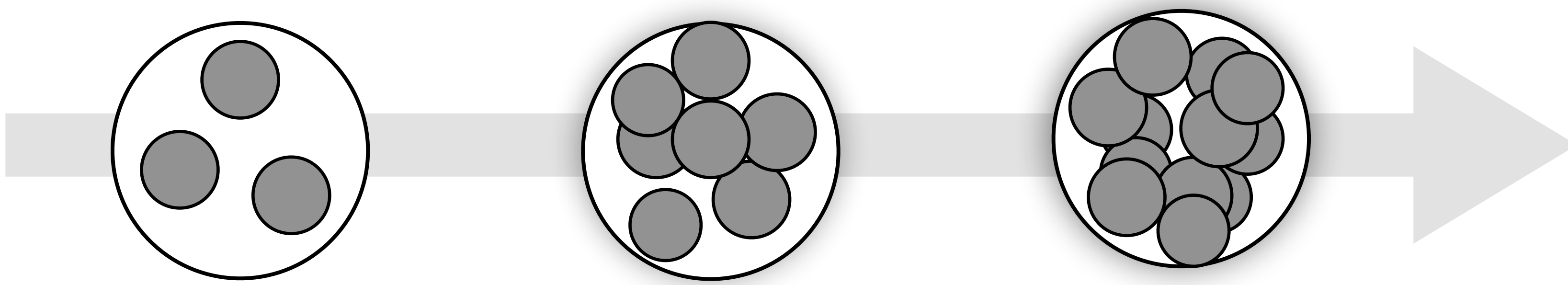
OUR WORK:



- We need **theoretical input** to constrain the contributions of each source

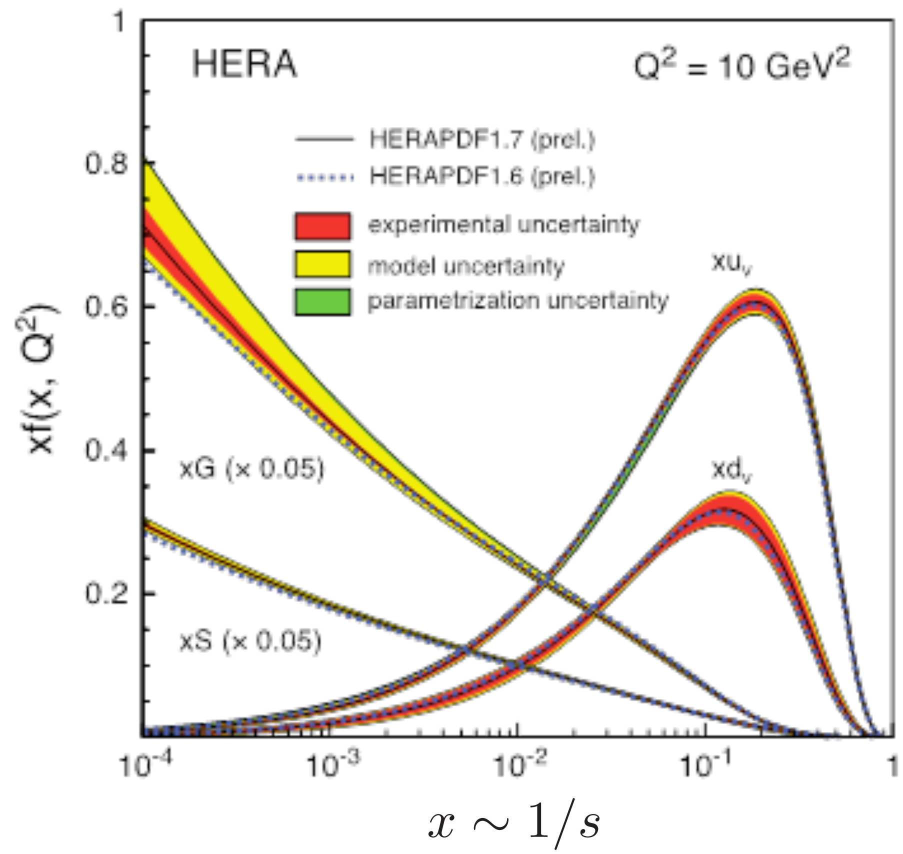
High-energy QCD: Saturation

- **High-energy limit** (or equivalently, low-x limit) governed by **large densities of soft gluons**.

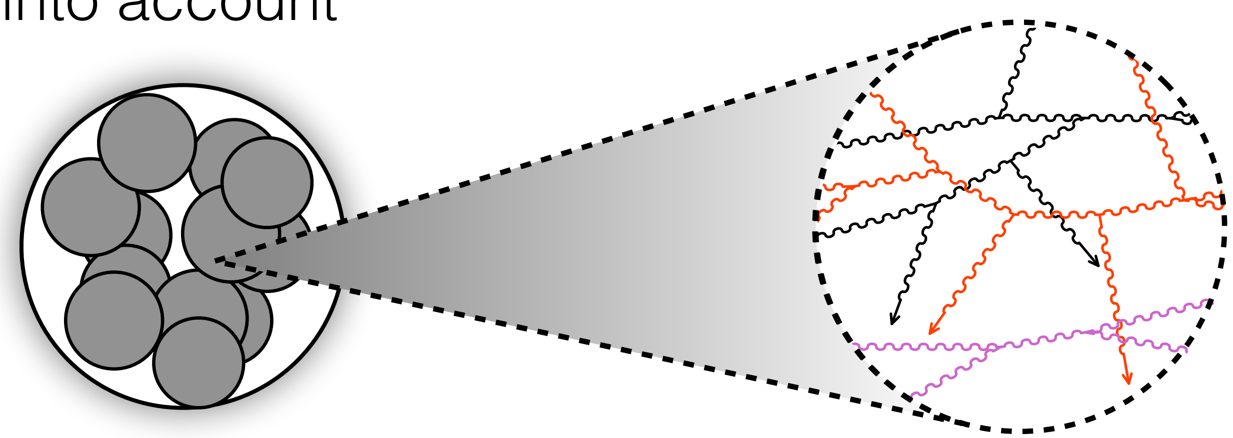


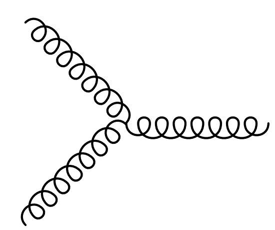
Moderate energy:
Dilute hadrons

High energy:
Gluon-dense hadrons



- Area density becomes large: **gluon recombination** must be taken into account





$$g\,g \to g$$

Non-linear effects as relevant as radiation processes

↓

Wave function of hadrons becomes SATURATED

High-energy QCD: Saturation

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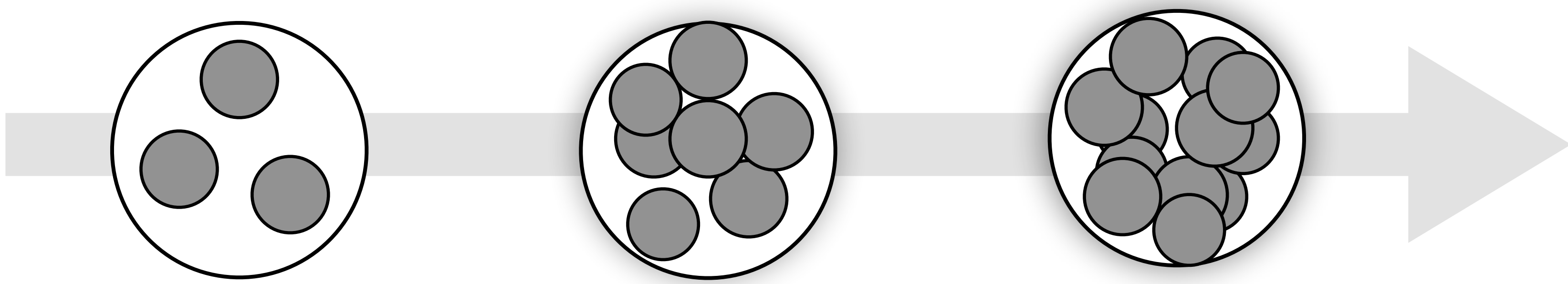
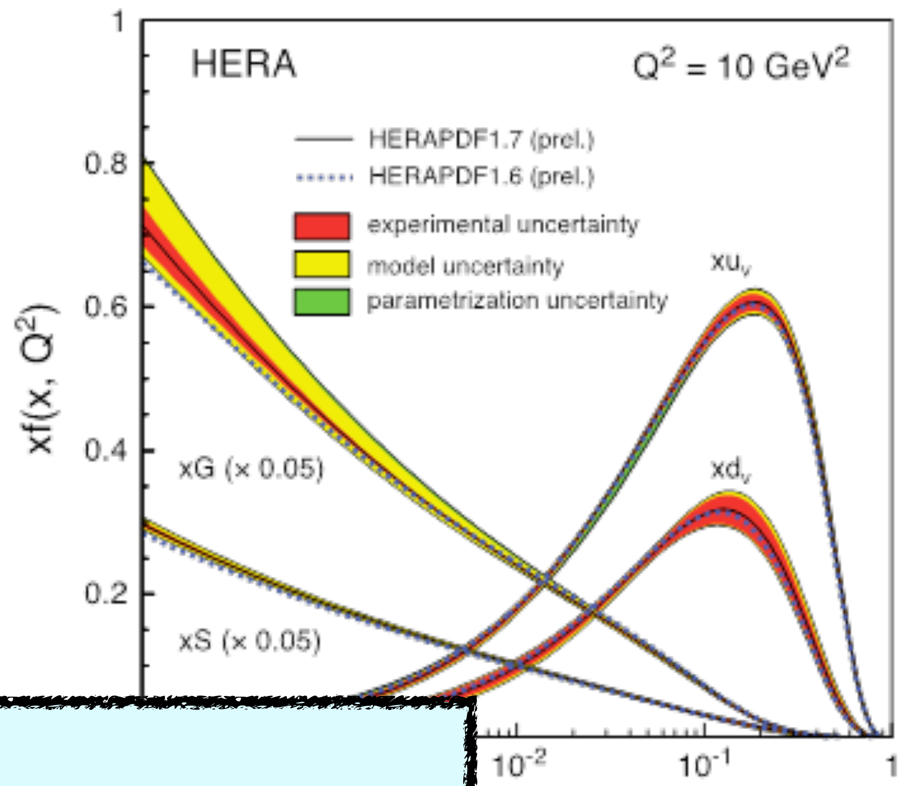


Moderate energy: Dilute hadrons

High energy: Gluon-dense hadrons



$\sim 1/s$

- Area density
must be take

- This effect is enhanced in nuclei due to **larger ab-initio gluon densities:**

$$\frac{xG_A(x,Q^2)}{\pi R_A^2} \sim A^{1/3}\frac{xG_N(x,Q^2)}{\pi R_N^2}$$

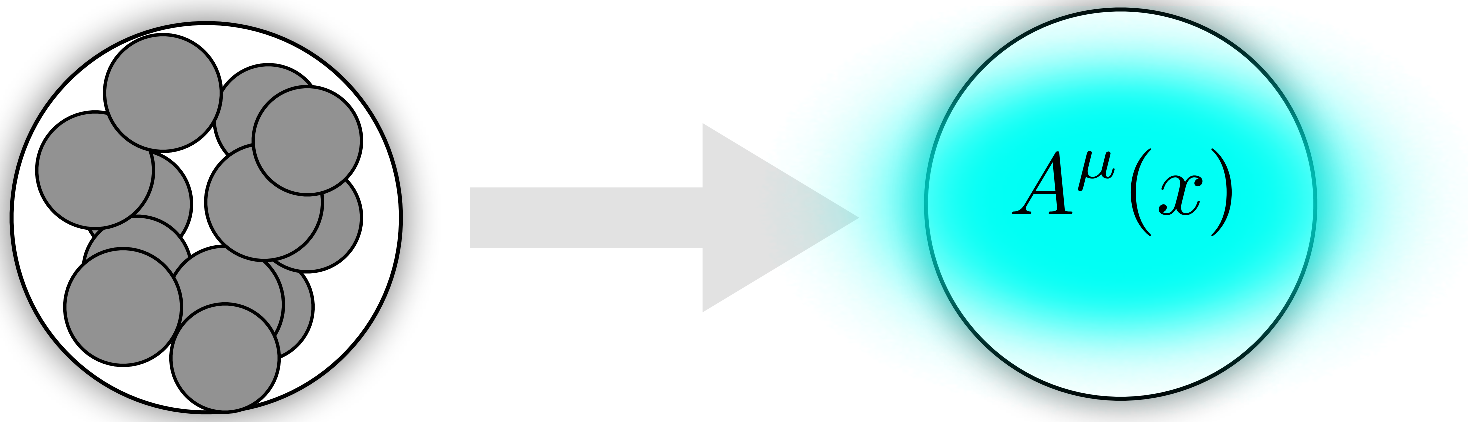
Non-linear effects as relevant as radiation processes

↓

Wave function of hadrons becomes SATURATED

High-energy QCD: The Color Glass Condensate

- In the **Color Glass Condensate** we replace the gluons with a **classical field** generated by the valence quarks



- Dynamics of the field described by **Yang-Mills** classical equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho^a(x)t^a$$

- Calculation of observables: **average** over background classical fields

$$\langle \mathcal{O}[\rho] \rangle = \int [d\rho]\mathcal{O}[\rho]W[\rho] \approx \int [d\rho]\mathcal{O}[\rho]e^{-\frac{\rho^2}{\mu^2}}$$

- Basic building block: MV model **2-point correlator**

$$\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)\delta^{ab}\delta(x^- - y^-)\delta^{(2)}(x_\perp - y_\perp)$$

PART II: CGC calculations

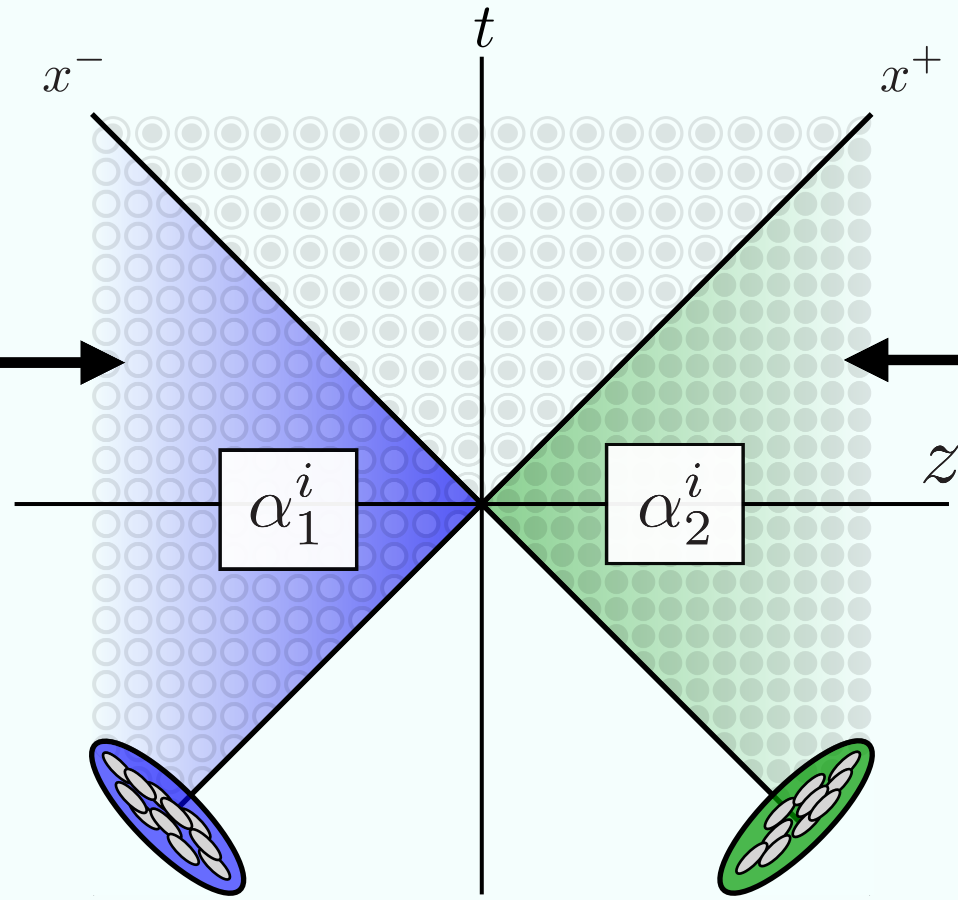
- **Computation of Glasma fields**
- **Correlators at** $\tau = 0^+$
- **Glasma fields at** $\tau > 0$**: linearized YM equations**
- **Correlators at** $\tau > 0$

Glasma fields at $\tau = 0^+$

- Yang-Mills equations **(with one source)**:

$$[D_\mu, F^{\mu\nu}] = J_1^\nu$$

$$\boxed{\alpha_1^i(x_\perp) = -\frac{1}{ig} U_1(x_\perp)\partial^i U_1^\dagger(x_\perp)}$$



$$[D_\mu, F^{\mu\nu}] = J_2^\nu$$

$$\boxed{\alpha_2^i(x_\perp) = -\frac{1}{ig} U_2(x_\perp)\partial^i U_2^\dagger(x_\perp)}$$

with: $U_{1,2}(x_\perp) = P^\pm \exp\left\{-ig\int_{-\infty}^{\infty} dz^- \frac{1}{\nabla^2}\rho_{1,2}(z^-, x_\perp)\right\}$

Previously computed in: Kovner, McLerran and Weigert, Phys. Rev. D 52 (1995)

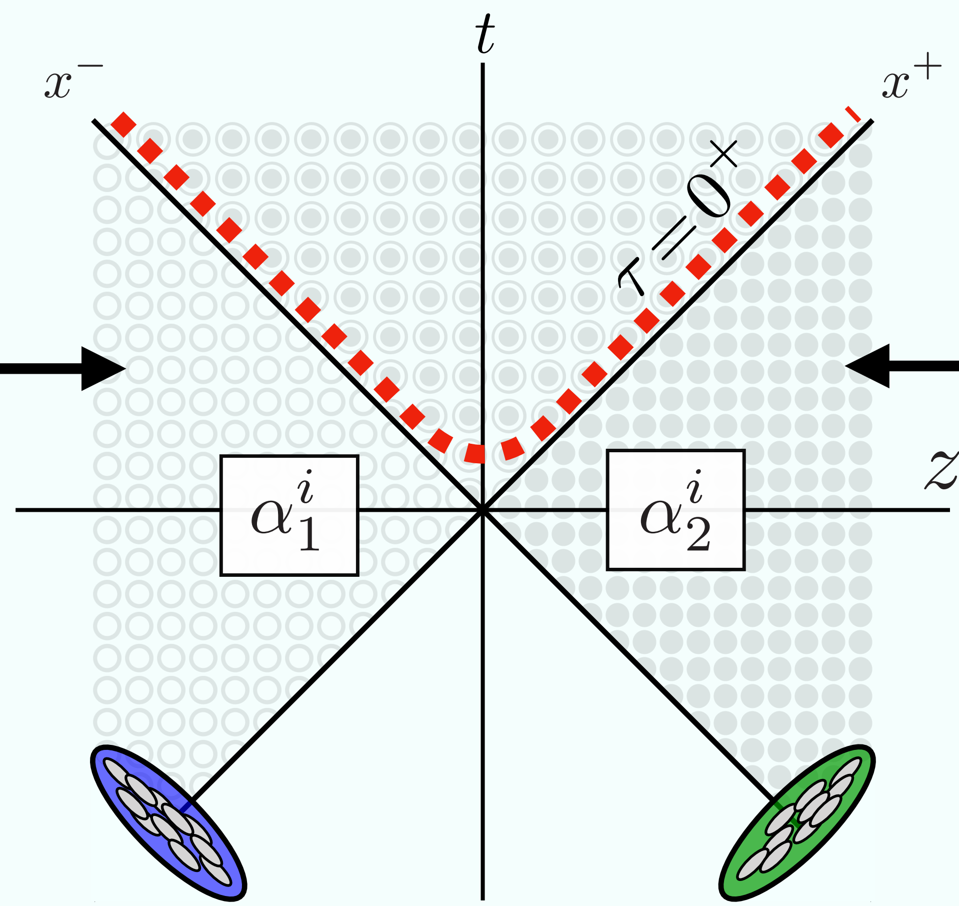
Glasma fields at $\tau = 0^+$

- Yang-Mills equations:

$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu$$

$$[D_\mu, F^{\mu\nu}] = J_1^\nu$$

$$[D_\mu, F^{\mu\nu}] = J_2^\nu$$



$$\boxed{\alpha_1^i(x_\perp) = -\frac{1}{ig}U_1(x_\perp)\partial^i U_1^\dagger(x_\perp)}$$

$$\boxed{\alpha_2^i(x_\perp) = -\frac{1}{ig}U_2(x_\perp)\partial^i U_2^\dagger(x_\perp)}$$

- Analytical solution at $\tau = 0^+$:

$$E^\eta(\tau = 0^+, x_\perp) = -ig\delta^{ij}[\alpha_1^i(x_\perp), \alpha_2^j(x_\perp)]$$

$$B^\eta(\tau = 0^+, x_\perp) = -ig\varepsilon^{ij}[\alpha_1^i(x_\perp), \alpha_2^j(x_\perp)]$$

- Energy density of the Glasma: $\varepsilon(\tau = 0^+, x_\perp) \equiv \varepsilon_0(x_\perp) = \mathrm{Tr}\{E^\eta E^\eta + B^\eta B^\eta\}$

We compute $\langle\varepsilon_0(x_\perp)\rangle$, $\langle\varepsilon_0(x_\perp)\varepsilon_0(y_\perp)\rangle$ in the CGC

Glasma correlators at $\tau = 0^+$

We compute $\langle \varepsilon_0(x_\perp) \rangle$, $\langle \varepsilon_0(x_\perp)\varepsilon_0(y_\perp) \rangle$ in the CGC

- For the 1-point correlator (i.e. the average energy density):

$$\langle \varepsilon_0(x_\perp) \rangle = \frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) f^{abm} f^{cdm} \left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp) \right\rangle \left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp) \right\rangle$$

The Weizsäcker-Williams gluon distribution

$$\boxed{\langle\alpha^{a,i}(x_\perp)\alpha^{b,j}(y_\perp)\rangle=\frac{\delta^{ab}}{2}\left(\delta^{ij}G(x_\perp,y_\perp)+\left(\delta^{ij}-2\frac{r^ir^j}{r^2}\right)h(x_\perp,y_\perp)\right)}$$

- Unpolarized gluon distribution: $G(x_\perp,y_\perp)=\frac{1}{2\pi}\int dkkJ_0(kr)\,\hat{G}\left(\frac{x_\perp+y_\perp}{2},k\right)$
- Linearly polarized gluon distribution: $h(x_\perp,y_\perp)=\frac{1}{2\pi}\int dkkJ_2(kr)\,\hat{h}\left(\frac{x_\perp+y_\perp}{2},k\right)$

In **Gaussian models** we can relate these objects to the **Dipole function**: $D(r)=\langle\mathrm{Tr}\left\{U(x_\perp)U(y_\perp)\right\}\rangle$

- $G(r)=\frac{1}{g^2N_c}\frac{1-D(r)}{\ln(D(r))}\left(\partial_r^2+\frac{1}{r}\partial_r\right)\ln(D(r))$
- $h(r)=\frac{1}{g^2N_c}\frac{1-D(r)}{\ln(D(r))}\left(\partial_r^2-\frac{1}{r}\partial_r\right)\ln(D(r))$

GBW model:

$$\boxed{D_{\mathrm{GBW}}(r)=\exp\left\{-\frac{N_c^2-1}{2N_c^2}\frac{Q_s^2r^2}{4}\right\}}$$

- $G_{\mathrm{GBW}}(r)=\frac{Q_s^2}{g^2N_c}\frac{1-\exp\left\{-\frac{Q_s^2r^2}{4}\right\}}{Q_s^2r^2/4}$
- $h_{\mathrm{GBW}}(r)=0$

MV model:

$$\boxed{D_{\mathrm{MV}}(r)=\exp\left\{\frac{N_c^2-1}{2N_c}\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}$$

- $G_{\mathrm{MV}}(r_\perp)=\frac{g^2\bar{\mu}^2}{4\pi N_c}\left(mrK_1(mr)-2K_0(mr)\right)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$
- $h_{\mathrm{MV}}(r_\perp)=-\frac{g^2\bar{\mu}^2}{4\pi N_c}mrK_1(mr)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$

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- $h_{\mathrm{GBW}}(r)=0$

MV model:

$$\boxed{D_{\mathrm{MV}}(r)=\exp\left\{\frac{N_c^2-1}{2N_c}\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}$$

- G … $r))\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$

$$\boxed{\lim_{k\to\infty}\hat{G}_{\mathrm{MV}}(k_\perp)\sim 1/k^2}$$

- $h_{\mathrm{MV}}(r_\perp)=-\frac{g^2\bar{\mu}^2}{4\pi N_c}mrK_1(mr)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$

Glasma correlators at $\tau=0^+$

We compute $\langle\varepsilon_0(x_\perp)\rangle$, $\langle\varepsilon_0(x_\perp)\varepsilon_0(y_\perp)\rangle$ in the CGC

- For the 1-point correlator (i.e. the average energy density):

$$\langle\varepsilon_0(x_\perp)\rangle = \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})f^{abm}f^{cdm}\left\langle\alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle\left\langle\alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle$$

Substituting the gluon 2-point functions:

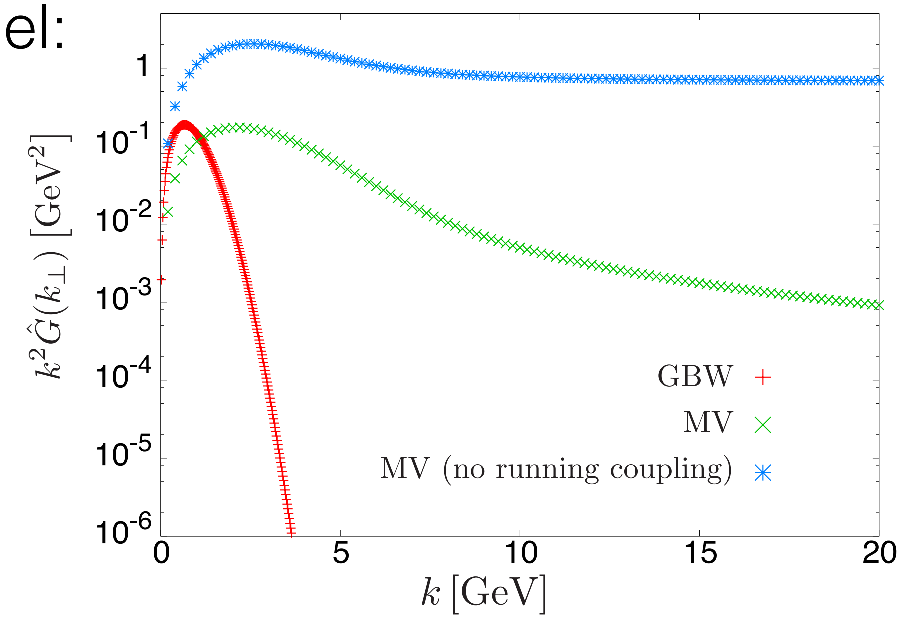
GBW:

$$=\frac{g^2}{2}N_c(N_c^2-1)G_1(0)G_2(0)\boxed{=\frac{C_F}{g^2}Q_{s1}^2Q_{s2}^2}$$

BUT: $G_{\mathrm{MV}}(0)$ is a logarithmically divergent quantity

The UV divergence of the energy density

- Divergence at $r \to 0$ related to perturbative tail of MV model:



- How do we treat this divergence?

We apply a **running coupling** prescription:

$$\boxed{g^2(r^2) = \frac{g^2(\bar{\mu}^2)}{\ln\left(\frac{4e^{-2\gamma_e - 1}}{m^2 r^2} + e\right)}}$$

$$G_{\mathrm{MV}}(r_\perp) = \frac{g^2\bar{\mu}^2}{4\pi N_c}\left(mrK_1(mr) - 2K_0(mr)\right)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2 N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$$

$$h_{\mathrm{MV}}(r_\perp) = -\frac{g^2\bar{\mu}^2}{4\pi N_c} mrK_1(mr)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2 N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$$

Yielding the following results:

$$\lim_{r\to 0} G_{\mathrm{MV}}(r_\perp) = \frac{g^2(\bar{\mu}^2)}{4\pi}\frac{\bar{\mu}^2}{N_c}$$

$$\lim_{r\to 0} h_{\mathrm{MV}}(r_\perp) = 0$$

- If we define $Q_s^2 = \alpha_s(\bar{\mu}^2)\bar{\mu}^2 N_c$:

$$\lim_{r\to 0} G_{\mathrm{MV}}(r_\perp) = \lim_{r\to 0} G_{\mathrm{GBW}}(r_\perp) = \frac{Q_s^2}{g^2 N_c}$$

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Substituting the gluon 2-point functions:

GBW:

$$= \frac{g^2}{2}N_c(N_c^2-1)G_1(0)G_2(0)\boxed{= \frac{C_F}{g^2}Q_{s1}^2Q_{s2}^2}$$

- For the 2-point correlator (i.e. the variance):

$$\langle\varepsilon_0(x_\perp)\varepsilon_0(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm}$$

$$\times\left\langle\alpha_x^{ia}\alpha_x^{kc}\alpha_y^{i'a'}\alpha_y^{k'c'}\right\rangle_1\left\langle\alpha_x^{jb}\alpha_x^{ld}\alpha_y^{j'b'}\alpha_y^{l'd'}\right\rangle_2$$

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Substituting the gluon 2-point functions:

GBW:

$$= \frac{g^2}{2}N_c(N_c^2-1)G_1(0)G_2(0) \boxed{= \frac{C_F}{g^2}Q_{s1}^2Q_{s2}^2}$$

Building block of the calculation:

$$\left\langle\alpha^{ia}(x_\perp)\alpha^{jb}(y_\perp)\alpha^{kc}(u_\perp)\alpha^{ld}(v_\perp)\right\rangle$$

- For the 2-point corr

$$\langle\varepsilon_0(x_\perp)\varepsilon_0(y_\perp)\rangle = \frac{g^4}{4}\cdots f^{cdn}f^{a'b'm}f^{c'd'm}\cdots\left\langle\alpha^{ib}\alpha^{ld}\alpha^{j'b'}\alpha^{l'd'}\right\rangle_2$$

Glasma Graph approximation:

$$\langle\alpha_x^{i,a}\alpha_y^{j,b}\alpha_u^{k,c}\alpha_v^{l,d}\rangle = \langle\alpha_x^{i,a}\alpha_y^{j,b}\rangle\langle\alpha_u^{k,c}\alpha_v^{l,d}\rangle + \langle\alpha_x^{i,a}\alpha_u^{k,c}\rangle\langle\alpha_y^{j,b}\alpha_v^{l,d}\rangle + \langle\alpha_x^{i,a}\alpha_v^{l,d}\rangle\langle\alpha_y^{j,b}\alpha_u^{k,c}\rangle$$

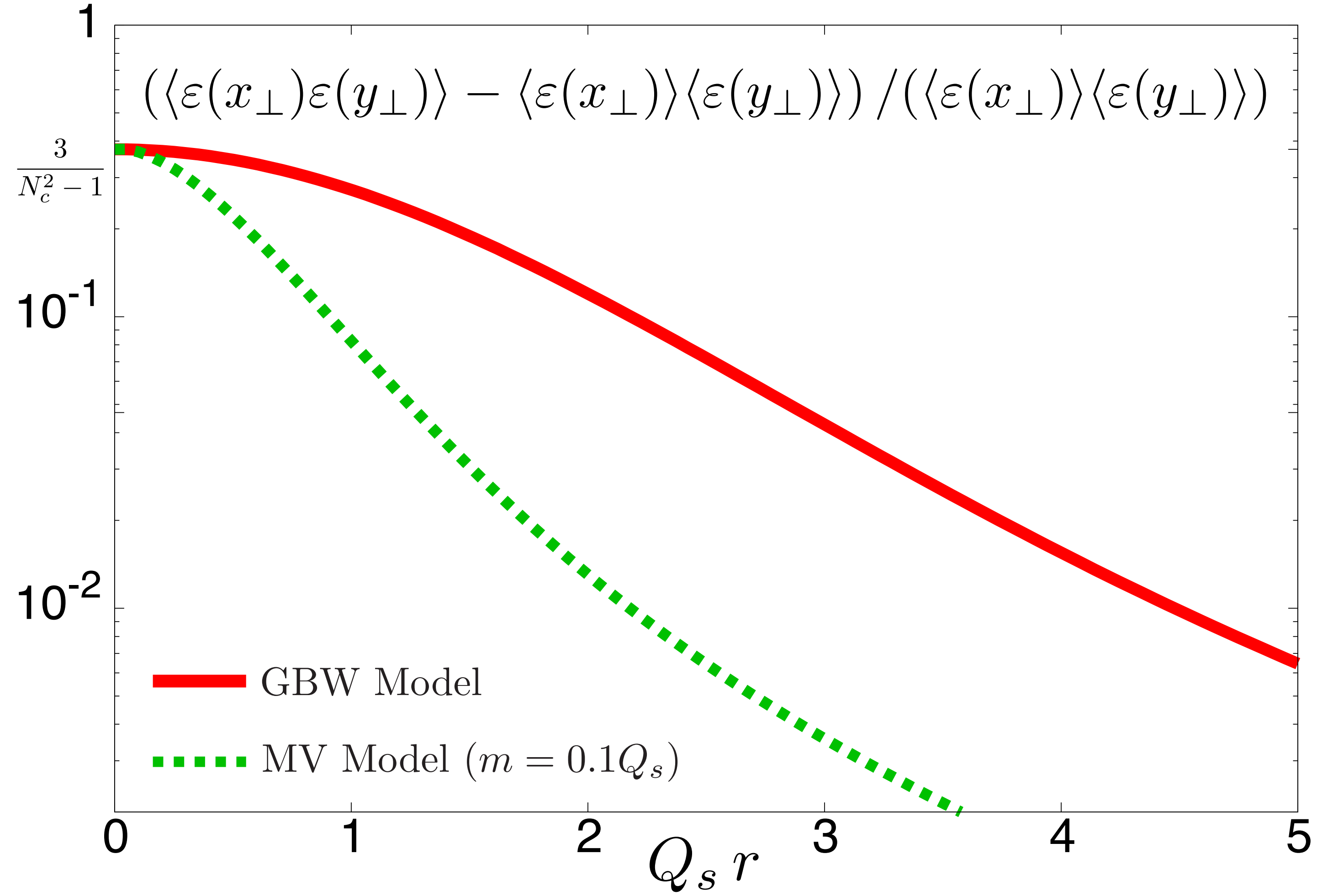
Glasma correlators at $\tau = 0^+$

- Compact expression in GBW model:

$$\frac{\langle \varepsilon_0(x_\perp)\varepsilon_0(y_\perp)\rangle - \langle \varepsilon_0(x_\perp)\rangle\langle \varepsilon_0(y_\perp)\rangle}{\langle \varepsilon_0(x_\perp)\rangle\langle \varepsilon_0(y_\perp)\rangle} = \frac{3}{N_c^2-1}\left[\frac{1}{3}\left(\frac{1-e^{-Q_s^2r^2/4}}{Q_s^2r^2/4}\right)^4 + \frac{2}{3}\left(\frac{1-e^{-Q_s^2r^2/4}}{Q_s^2r^2/4}\right)^2\right]$$

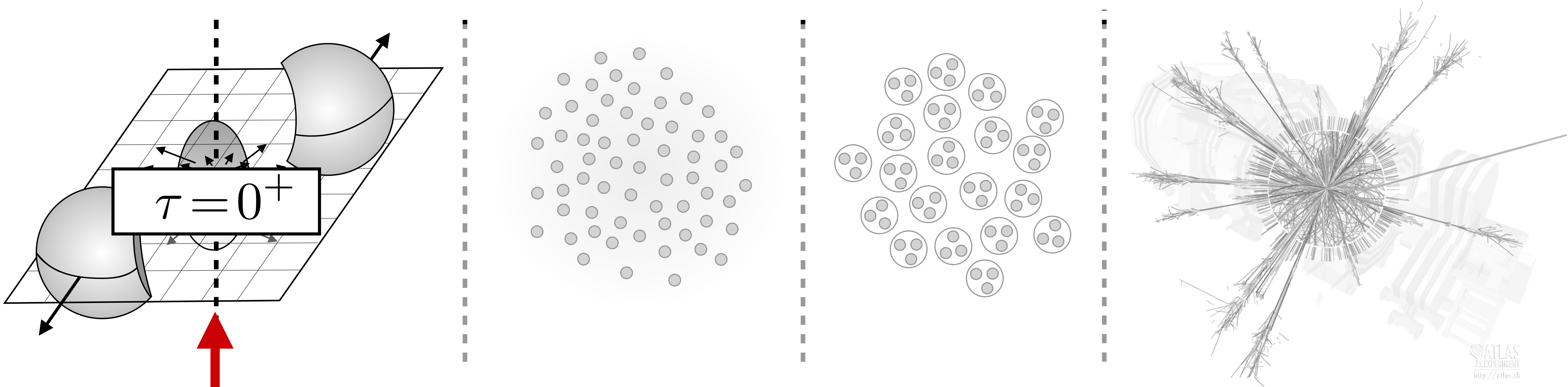
Previously obtained in: **T.Lappi and S.Schlichting, Phys.Rev.D 97, 034034 (2018)**

- Comparison between GBW and MV model:



Heavy Ion Collisions

- Let’s look at the bigger picture of HICs:



- **Calculations at $\tau=0^+$:** T.Lappi and S.Schlichting, Phys.Rev.D 97, 034034 (2018)
  J.L.Albacete, Cyrille Marquet, PGR, JHEP 1901 (2019) 073 [1808.00795]
  PGR, JHEP 1908 (2019) 026 [1903.11602]

Glasma correlators at $\tau=0^+$: beyond Glasma Graph

- For the 2-point correlator:

$$\langle\epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm} \times \langle\alpha_{1\,x}^{i\,a}\alpha_{1\,x}^{k\,c}\alpha_{1\,y}^{i'a'}\alpha_{1\,y}^{k'c'}\rangle\langle\alpha_{2\,x}^{j\,b}\alpha_{2\,x}^{l\,d}\alpha_{2\,y}^{j'b'}\alpha_{2\,y}^{l'd'}\rangle$$

- The **building block**:

$$\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle=\int_{-\infty}^{\infty}dz^-dw^-dz^{-\prime}dw^{-\prime}\Big\langle\frac{\partial^i\tilde\rho^e(z^-,x_\perp)}{\nabla^2}U^{ea}(z^-,x_\perp) \frac{\partial^k\tilde\rho^f(w^-,x_\perp)}{\nabla^2}U^{fc}(w^-,x_\perp)\frac{\partial^{i'}\tilde\rho^{e'}(z^{-\prime},y_\perp)}{\nabla^2}U^{e'a'}(z^{-\prime},y_\perp)\frac{\partial^{k'}\tilde\rho^{f'}(w^{-\prime},y_\perp)}{\nabla^2}U^{f'c'}(w^{-\prime},y_\perp)\Big\rangle.$$

$$=\int_{-\infty}^{\infty}dz^-dw^-dz^{-\prime}dw^{-\prime}\left(\#3\left\langle\rho^4\right\rangle\left\langle U^4\right\rangle+\#4\left\langle\rho^2\right\rangle\left\langle\rho^2U^4\right\rangle\right)$$

$$\boxed{\alpha_1^{i,b}(x_\perp)=\int_{-\infty}^{\infty}dz^-\frac{\partial^i\tilde\rho_1^a(z^-,z_\perp)}{\nabla^2}U_1^{ab}(z^-,x_\perp)}$$

Glasma correlators at $\tau=0^+$: beyond Glasma Graph

- For the 2-point correlator:

$$\langle\epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm} \times\langle\alpha_{1x}^{ia}\alpha_{1x}^{kc}\alpha_{1y}^{i'a'}\alpha_{1y}^{k'c'}\rangle\langle\alpha_{2x}^{jb}\alpha_{2x}^{ld}\alpha_{2y}^{j'b'}\alpha_{2y}^{l'd'}\rangle$$

- The **building block**:

$$\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle=\int_{-\infty}^{\infty}dz^-dw^-dz^{-\prime}dw^{-\prime}\left\langle\frac{\partial^i\tilde{\rho}^e(z^-,x_\perp)}{\nabla^2}U^{ea}(z^-,x_\perp)\frac{\partial^k\tilde{\rho}^f(w^-,x_\perp)}{\nabla^2}U^{fc}(w^-,x_\perp)\frac{\partial^{i'}\tilde{\rho}^{e'}(z^{-\prime},y_\perp)}{\nabla^2}U^{e'a'}(z^{-\prime},y_\perp)\frac{\partial^{k'}\tilde{\rho}^{f'}(w^{-\prime},y_\perp)}{\nabla^2}U^{f'c'}(w^{-\prime},y_\perp)\right\rangle.$$

$$=\int_{-\infty}^{\infty}dz^-dw^-dz^{-\prime}dw^{-\prime}\left(\#3\left\langle\rho^4\right\rangle\left\langle U^4\right\rangle+\#4\left\langle\rho^2\right\rangle\left\langle\rho^2U^4\right\rangle\right)$$

- The calculation requires computing projections of the **Wilson line quadrupole**:

$$\left\langle U^{ab}(z^-,x_\perp)U^{cd}(z^-,y_\perp)U^{ef}(z^-,x'_\perp)U^{gh}(z^-,y'_\perp)\right\rangle$$

- The **contraction of the color indices** demands a **computational treatment** (via FeynCalc).

Glasma correlators at $\tau=0^+$: beyond Glasma Graph

$$\mathrm{Cov}[\epsilon](x_\perp,y_\perp)=\langle\epsilon(x_\perp)\epsilon(y_\perp)\rangle-\langle\epsilon(x_\perp)\rangle\langle\epsilon(y_\perp)\rangle$$

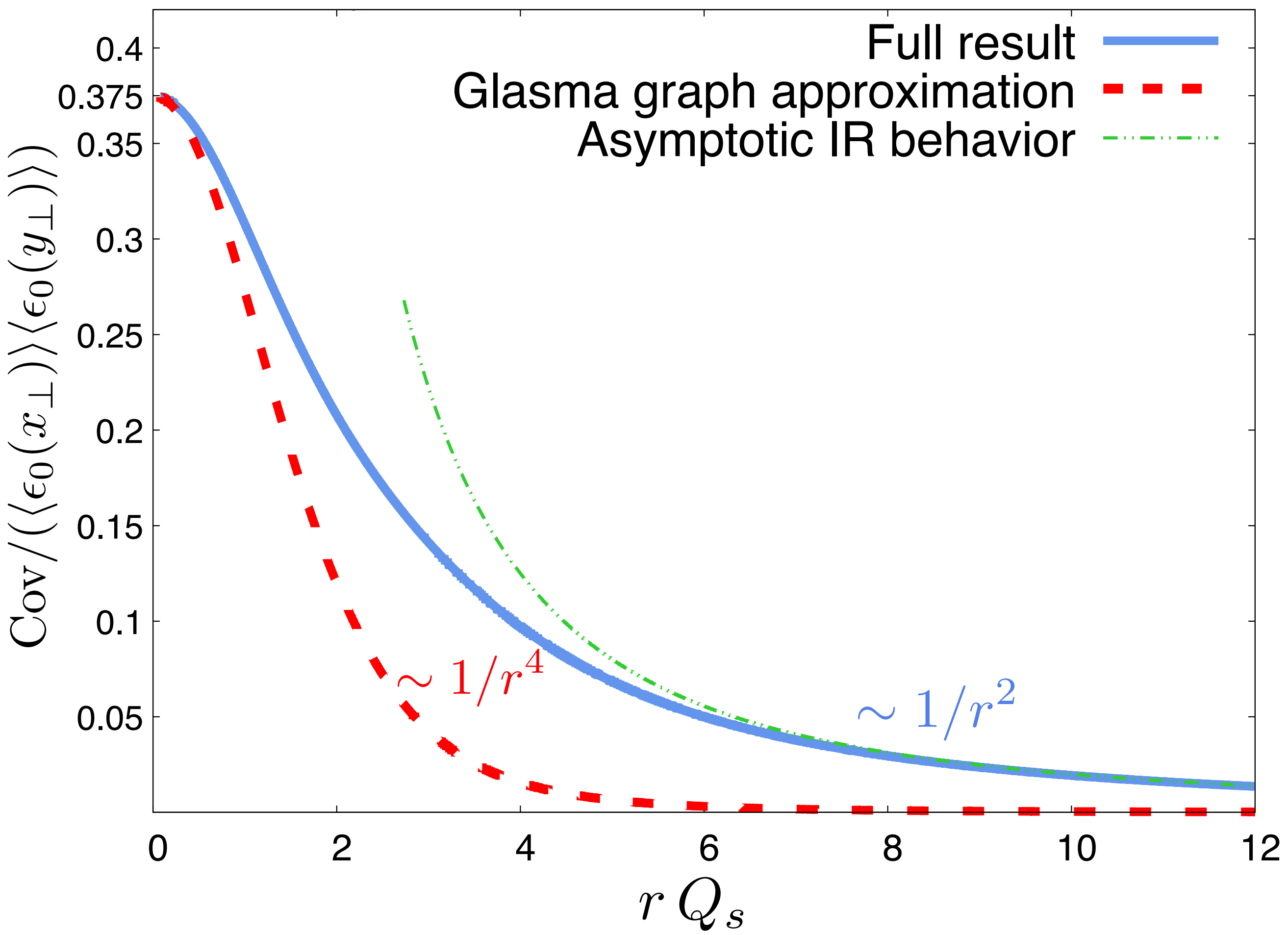
$$\begin{aligned}\mathrm{Cov}[\epsilon](\tau=0^+;x_\perp,y_\perp)=&\frac{\partial_x^i\Gamma\partial_y^i\Gamma(N_c^2-1)A(4A^2-B^2)}{16N_c^2\Gamma^5g^4}(p_1q_2+p_2q_1)\\&+\frac{(N_c^2-1)(16A^4+B^4)}{2N_c^2\Gamma^4g^4}p_1p_2+\frac{(\partial_x^i\Gamma\partial_y^i\Gamma)^2(N_c^2-1)A^2}{64N_c^2\Gamma^6g^4}q_1q_2\\&+\frac{(N_c^2-1)(4A^2+B^2)}{2N_c^2\Gamma^2g^4}\left(\left[\bar{Q}_{s1}^4(Q_{s2}^2r^2-4+4e^{-\frac{Q_{s2}^2r^2}{4}})\right]+[1\leftrightarrow2]\right)\\&+\frac{(4A^2+B^2)^2}{g^4\Gamma^4N_c^2}\Bigg(\Bigg[\frac{N_c^6+2N_c^4-19N_c^2+8}{(N_c^2-1)^2}-4\frac{N_c^6-3N_c^4-26N_c^2+16}{(N_c^2-1)(N_c^2-4)}e^{-\frac{Q_{s1}^2r^2}{4}}\\&+\frac{(N_c-1)(N_c+3)N_c^3}{(N_c+1)^2(N_c+2)^2}\left(\frac{N_c}{2}e^{-\frac{(N_c+1)r^2Q_{s2}^2}{2N_c}}+(N_c+2)-2(N_c+1)e^{-\frac{Q_{s2}^2r^2}{4}}\right)e^{-\frac{(N_c+1)r^2Q_{s1}^2}{2N_c}}\\&+\frac{(N_c+1)(N_c-3)N_c^3}{(N_c-1)^2(N_c-2)^2}\left(\frac{N_c}{2}e^{-\frac{(N_c-1)r^2Q_{s2}^2}{2N_c}}+(N_c-2)-2(N_c-1)e^{-\frac{Q_{s2}^2r^2}{4}}\right)e^{-\frac{(N_c-1)r^2Q_{s1}^2}{2N_c}}\\&+\frac{r^4}{2}Q_{s1}^2Q_{s2}^2-4r^2Q_{s1}^2\left(1-e^{-\frac{Q_{s2}^2r^2}{4}}\right)+4\frac{(N_c^2-8)(N_c^2-1)(N_c^2+4)}{(N_c^2-4)^2}e^{-\frac{(Q_{s1}^2+Q_{s2}^2)r^2}{4}}\Bigg]+[1\leftrightarrow2]\Bigg)\end{aligned}$$

With: $p_{1,2}\equiv e^{-\frac{Q_{s1,2}^2r^2}{4}}(Q_{s1,2}^2r^2+4)-4$, $q_{1,2}\equiv e^{-\frac{Q_{s1,2}^2r^2}{4}}\left(Q_{s1,2}^4r^4+8Q_{s1,2}^2r^2+32\right)-32$.

And: $\dfrac{r^2Q_s^2}{4}=g^2\dfrac{N_c}{2}\Gamma(r_\perp)\bar{\lambda}(b_\perp)$.

Glasma correlators at $\tau=0^+$: beyond Glasma Graph

- Agreement in the $r \to 0$ limit. **Strong discrepancies** in the $rQ_s \to \infty$ limit.



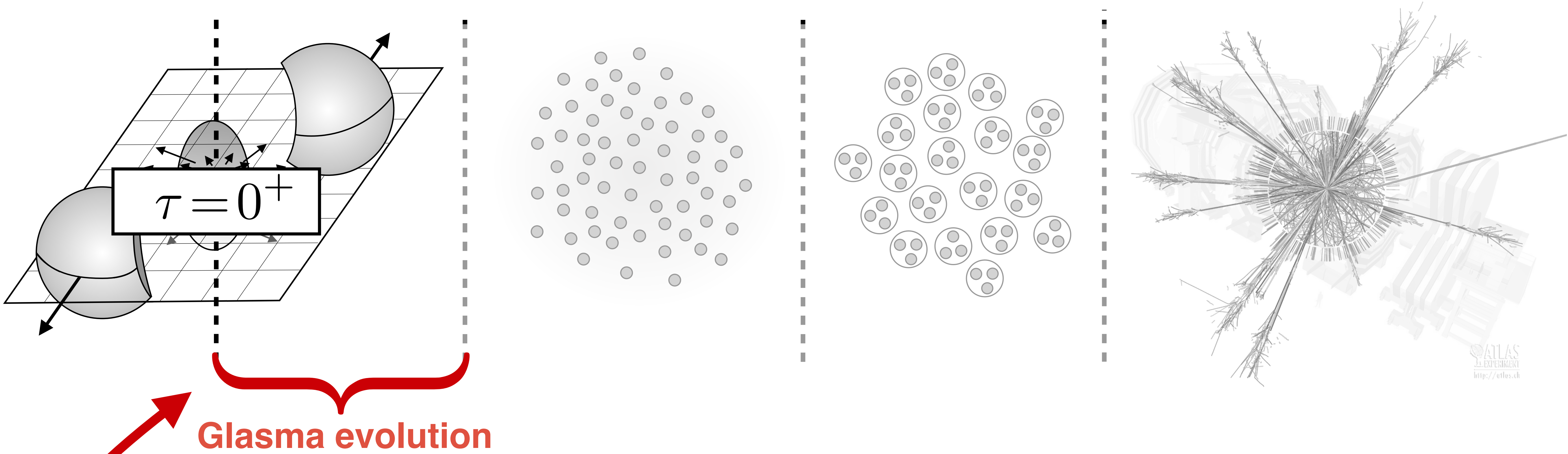
Spoiler alert:

- This slowly decaying behavior could potentially have an impact in **infrared-sensitive observables** built from this quantity.

J.L.Albacete, Cyrille Marquet, PGR, JHEP 1901 (2019) 073 [1808.00795]

Heavy Ion Collisions

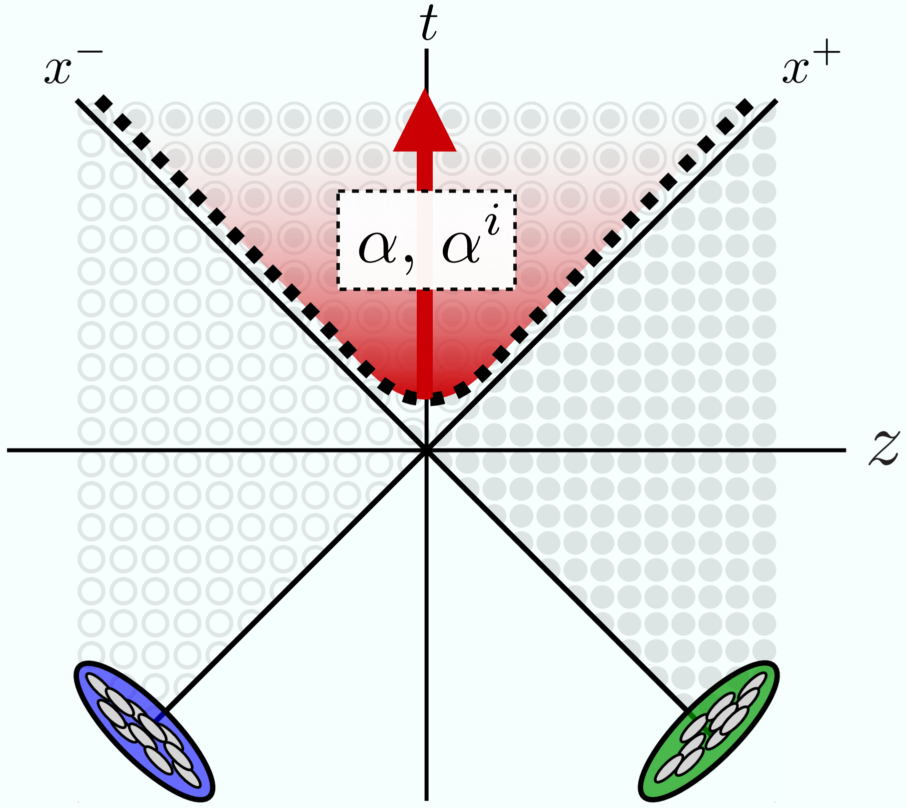
- Let's look at the bigger picture of HICs:



- **Calculations at** $\tau=0^+$**:** **T.Lappi and S.Schlichting, Phys.Rev.D 97, 034034 (2018)**
  J.L.Albacete, Cyrille Marquet, PGR, JHEP 1901 (2019) 073 [1808.00795]
  PGR, JHEP 1908 (2019) 026 [1903.11602]

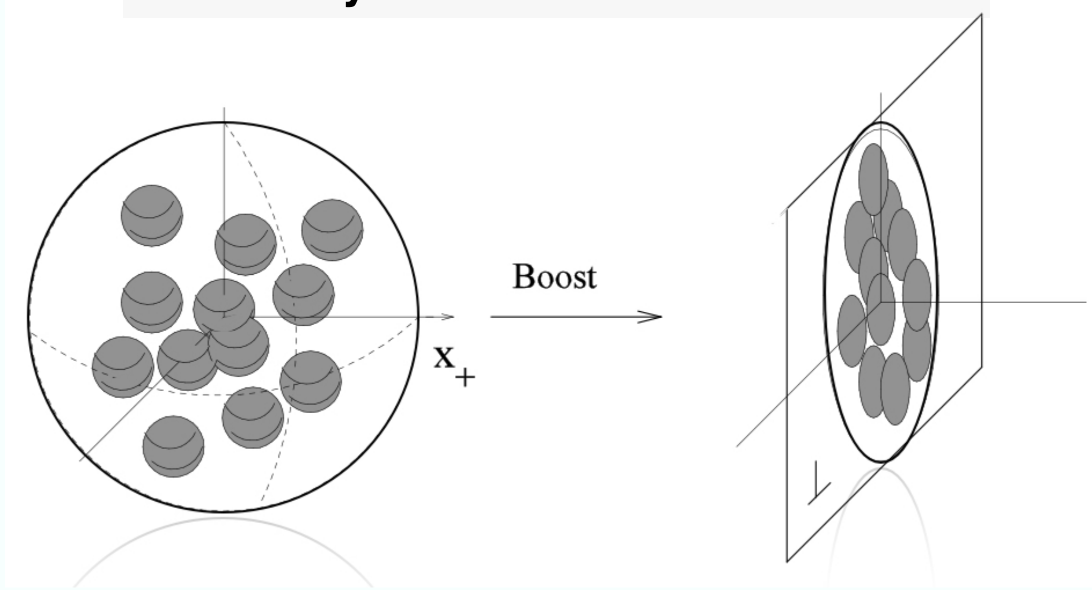
- **Calculations at** $\tau>0$**:** **T.Lappi, PGR, [2102.09993]**

The Glasma field at $\tau > 0$

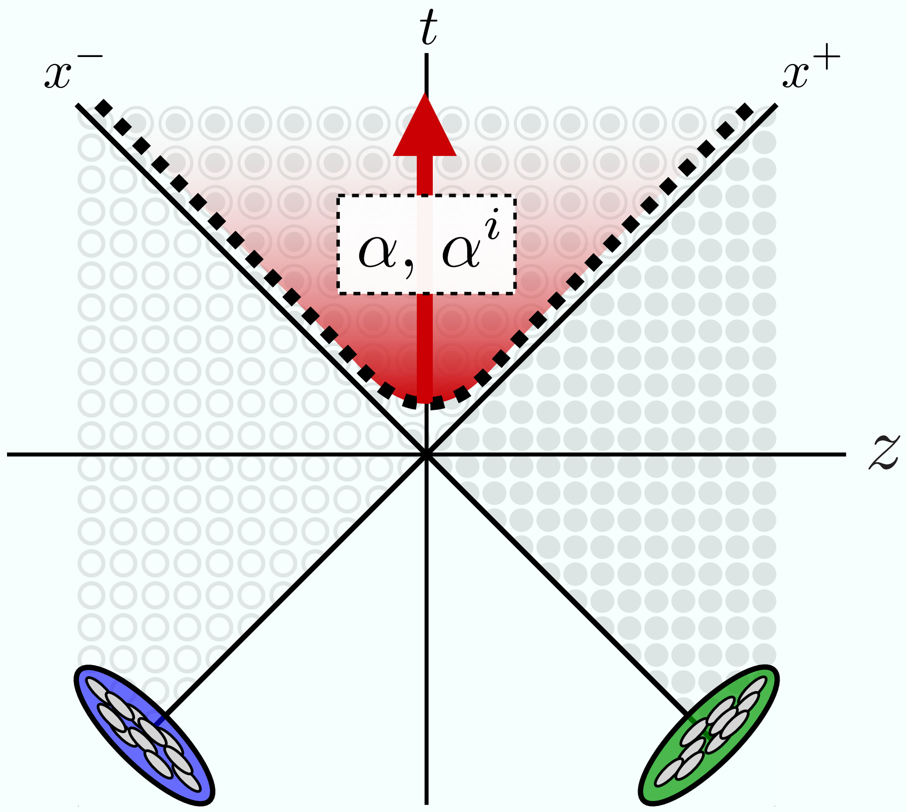


$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu \approx \rho_1(x_\perp)\delta(x^-)\delta^{\nu+} + \rho_2(x_\perp)\delta(x^+)\delta^{\nu-}$$

Extremely Lorentz-contracted nuclei



The Glasma field at $\tau > 0$



$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu \approx \rho_1(x_\perp)\delta(x^-)\delta^{\nu+} + \rho_2(x_\perp)\delta(x^+)\delta^{\nu-}$$

$$\tau = \sqrt{2x^+x^-} > 0 \longrightarrow [D_\mu, F^{\mu\nu}] = 0$$

$$\nu = \tau \longrightarrow \ ig\tau\,[\alpha, \partial_\tau \alpha] - \frac{1}{\tau}\left[D_i, \partial_\tau \alpha^i\right] = 0$$

$$\nu = \eta \longrightarrow \ \frac{1}{\tau}\partial_\tau \frac{1}{\tau}\partial_\tau(\tau^2\alpha) - \left[D_i, \left[D^i, \alpha\right]\right] = 0$$

$$\nu = i \longrightarrow \ \frac{1}{\tau}\partial_\tau(\tau\partial_\tau\alpha^i) - ig\tau^2\left[\alpha, \left[D^j, \alpha\right]\right] - \left[D^j, F^{ji}\right] = 0$$

- Analytical approximation in the forward light cone: **linearized Yang-Mills equations**

$$\alpha(\tau, x_\perp) = U(x_\perp)\beta(\tau, x_\perp)U^\dagger(x_\perp)$$

$$\alpha^i(\tau, x_\perp) = U(x_\perp)\left(\beta^i(\tau, x_\perp) - \frac{1}{ig}\partial^i\right)U^\dagger(x_\perp)$$

$$\longrightarrow$$

$$\partial_\tau \frac{1}{\tau}\partial_\tau(\tau^2\beta) = \tau\partial_i\partial^i\beta$$

$$\partial_\tau(\tau\partial_\tau\beta^i) = \tau\partial^k\partial_k\beta^i$$

- We consider fields in the **Coulomb gauge:** $\partial_i\beta^i = 0$

- General solution in momentum space: **free plane waves** (dispersion relation: $\omega(k_\perp) = |k_\perp| \equiv k$)

$$\beta(\tau, k_\perp) = \beta_0(k_\perp)\frac{2J_1(k\tau)}{k\tau}$$

$$\beta^i(\tau, k_\perp) = \beta_0^i(k_\perp)J_0(k\tau)$$

The Glasma field at $\tau > 0$

- Initial conditions: **matching to** $\tau = 0^+$ **solution**

$$\beta(\tau, k_\perp) = \frac{\tau}{k} E_0^\eta(k_\perp) J_1(k\tau) \qquad \beta^i(\tau, k_\perp) = -i\frac{\epsilon^{ij}k^j}{k^2} B_0^\eta(k_\perp) J_0(k\tau)$$

- Electric and magnetic fields at $\tau > 0$:

$$E^\eta(\tau, k_\perp) = E_0^\eta(k_\perp) J_0(k\tau)$$
$$E^i(\tau, k_\perp) = -i\epsilon^{ij}\frac{k^j}{k} B_0^\eta(k_\perp) J_1(k\tau)$$
$$B^\eta(\tau, k_\perp) = B_0^\eta(k_\perp) J_0(k\tau)$$
$$B^i(\tau, k_\perp) = -i\epsilon^{ij}\frac{k^j}{k} E_0^\eta(k_\perp) J_1(k\tau)$$

- Energy density and divergence of the Chern-Simons current at $\tau > 0$:

$$\varepsilon(\tau, x_\perp) = \mathrm{Tr}\{E^\eta E^\eta + B^\eta B^\eta + E^i E^i + B^i B^i\}$$
$$\dot{\nu}(\tau, x_\perp) = \mathrm{Tr}\{E^\eta B^\eta + E^i B^i\}$$

- τ-dependence in coordinate space:

$$E_0^\eta(x_\perp) = -ig\delta^{ij}\int \frac{d^2k_\perp}{(2\pi)^2}\int d^2u_\perp [\alpha_1^i(u_\perp), \alpha_2^j(u_\perp)] e^{ik_\perp (x-u)_\perp} \equiv \int \frac{d^2k_\perp}{(2\pi)^2} E_0^\eta(k_\perp) e^{ik_\perp x_\perp}$$

$$E^\eta(\tau, x_\perp) = -ig\delta^{ij}\int \frac{d^2k_\perp}{(2\pi)^2}\int d^2u_\perp [\alpha_1^i(u_\perp), \alpha_2^j(u_\perp)] J_0(k\tau) e^{ik_\perp (x-u)_\perp}$$

Glasma correlators at $\tau > 0$

- For the 1-point correlator (i.e. the average energy density):

$$\langle \varepsilon(\tau, x_\perp) \rangle = \frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) f^{abn} f^{cdn} \int_{p,k} \int_{u,v} \langle \alpha_u^{i,a} \alpha_v^{k,c} \rangle_1 \langle \alpha_u^{j,b} \alpha_v^{l,d} \rangle_2$$

$$\times \left(J_0(p\tau) J_0(k\tau) - \frac{p_\perp \cdot k_\perp}{p\,k} J_1(p\tau) J_1(k\tau) \right) e^{ip_\perp (x-u)_\perp} e^{ik_\perp (x-v)_\perp}$$

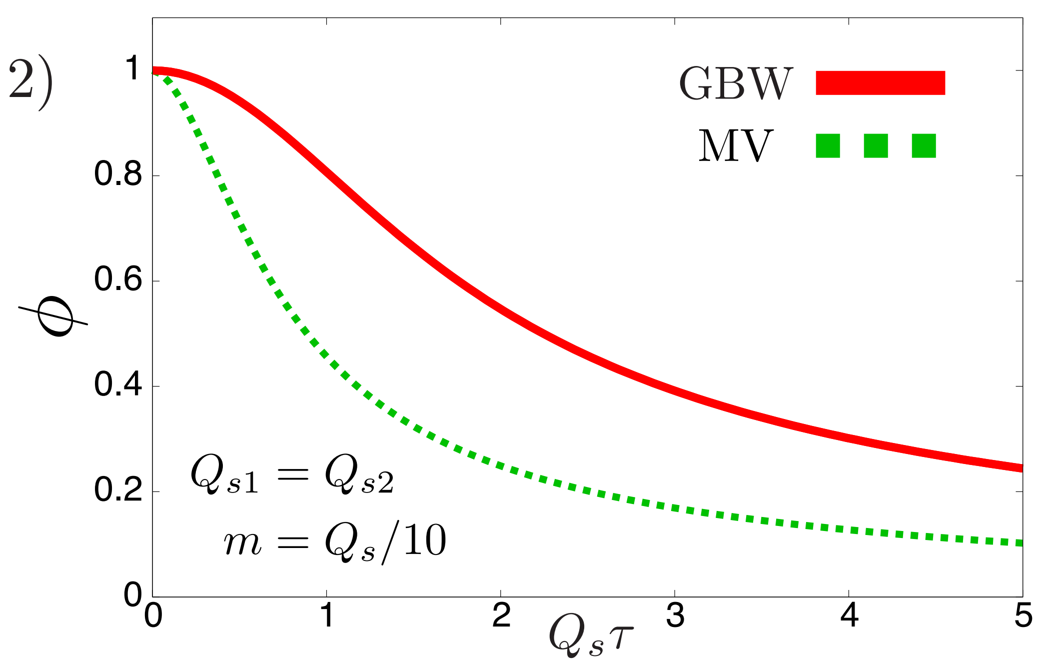
$$= \frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) f^{abn} f^{cdn} \int_{u,v} \langle \alpha_u^{i,a} \alpha_v^{k,c} \rangle_1 \langle \alpha_u^{j,b} \alpha_v^{l,d} \rangle_2$$

$$\times \frac{\delta(|x_\perp - u_\perp| - \tau)}{2\pi\tau} \frac{\delta(|x_\perp - v_\perp| - \tau)}{2\pi\tau} (1 + \cos(\theta_{x-u} - \theta_{x-v}))$$

$$= \frac{g^2}{2} N_c (N_c^2 - 1) \int \frac{d\Theta}{2\pi} (1 + \cos(\Theta))$$

$$\times \left(\frac{Q_{s1}^2}{g^2 N_c} \frac{1 - \exp\left\{ -\frac{Q_{s1}^2 \tau^2}{2} (1 - \cos(\Theta)) \right\}}{\frac{Q_{s1}^2 \tau^2}{2} (1 - \cos(\Theta))} \right) \times (1 \to 2)$$

$$\equiv \langle \varepsilon_0 \rangle \times \phi(Q_{s1}\tau, Q_{s2}\tau)$$



Glasma correlators at $\tau > 0$

- One-point function: $\langle \varepsilon(\tau, x_\perp) \rangle = \langle \varepsilon_0 \rangle \times \phi(Q_{s1}\tau, Q_{s2}\tau)$
- Two-point function:

$$\langle \varepsilon(\tau, x_\perp)\varepsilon(\tau, y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'} + \epsilon^{i'j'}\epsilon^{k'l'}) f^{abn} f^{cdn} f^{a'b'm} f^{c'd'm} \int_{p,k}\int_{\bar{p},\bar{k}}\int_{u,v}\int_{\bar{u},\bar{v}} \langle \alpha_u^{i,a}\alpha_{\bar{u}}^{k,c}\alpha_v^{i',a'}\alpha_{\bar{v}}^{k',c'}\rangle_1 \langle \alpha_u^{j,b}\alpha_{\bar{u}}^{l,d}\alpha_v^{j',b'}\alpha_{\bar{v}}^{l',d'}\rangle_2$$

$$\times \left(J_0(p\tau)J_0(\bar{p}\tau) - \frac{p_\perp \cdot \bar{p}_\perp}{p\,\bar{p}} J_1(p\tau)J_1(\bar{p}\tau)\right)\left(J_0(k\tau)J_0(\bar{k}\tau) - \frac{k_\perp \cdot \bar{k}_\perp}{k\,\bar{k}} J_1(k\tau)J_1(\bar{k}\tau)\right)$$

$$\times e^{ip_\perp(x-u)_\perp} e^{ik_\perp(y-v)_\perp} e^{i\bar{p}_\perp(x-\bar{u})_\perp} e^{i\bar{k}_\perp(y-\bar{v})_\perp}$$

$$= \frac{g^4}{4}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'} + \epsilon^{i'j'}\epsilon^{k'l'}) f^{abn} f^{cdn} f^{a'b'm} f^{c'd'm} \int_{u,v}\int_{\bar{u},\bar{v}} \langle \alpha_u^{i,a}\alpha_{\bar{u}}^{k,c}\alpha_v^{i',a'}\alpha_{\bar{v}}^{k',c'}\rangle_1 \langle \alpha_u^{j,b}\alpha_{\bar{u}}^{l,d}\alpha_v^{j',b'}\alpha_{\bar{v}}^{l',d'}\rangle_2$$

$$\times \frac{\delta(|x_\perp - u_\perp| - \tau)}{2\pi\tau} \frac{\delta(|x_\perp - \bar{u}_\perp| - \tau)}{2\pi\tau} \frac{\delta(|y_\perp - v_\perp| - \tau)}{2\pi\tau} \frac{\delta(|y_\perp - \bar{v}_\perp| - \tau)}{2\pi\tau}$$

$$\times (1 + \cos(\theta_{x-u} - \theta_{x-\bar{u}}))(1 + \cos(\theta_{y-v} - \theta_{y-\bar{v}}))$$

Glasma Graph Approximation:

$$\langle \alpha^i\alpha^j\alpha^k\alpha^l\rangle = \langle \alpha^i\alpha^j\rangle\langle \alpha^k\alpha^l\rangle + \langle \alpha^i\alpha^k\rangle\langle \alpha^j\alpha^l\rangle + \langle \alpha^i\alpha^l\rangle\langle \alpha^j\alpha^k\rangle$$

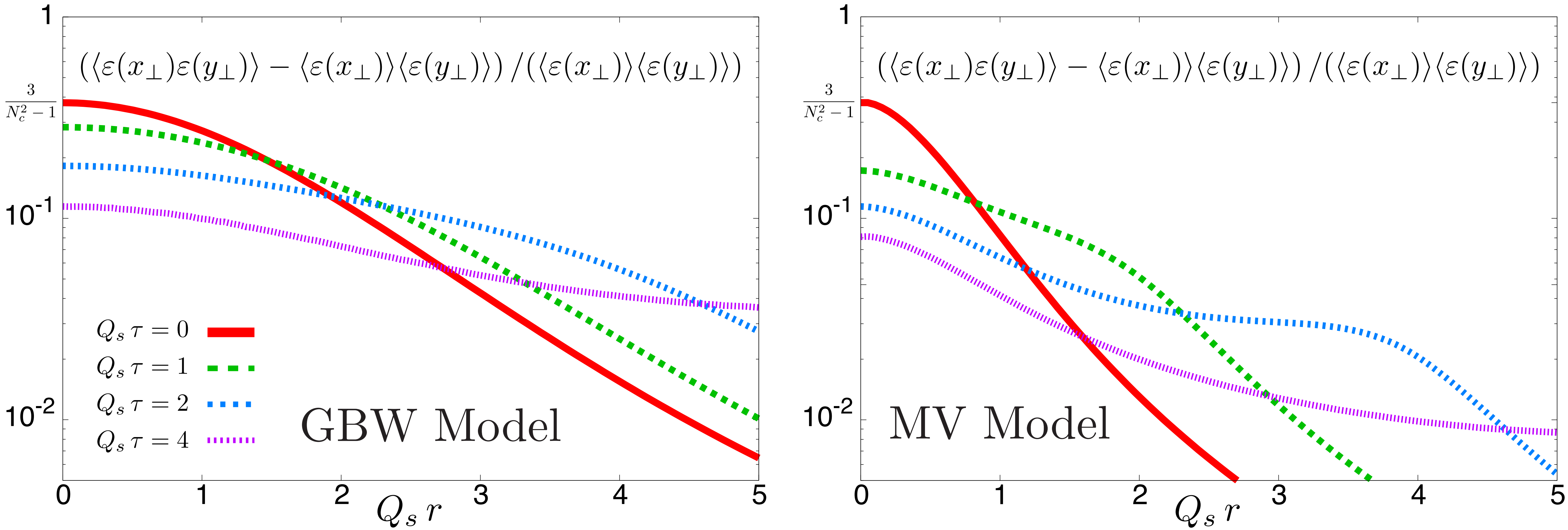
Glasma correlators at $\tau > 0$

- One-point function: $\langle\varepsilon(\tau,x_\perp)\rangle = \langle\varepsilon_0\rangle\times\phi(Q_{s1}\tau,Q_{s2}\tau)$
- Two-point function:

$$
\begin{aligned}
\langle\varepsilon(\tau,x_\perp)\varepsilon(\tau,y_\perp)\rangle &= \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm}\int_{p,k}\int_{\bar p,\bar k}\int_{u,v}\int_{\bar u,\bar v}\langle\alpha_u^{i,a}\alpha_{\bar u}^{k,c}\alpha_v^{i',a'}\alpha_{\bar v}^{k',c'}\rangle_1\langle\alpha_u^{j,b}\alpha_{\bar u}^{l,d}\alpha_v^{j',b'}\alpha_{\bar v}^{l',d'}\rangle_2 \\
&\times\left(J_0(p\tau)J_0(\bar p\tau)-\frac{p_\perp\cdot\bar p_\perp}{p\,\bar p}J_1(p\tau)J_1(\bar p\tau)\right)\left(J_0(k\tau)J_0(\bar k\tau)-\frac{k_\perp\cdot\bar k_\perp}{k\,\bar k}J_1(k\tau)J_1(\bar k\tau)\right) \\
&\times e^{ip_\perp(x-u)_\perp}e^{ik_\perp(y-v)_\perp}e^{i\bar p_\perp(x-\bar u)_\perp}e^{i\bar k_\perp(y-\bar v)_\perp} \\
&= \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm}\int_{u,v}\int_{\bar u,\bar v}\langle\alpha_u^{i,a}\alpha_{\bar u}^{k,c}\alpha_v^{i',a'}\alpha_{\bar v}^{k',c'}\rangle_1\langle\alpha_u^{j,b}\alpha_{\bar u}^{l,d}\alpha_v^{j',b'}\alpha_{\bar v}^{l',d'}\rangle_2 \\
&\times\frac{\delta(|x_\perp-u_\perp|-\tau)}{2\pi\tau}\frac{\delta(|x_\perp-\bar u_\perp|-\tau)}{2\pi\tau}\frac{\delta(|y_\perp-v_\perp|-\tau)}{2\pi\tau}\frac{\delta(|y_\perp-\bar v_\perp|-\tau)}{2\pi\tau} \\
&\times(1+\cos(\theta_{x-u}-\theta_{x-\bar u}))(1+\cos(\theta_{y-v}-\theta_{y-\bar v}))
\end{aligned}
$$

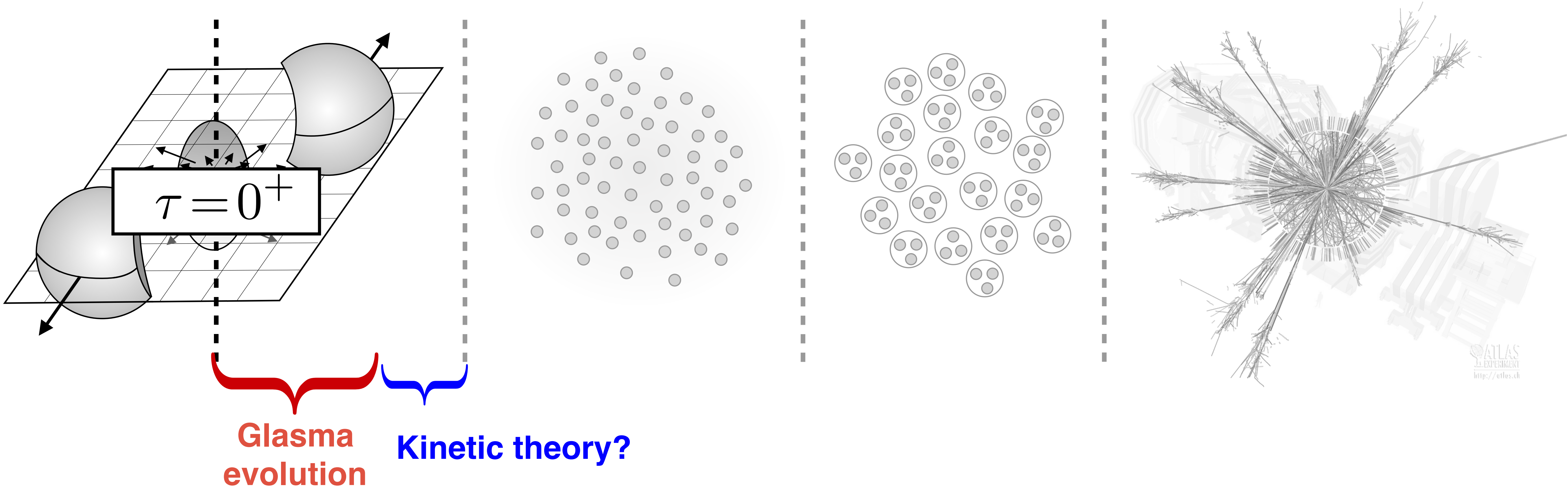
$$
\boxed{
\begin{aligned}
&= \frac{g^4}{8}N_c^2(N_c^2-1)\int_0^{2\pi}\frac{d\theta_s}{2\pi}\frac{d\theta_{\bar s}}{2\pi}\frac{d\theta_t}{2\pi}\frac{d\theta_{\bar t}}{2\pi}(1+\cos(\theta_s-\theta_{\bar s}))(1+\cos(\theta_t-\theta_{\bar t})) \\
&\times\Bigg[\Big((N_c^2-1)G_1((s-\bar s)_\tau)G_1((t-\bar t)_\tau)G_2((s-\bar s)_\tau)G_2((t-\bar t)_\tau) \\
&+2G_1((s-\bar s)_\tau)G_1((t-\bar t)_\tau)G_2((s-t)_\tau-r_\perp)G_2((\bar s-\bar t)_\tau-r_\perp) \\
&+G_1((s-t)_\tau-r_\perp)G_1((\bar s-\bar t)_\tau-r_\perp)G_2((s-t)_\tau-r_\perp)G_2((\bar s-\bar t)_\tau-r_\perp)\Big)+\Big(1\leftrightarrow 2\Big)\Bigg]
\end{aligned}
}
$$

Glasma correlators at $\tau > 0$



- Correlator decay significantly more pronounced under the MV model (both in space and time)
- "Correlation length" growth: sign of a system that is approaching the hydrodynamical regime
- But: this is not the whole story…

Glasma correlators at $\tau > 0$



- **Calculations at $\tau=0^+$:** T.Lappi and S.Schlichting, Phys.Rev.D 97, 034034 (2018)
  J.L.Albacete, Cyrille Marquet, PGR, JHEP 1901 (2019) 073 [1808.00795]
  PGR, JHEP 1908 (2019) 026 [1903.11602]
- **Calculations at $\tau > 0$:** T.Lappi, PGR, Phys. Rev. D 104, 014011 (2021) [2102.09993]

- **After CGC breakdown ("late" pre-equilibrium?):**
  A. Kurkela and E. Lu, Phys. Rev. Lett. 113 (2014) 182301 [1405.6318]
  A. Kurkela and Y. Zhu, Phys. Rev. Lett. 115 (2015) 182301 [1506.06647]
  L. Keegan, A. Kurkela, A. Mazeliauskas and D. Teaney, JHEP 08 (2016) 171 [1605.04287]
  A. Kurkela, A. Mazeliauskas, J.-F. Paquet, S. Schlichting and D. Teaney, Phys. Rev. C 99 (2019) 034910 [1805.00961]

PART III: Phenomenology

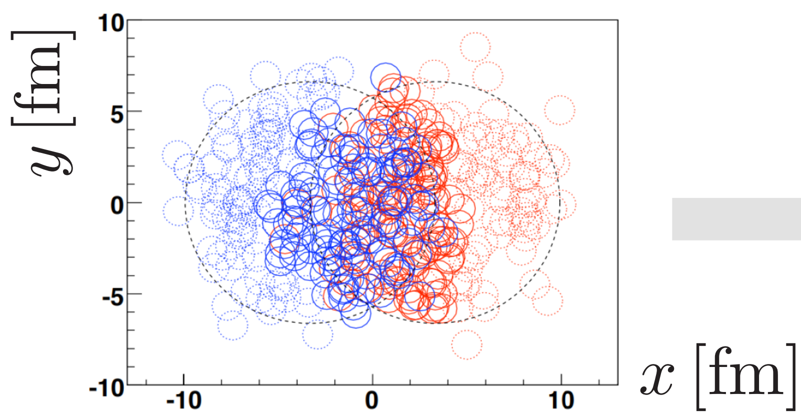
→ **Calculation of eccentricities**

→ **Comparison to Monte Carlo model**

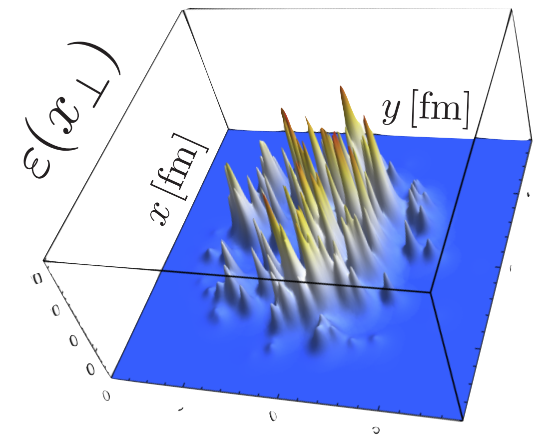
→ **Comparison to experimental data**

Calculation of eccentricities

- Traditional modelization of the initial stage: **Glauber Montecarlo Ansatz**



Random sampling of nucleon positions

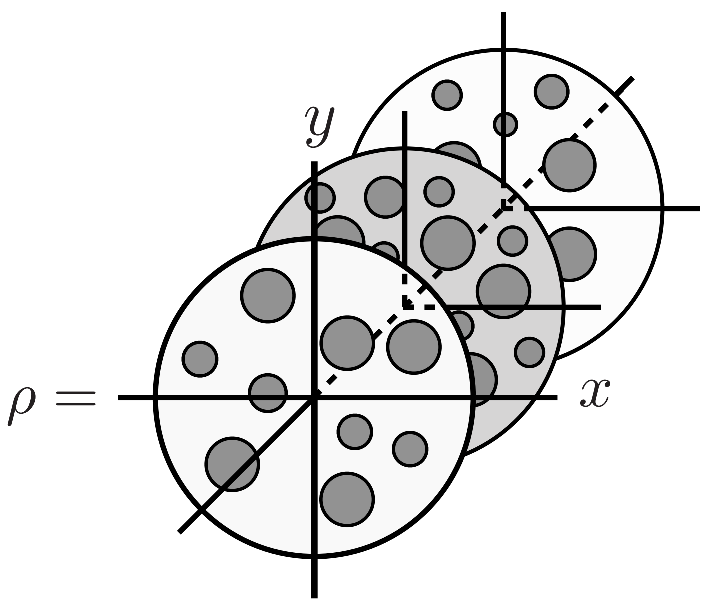


Mapping of nucleons/ partons into energy density profile.

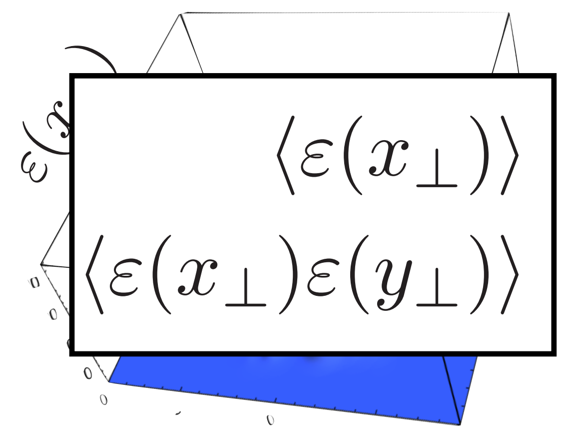
$$\epsilon_n = \frac{\int_{\mathbf{s}} \mathbf{s}^n \varepsilon(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}|^n \varepsilon(\mathbf{s})}$$

Initial state deformation characterized by **eccentricities**

- **Our approach:**



Primordial fluctuations



CGC calculation of energy density correlators

$$\epsilon_n = \frac{\int_{\mathbf{s}} \mathbf{s}^n \varepsilon(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}|^n \varepsilon(\mathbf{s})}$$

Initial state deformation characterized by **eccentricities**

Calculation of eccentricities

$$\boxed{\varepsilon(x_\perp) = \langle \varepsilon(x_\perp)\rangle + \delta\varepsilon(x_\perp) \text{ with } \langle \varepsilon(x_\perp)\rangle \gg \delta\varepsilon(x_\perp)}$$

- Under this assumption we can obtain analytical expressions for the **eccentricity cumulants:**

$$\epsilon_2\{4\} \sim \frac{\int_{\mathbf{s}} \mathbf{s}^2 \langle \varepsilon(\mathbf{s})\rangle}{\int_{\mathbf{s}} |\mathbf{s}|^2 \langle \varepsilon(\mathbf{s})\rangle}$$

$$\langle \epsilon_3 \epsilon_3^* \rangle = \epsilon_3\{2\}^2 = \frac{\int_{\mathbf{s}_1,\mathbf{s}_2} (\mathbf{s}_1)^3 (\mathbf{s}_2^*)^3 \mathrm{Cov}[\varepsilon](\mathbf{s}_1,\mathbf{s}_2)}{\left(\int_{\mathbf{s}} |\mathbf{s}|^3 \langle \varepsilon(\mathbf{s})\rangle\right)^2}$$

$$\langle \epsilon_2 \epsilon_2^* \rangle = \epsilon_2\{2\}^2 = \frac{\int_{\mathbf{s}_1,\mathbf{s}_2} |\mathbf{s}|^4 \mathrm{Cov}[\varepsilon](\mathbf{s_1},\mathbf{s_2})}{\left(\int_{\mathbf{s}} |\mathbf{s}|^2 \langle \varepsilon(\mathbf{s})\rangle\right)^2} + \frac{\int_{\mathbf{s}} \mathbf{s}^2 \langle \varepsilon(\mathbf{s})\rangle}{\int_{\mathbf{s}} |\mathbf{s}|^2 \langle \varepsilon(\mathbf{s})\rangle} \quad \text{with} \quad \mathbf{s} = x + iy$$

where $\boxed{\langle \varepsilon(\mathbf{s})\rangle}$ is the **average** energy density and $\boxed{\mathrm{Cov}\left[\varepsilon\right]\left(\mathbf{s}_1,\mathbf{s}_2\right) = \langle \varepsilon(\mathbf{s}_1)\varepsilon(\mathbf{s}_2)\rangle - \langle \varepsilon(\mathbf{s}_1)\rangle\langle \varepsilon(\mathbf{s}_2)\rangle}$

encodes the **fluctuations** around the average.

- We make our saturation scale proportional to the integrated nuclear density: $Q_s^2(\mathbf{s}) = Q_{s0}^2 T(\mathbf{s})/T(\mathbf{0})$

Blaizot, Broniowski, Ollitrault [1405.3572]

Calculation of eccentricities

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$$\langle\epsilon_2\epsilon_2^*\rangle = \epsilon_2\{2\}^2 = \frac{\int_{\mathbf{s}_1,\mathbf{s}_2} |\mathbf{s}|^4 \mathrm{Cov}[\varepsilon](\mathbf{s_1},\mathbf{s_2})}{\left(\int_{\mathbf{s}} |\mathbf{s}|^2 \langle\varepsilon(\mathbf{s})\rangle\right)^2} + \frac{\int_{\mathbf{s}} \mathbf{s}^2 \langle\varepsilon(\mathbf{s})\rangle}{\int_{\mathbf{s}} |\mathbf{s}|^2 \langle\varepsilon(\mathbf{s})\rangle}$$

Dominated by $\lim_{Q_s r \gg 1} \mathrm{Cov}\left[\varepsilon\right](r)$

where $\boxed{\langle\varepsilon(\mathbf{s})\rangle}$ is the **average** energy density and $\boxed{\mathrm{Cov}\left[\varepsilon\right](\mathbf{s}_1,\mathbf{s}_2) = \langle\varepsilon(\mathbf{s}_1)\varepsilon(\mathbf{s}_2)\rangle - \langle\varepsilon(\mathbf{s}_1)\rangle\langle\varepsilon(\mathbf{s}_2)\rangle}$

encodes the **fluctuations** around the average.

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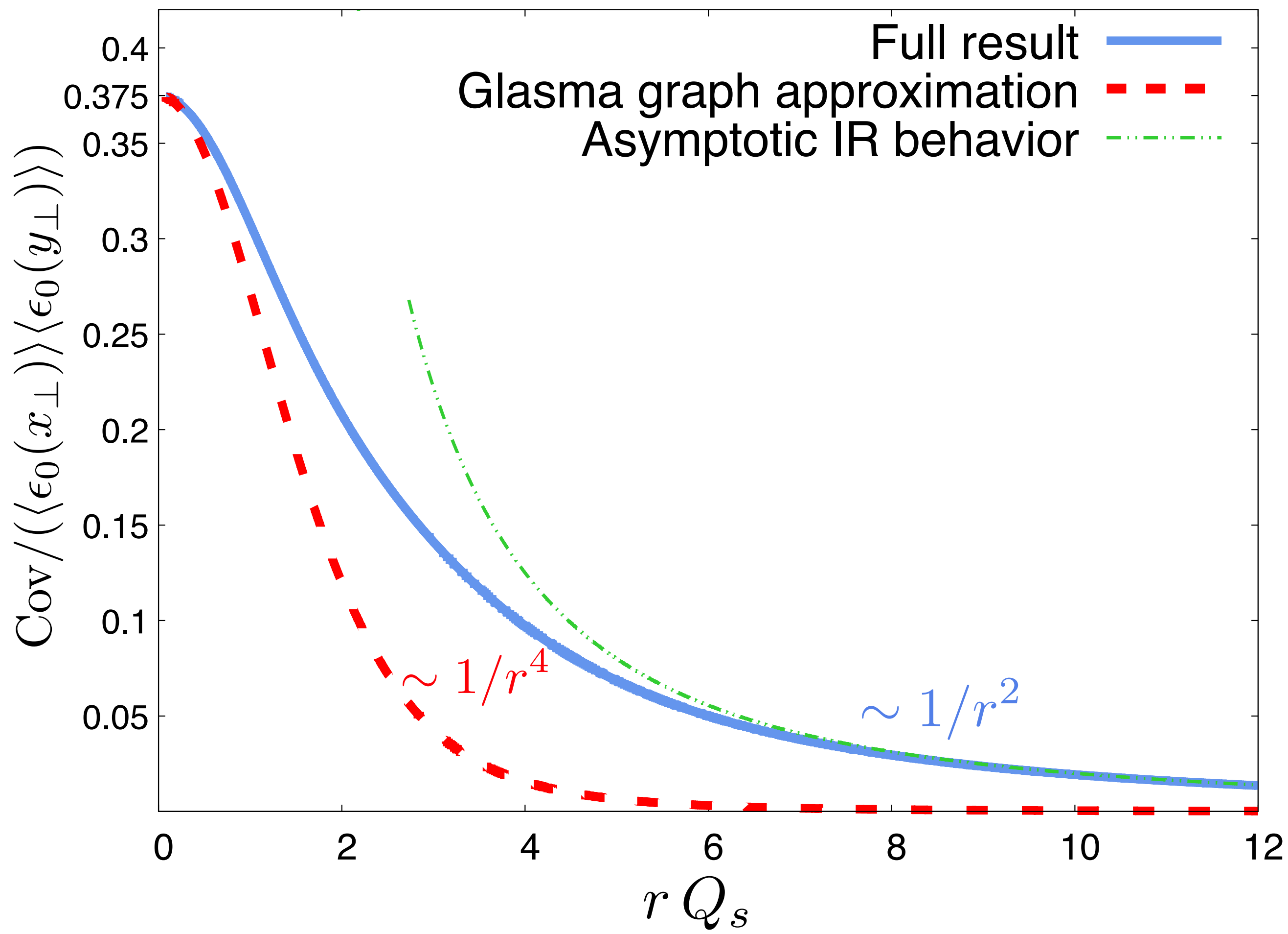
Blaizot, Broniowski, Ollitrault [1405.3572]

Glasma correlators at $\tau = 0^+$

- Agreement in the $r \to 0$ limit. **Strong discrepancies** in the $rQ_s \to \infty$ limit.

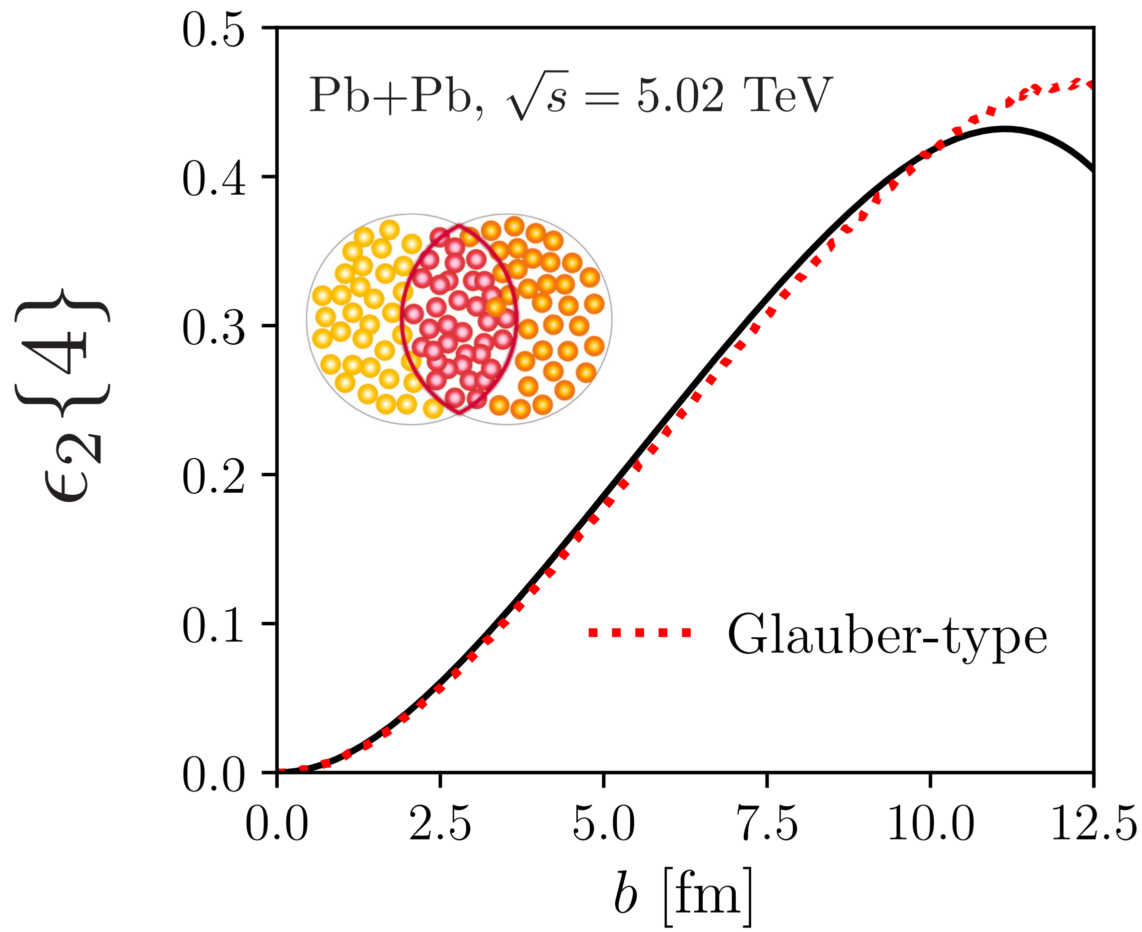


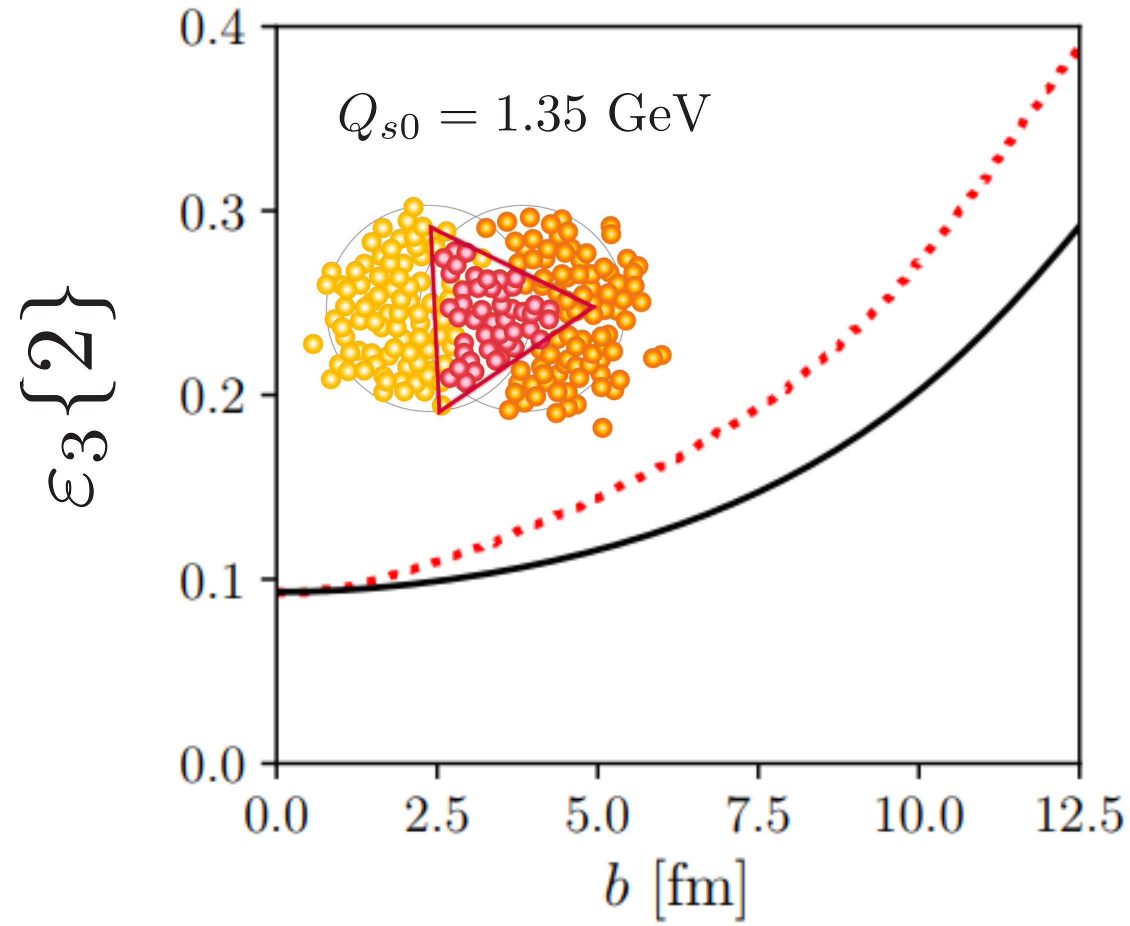
- This slowly decaying behavior could potentially have an impact in **infrared-sensitive observables** built from this quantity, **such as the eccentricities**

J.L.Albacete, Cyrille Marquet, PGR, JHEP 1901 (2019) 073 [1808.00795]

Comparison to Monte Carlo Glauber Model

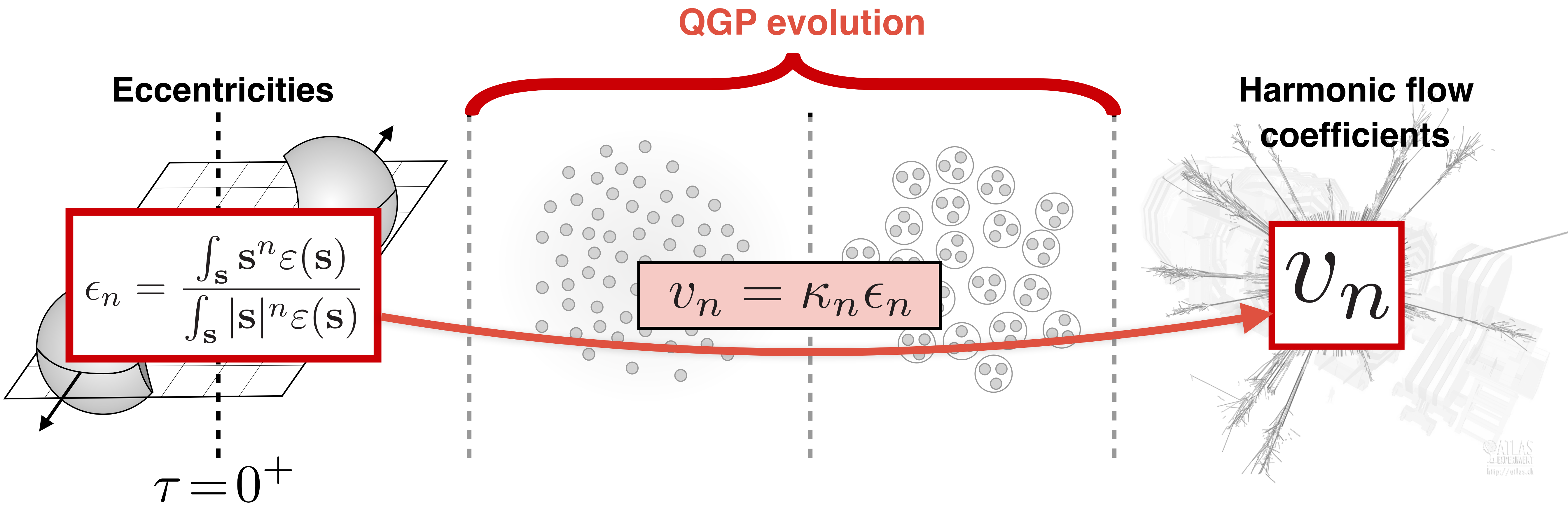
LHC eccentricities: Pb+Pb, $\sqrt{s} = 5.02$ TeV





- Elliptic deformation largely determined by the geometry of the collision
- CGC-based method differs from MC model in description of energy density fluctuations

Comparison to data

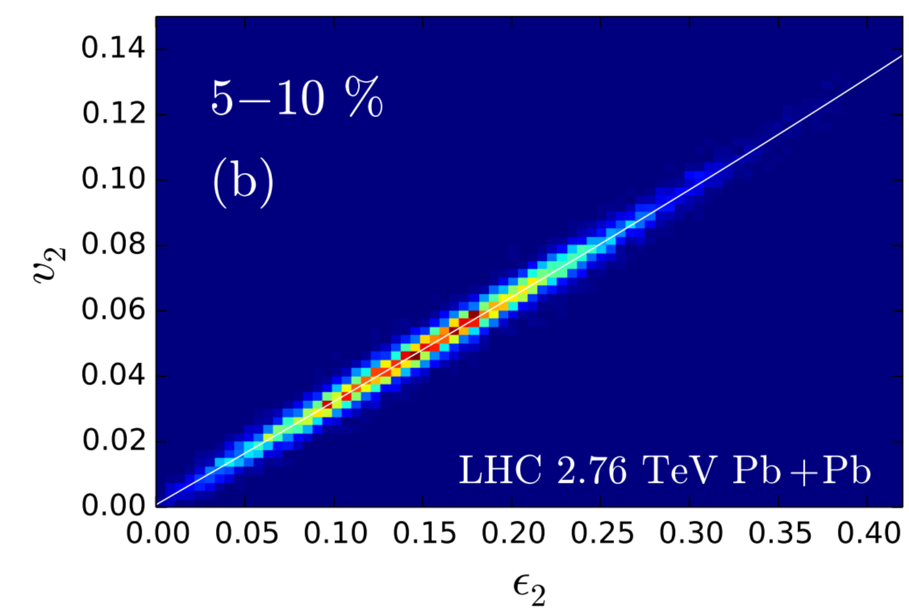


- We want to relate our calculations to experimental data on azimuthal anisotropy
- A **linear relation** is observed (in central collisions)

$$\sqrt{\langle v_2^2 \rangle} = v_2\{2\} = \kappa_2 \varepsilon_2\{2\}$$

$$\sqrt[4]{2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle} = v_2\{4\} = \kappa_2 \varepsilon_2\{4\}$$

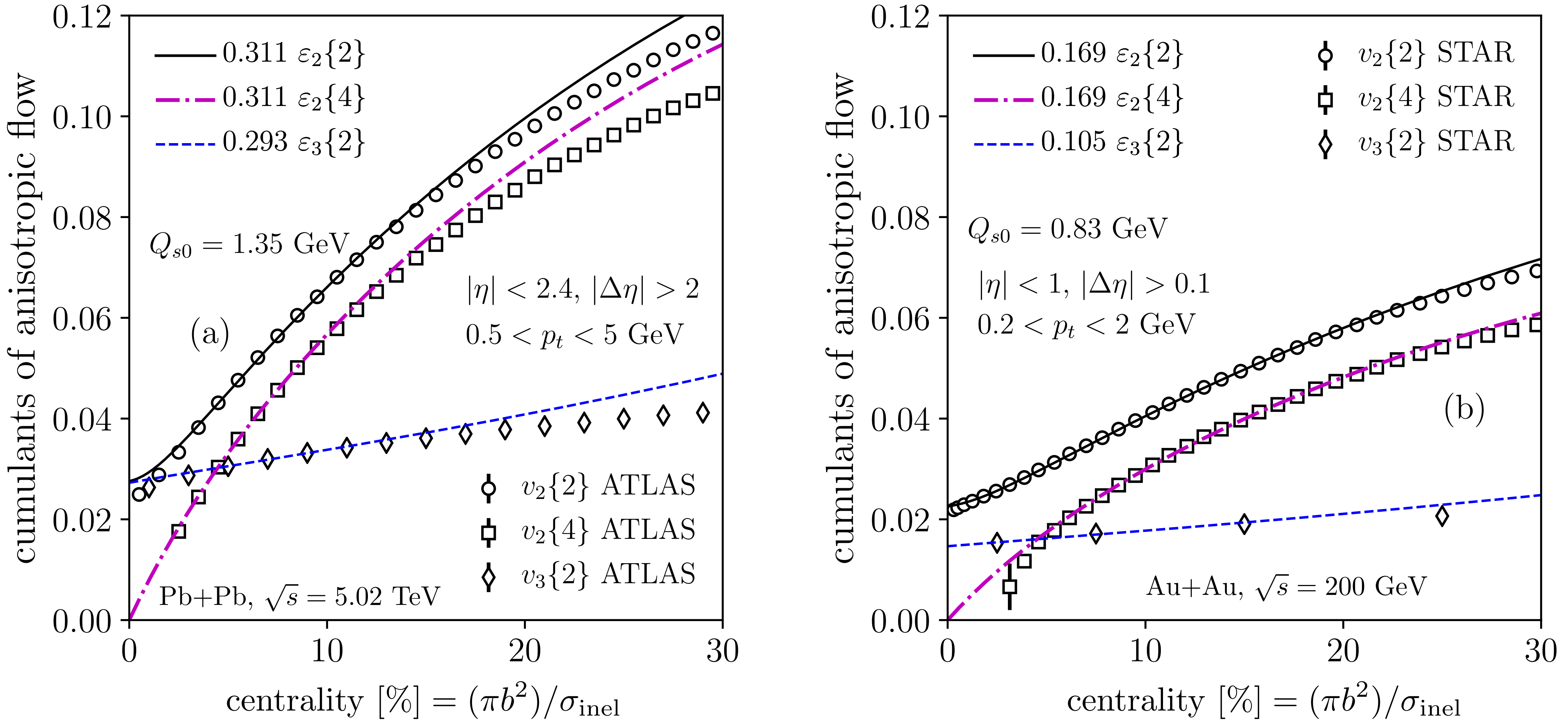
$$\sqrt{\langle v_3^2 \rangle} = v_3\{2\} = \kappa_3 \varepsilon_3\{2\}$$



Niemi, Eskola and Patelaainen [1505.02677]

Comparison to data

LHC data: Pb+Pb, $\sqrt{s} = 5.02$ TeV   RHIC data: Au+Au, $\sqrt{s} = 200$ GeV



- The value of κ_2 is fixed by fitting to $v_2\{4\}$ data (which probes average geometry)
- The resulting response coefficients are comparable to latest results from state-of-the-art hydrodynamical simulations

G.Giacalone, M.Luzum, C.Marquet, J-Y.Ollitrault, PGR, Phys.Rev.C 100 (2019) 024905 [1902.07168]

PART IV: Future prospects and conclusions

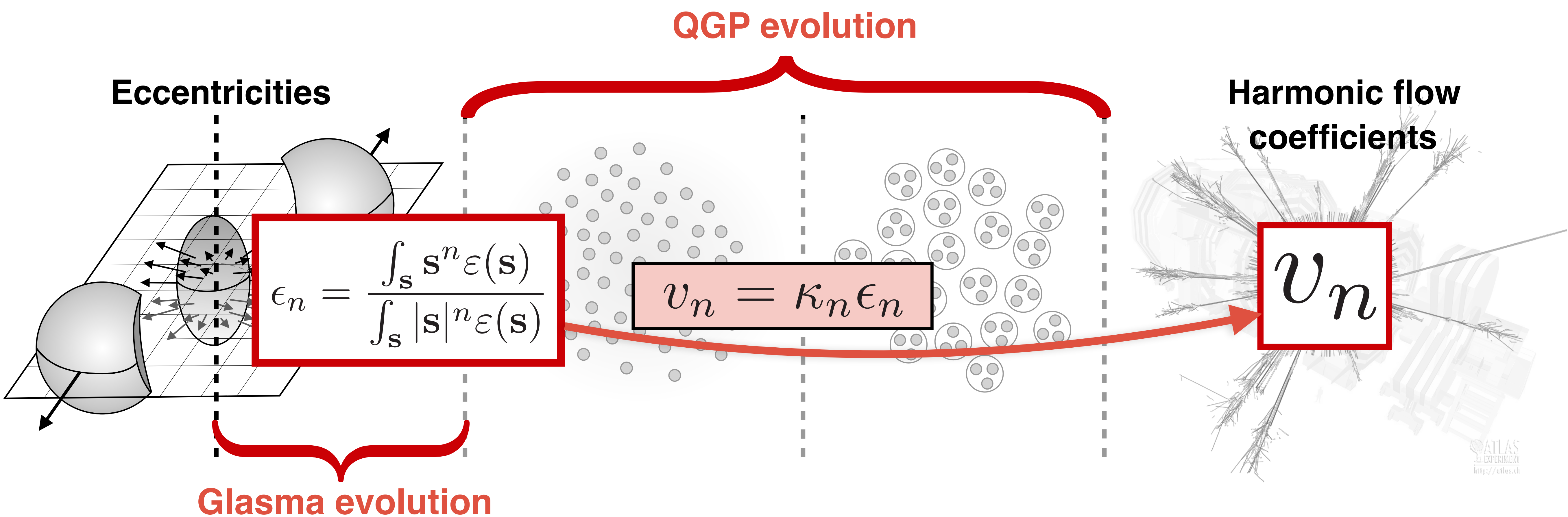
→ **Eccentricities: τ -evolution**

→ **Hot Spots + gluon field fluctuations: τ -evolution**

→ **Applications in Chiral Magnetic Effect studies**

→ **Conclusions**

Eccentricities: τ-evolution



- **Calculations at** $\tau = 0^+$ **:** T.Lappi and S.Schlichting, Phys.Rev.D 97, 034034 (2018)
  J.L.Albacete, Cyrille Marquet, PGR, JHEP 1901 (2019) 073 [1808.00795]
  PGR, JHEP 1908 (2019) 026 [1903.11602]
- **Calculations at** $\tau > 0$ **:** T.Lappi, PGR, [2102.09993]
- **Phenomenology:** F.Gelis, G.Giacalone, C.Marquet, J-Y.Ollitrault, PGR [1907.10948]
  G.Giacalone, M.Luzum, C.Marquet, J-Y.Ollitrault, PGR, Phys.Rev.C 100 (2019) 024905 [1902.07168]

- In order to perform a similar study based on our tau-evolved correlators, we require going **beyond the Glasma Graph approximation**

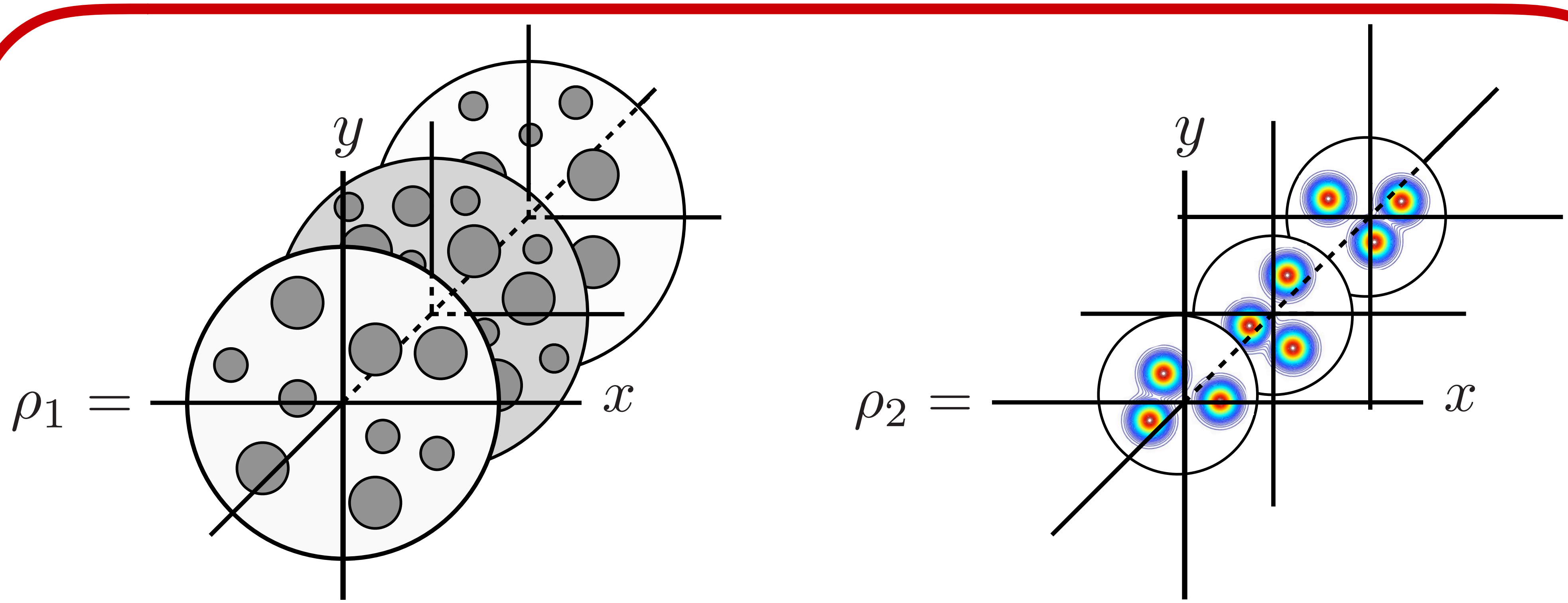
CGC + Hot Spots

- Sources of fluctuations:

→ **Random positions of nucleons**

→ **Subnucleonic structure (e.g. Hot Spots)**

→ **Quantum fluctuations of the wave functions (i.e. primordial fluctuations)**



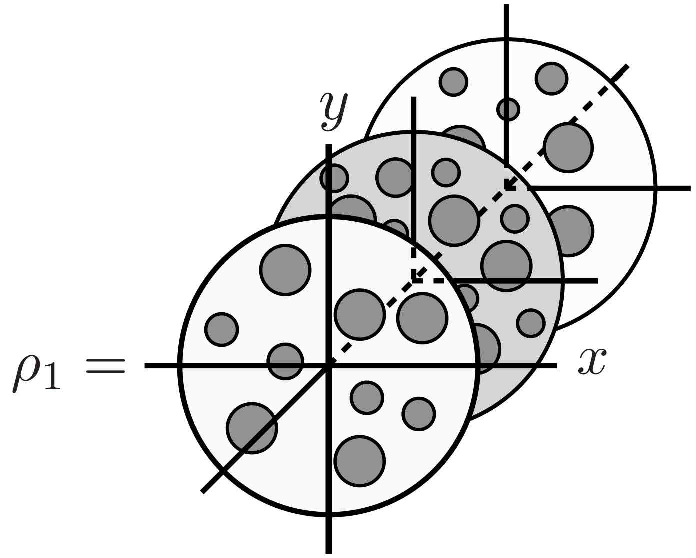
Dense nucleus: event-by-event fluctuations of **color source densities**

Dilute nucleus: event-by-event fluctuations of **positions of Hot Spots**

Calculations at $\tau = 0^+$**:** **S.Demirci, T.Lappi and S.Schlichting, Phys. Rev. D 103, 094025 (2021) [2101.03791]**

CGC + Hot Spots calculation

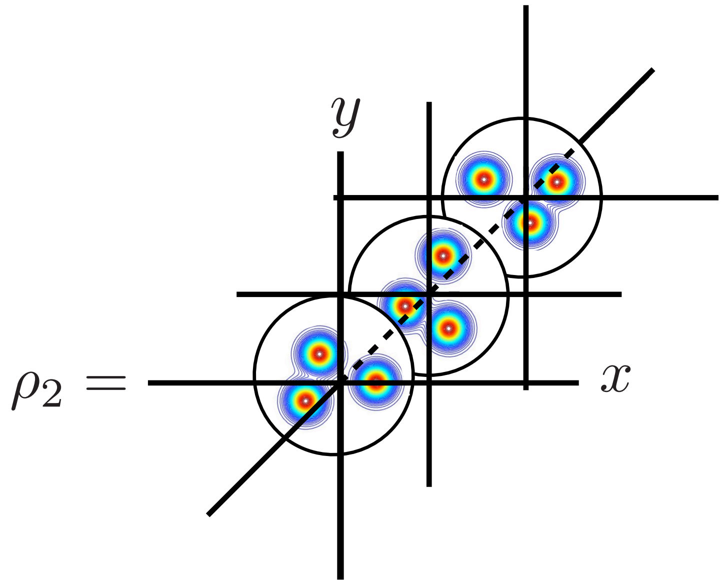
- Initial fluctuations from **dense nucleus** described by **CGC correlators**:



$$\langle \mathcal{O}(\rho_1)\rangle = \int [\mathcal{D}\rho_1] W(\rho_1)\mathcal{O}(\rho_1) = \int [\mathcal{D}\rho_1] \exp\left\{-\int dx\, \mathrm{Tr}\left[\rho_1^2\right]\right\}\mathcal{O}(\rho_1)$$

$$\langle \rho_1^a(x_\perp)\rho_1^b(y_\perp)\rangle \propto \delta^{ab}\delta(x_\perp - y_\perp)$$

- Initial fluctuations from **dilute nucleus** described by **double correlators (CGC + Hot Spots)**:



$$\langle\langle \mathcal{O}(\rho_2)\rangle\rangle = \left(\frac{2\pi R^2}{N_q}\right)\int \prod_{i=1}^{N_q}\left[d^2\mathbf{b}_i T(\mathbf{b}_i - \mathbf{B})\right]\delta\left(\frac{1}{N_q}\sum_{i=1}^{N_q}\mathbf{b}_i - \mathbf{B}\right)\langle \mathcal{O}(\rho_2)\rangle_{\text{CGC}}$$

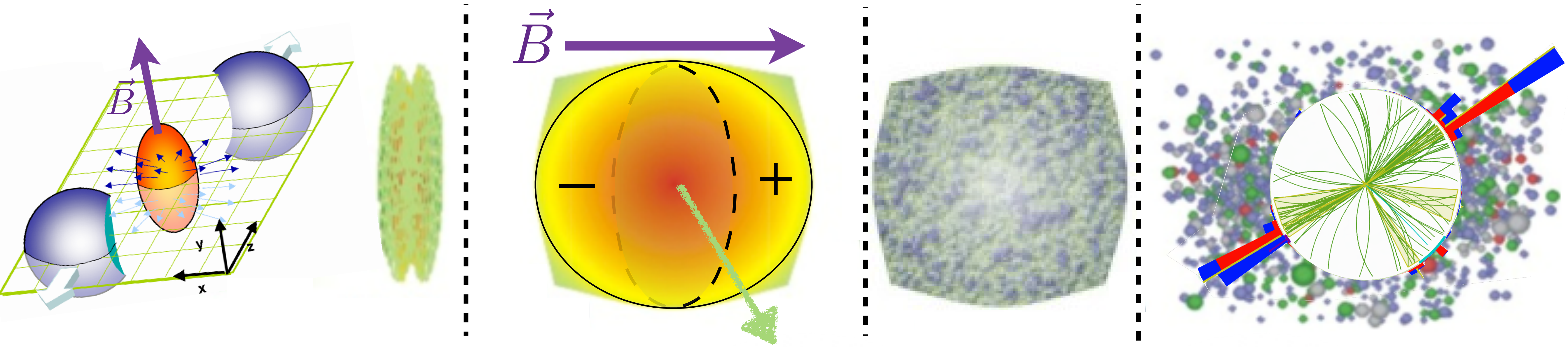
with:

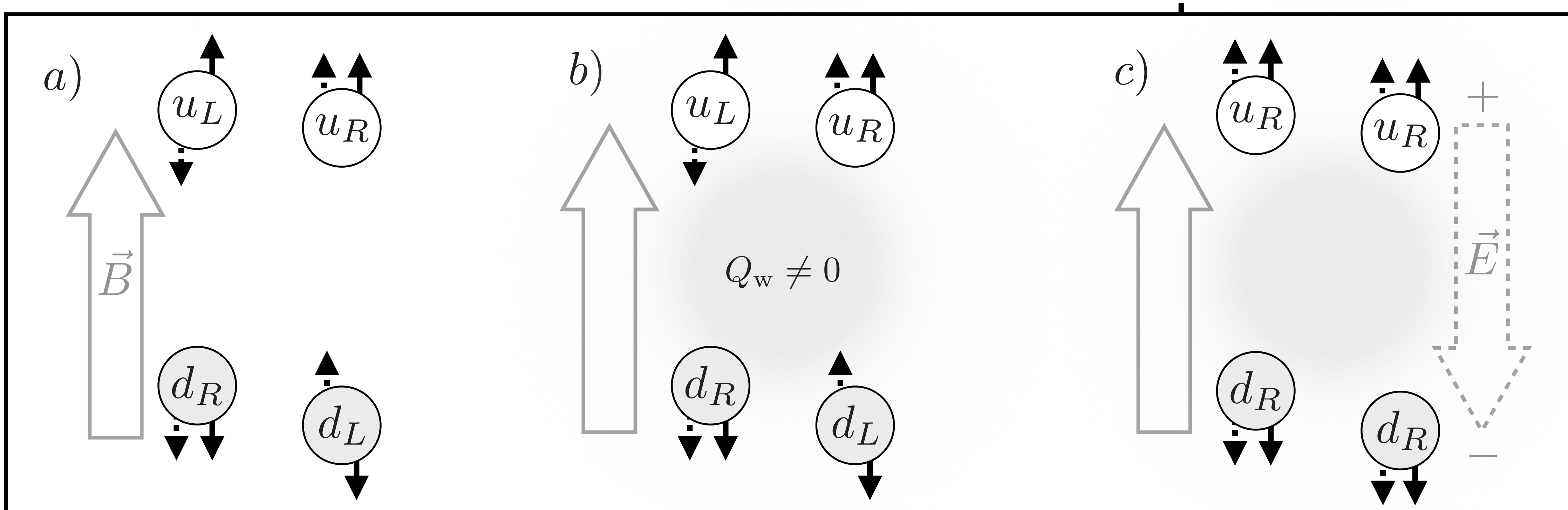
$$T(\mathbf{b}) = \frac{1}{2\pi R^2}\exp\left[-\frac{\mathbf{b}^2}{2R^2}\right]$$

Fluctuations of color charge densities within each Hot Spot

Calculations at $\tau = 0^+$**:** S.Demirci, T.Lappi and S.Schlichting, Phys. Rev. D 103, 094025 (2021) [2101.03791]

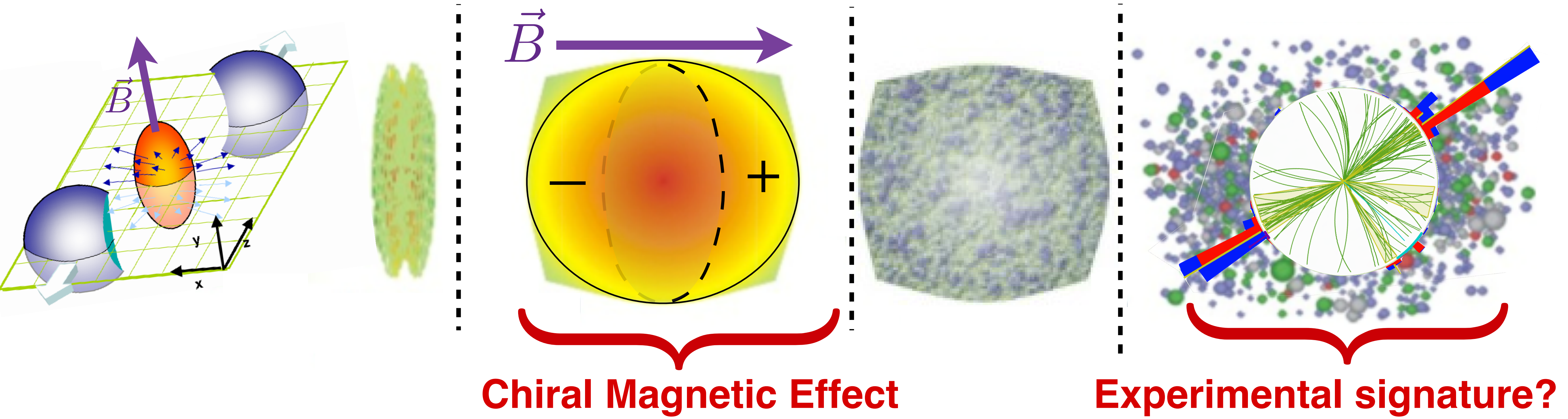
CP violation in the Quark Gluon Plasma: the Chiral Magnetic Effect





- Chirally-imbalanced matter in the presence of a background magnetic field will induce a separation of positive and negative charges (**Chiral Magnetic Effect**).

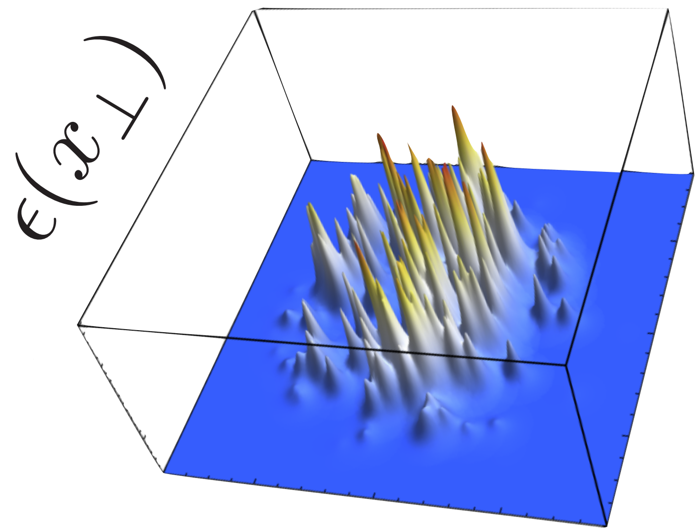
CP violation in the Quark Gluon Plasma: the Chiral Magnetic Effect

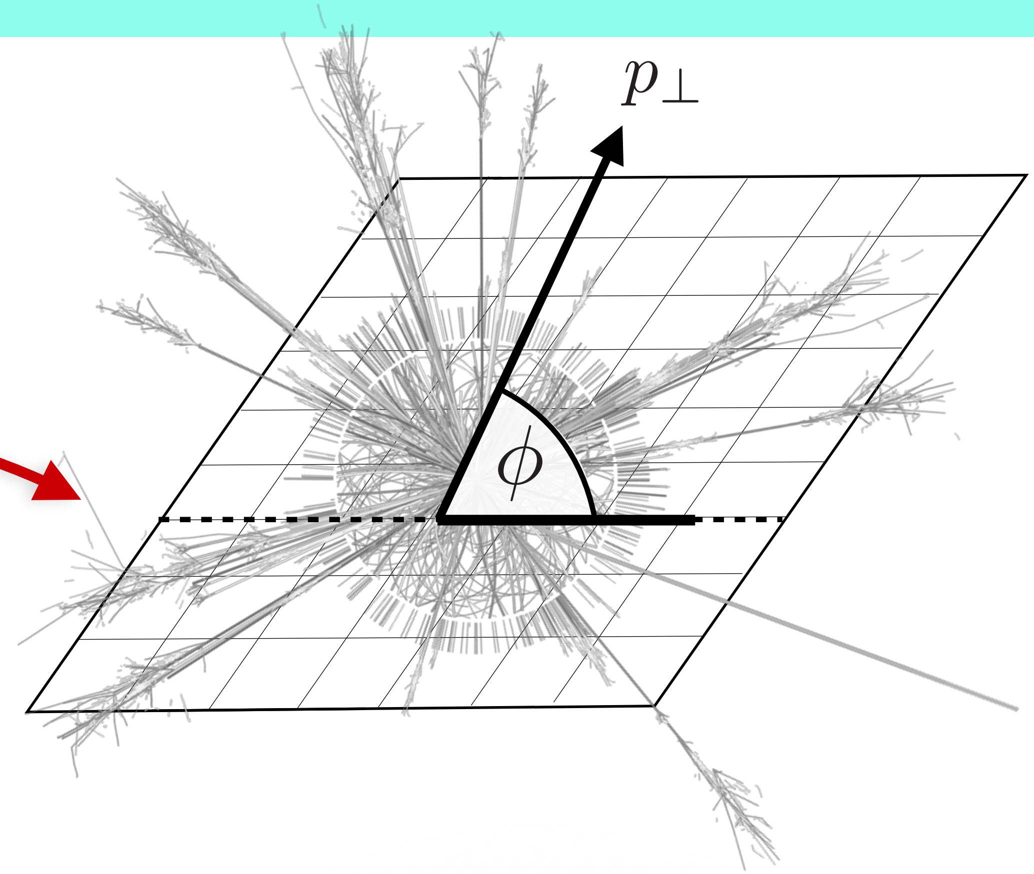


- Off-central HICs give rise to **large background electromagnetic fields**.
- Parity and Charge-Parity violating fluctuations are expected to happen with relatively high probability in the QGP.
- Chirally-imbalanced matter in the presence of a background magnetic field will induce a separation of positive and negative charges (**Chiral Magnetic Effect**).
- The search for signatures of this and other anomalous transport effects is affected by the presence of **large background effects**.

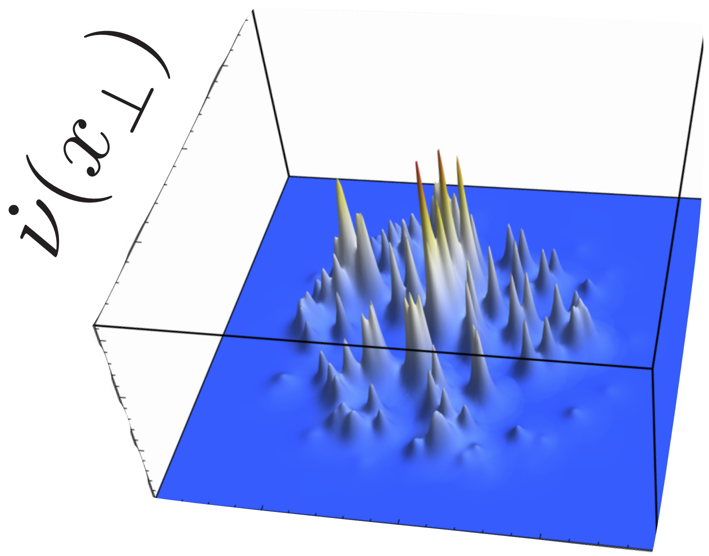
The Chiral Magnetic effect in the CGC

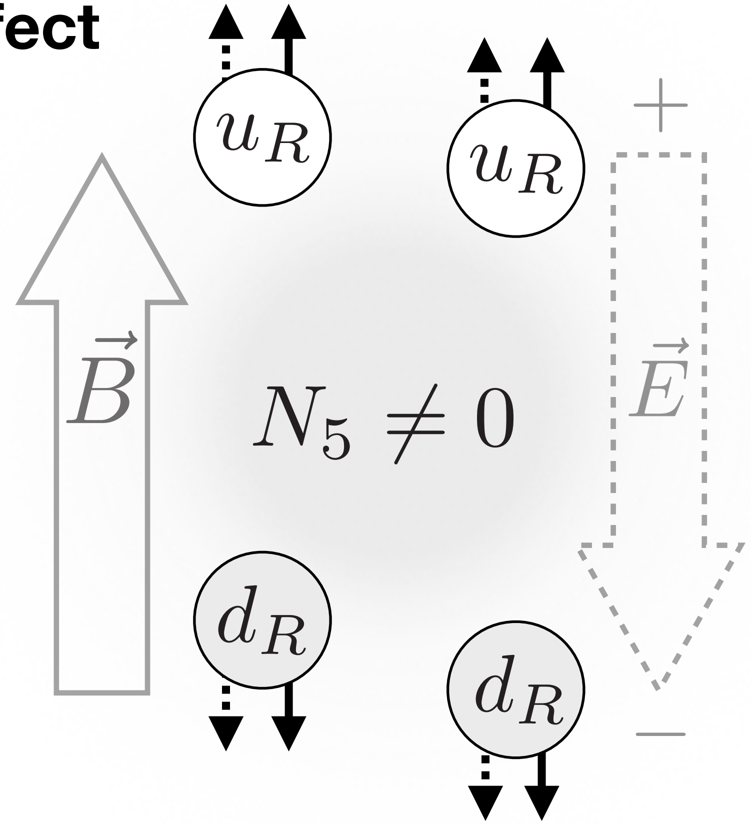
- Fluctuations of energy density and pressure gradients → **Azimuthal anisotropy**





- Fluctuations of axial charge → **Chiral Magnetic Effect** (defined as $N_5 = N_R - N_L$)





Conclusions

- We apply the **Color Glass Condensate** effective theory to describe fluctuations of color charge densities (**primordial fluctuations**) in the **initial stage of Heavy Ion Collisions**
- We do so through the computation of **one- and two-point correlators** of the **energy density** deposited in the **Glasma state** at $\tau = 0^+$ *J.L.Albacete, Cyrille Marquet and PGR, JHEP 1901 (2019) 073*
  T.Lappi and S.Schlichting, Phys.Rev.D97 (2018) 3 034034
- We perform a study of **azimuthal anisotropy** based on these first-principle calculations. Our method provides a satisfactory description of $v_2\{2\}, v_2\{4\}, v_3\{2\}$ without having to rely on ad-hoc sources of fluctuations. *F.Gelis, G.Giacalone, C.Marquet, J-Y.Ollitrault, PGR [1907.10948]*
- By assuming a **free field propagation**, we perform an **analytical calculation** of the evolution of said correlators at **finite proper times** *T.Lappi, PGR, Phys. Rev. D 104, 014011 (2021) [2102.09993]*
- Our results provide theoretical insight into the **thermalization phase of Heavy Ion Collisions**, at least while the system can be described classically
- **Another source of fluctuations** can be considered in the **dilute-dense regime**: the changing positions of **Hot Spots** *S.Demirci, T.Lappi and S.Schlichting, Phys. Rev. D 103, 094025 (2021) [2101.03791]*
- **Work in progress:** We intend to perform a similar study of the evolution of correlators considering **both sources of fluctuations simultaneously**
- Once completed, we expect this calculation to provide an extra constraint to the phenomenological Hot Spots model, which includes parameters such as the proton radius, the size and number of hot spots, etc.