Coimbra, July of 2022

Tidal evolution of **circumbinary planets**

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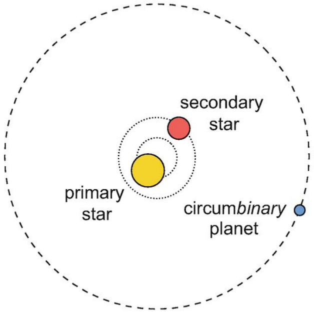
Introduction

~ 15 CB planets
(*Kepler* + *TESS*)

BINARIES	PLANETS
$P_B < 40$ days	$a_B/a_p > 0.25$
$m_{star} \sim m_{Sun}$	m_p in $[m_{earth}, m_{Jup}]$
e_B in [0,0.5]	e_p in [0,0.2]

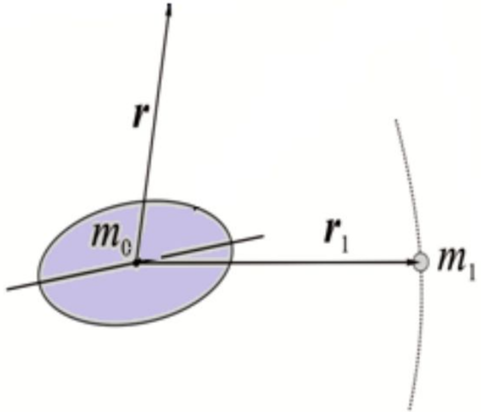


- Circumbinary (CB) planets observed very close to the central binaries → tidal forces
- Tides exerted by two comparable masses → different from the case of a single star
- Stars dynamics expected to be little affected by the presence of the planet

›› **Study the role of tidal forces in the dynamical (orbital + rotational) evolution of the planet**

What do I call *tide*?

m_0 deformed body
m_1 deforming body



m_0 is NOT a perfectly elastic body → is lagged with respect to **m_1**

Tidal Potential ❯ Force ❯ Torque

Longer timescales
Decrease *a* and *e*

Orbital evolution

Spin evolution

Shorter timescales
Pseudo-synchronism

$$\langle \Omega_2 \rangle_{\rm stat} = (1 + 6e_2^2) n_2$$

Δt ∝ 1/Q

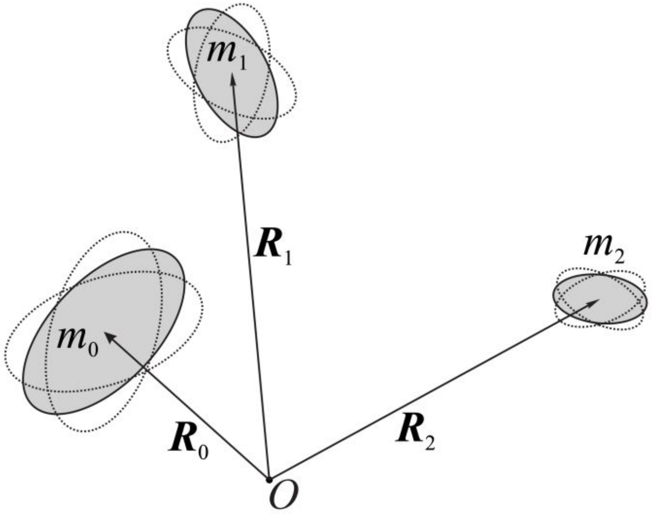
Tidal models

γ ∝ 1/η

Constant Time Lag: the tidal bulge is time lagged by a constant and small **Δt** with respect to m_1 → weak friction model expected to be valid for "gaseous" bodies *(e.g. Efroimsky 2012)*

Creep tide model: the fluid behaves like a Maxwellian body *(e.g. Correia+ 2014)* without the elastic component → expected to be valid also for Maxwellian "stiff" bodies

Our problem



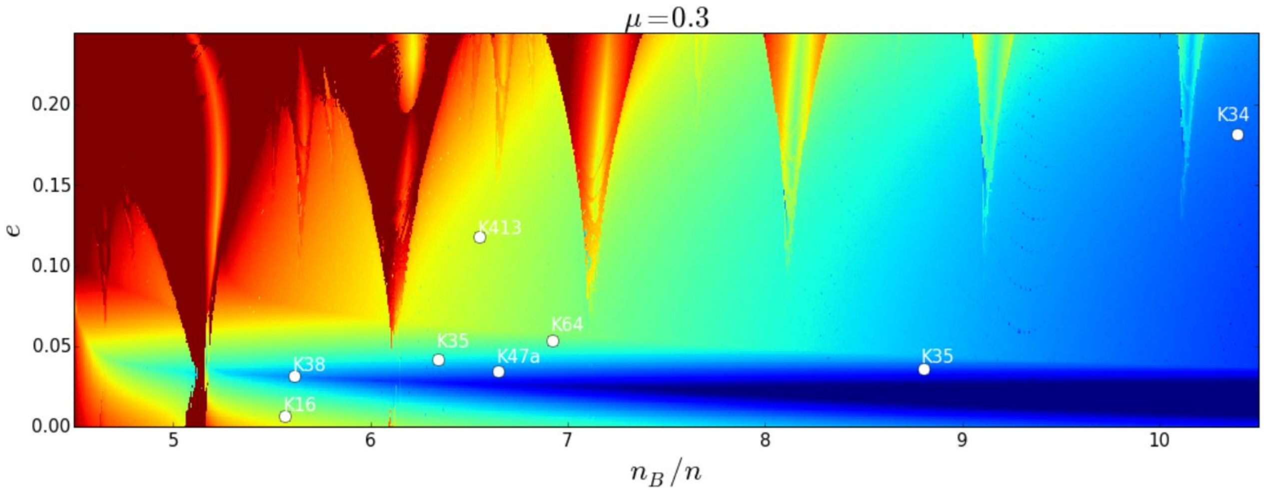
- $\mathbf{m_0}$ and $\mathbf{m_1}$ stellar bodies
- $\mathbf{m_2}$ planetary body

Each body is deformed by each of its two companions ❯ two tidal bulges on each body

Each tidal bulge is lagged and exerts a tidal force (and torque) on each of its two companions

Tides and Resonances (Zoppetti+ 2018)

Motivation



$e_B = 0.1$

- *In situ formation very difficult (e.g. Lines et al. 2014) → formation in outer region + migration*
- *Mechanisms to stall the inward migration (HD simulations): inner disk cavity (e.g. Masset et al. 2006) and resonance trapping (e.g. Nelson 2003)*
- *Tidal effects of planets around single stars responsible of divergence from the exact commensurabilities (e.g. Delisle et al. 2014)*

Tides and Resonances

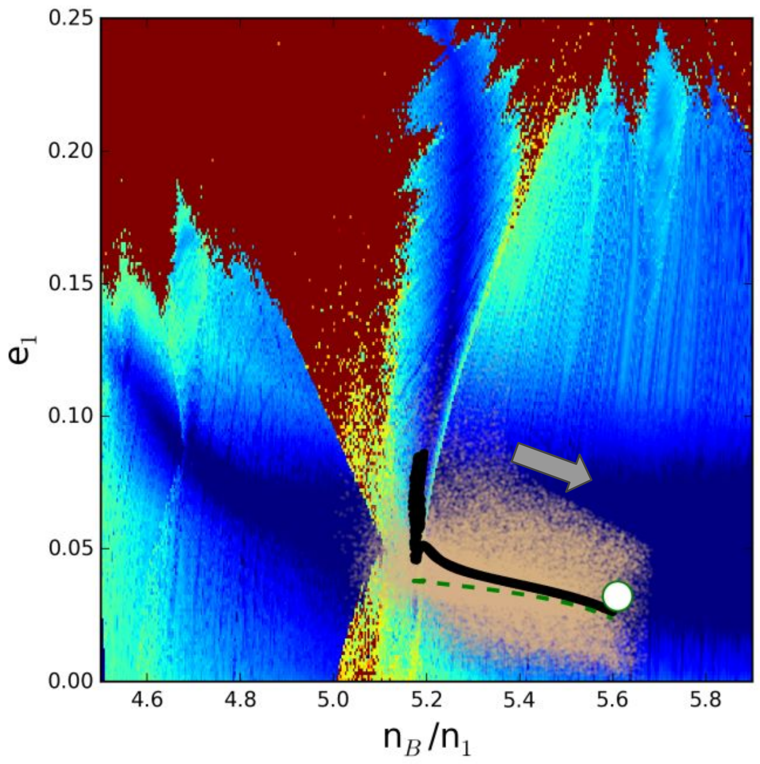
Kepler 38 (Orosz et al. 2012)

Body	Radius	$m\ [M_\odot]$	P [d]	a [AU]	e
m_A	$1.757\ R_\odot$	0.949			
m_B	$0.272\ R_\odot$	0.249	18.7954	0.1469	0.1032
m_1	$4.35\ R_\oplus$	?	105.595	0.4644	≤ 0.032

$n_B / n_1 = 5.6$ → far from the exact MMR

T = 12 Gyrs

- We simulated the interaction of the CB planet with a protoplanetary disc and obtained stable captures en the 5:1 MMR
- After the capture, we turn on “rustic” tidal effects and observed that the planet migrates outwards



Tides and Resonances

MAIN CONCLUSIONS:

The tidal evolution of the planet is outward

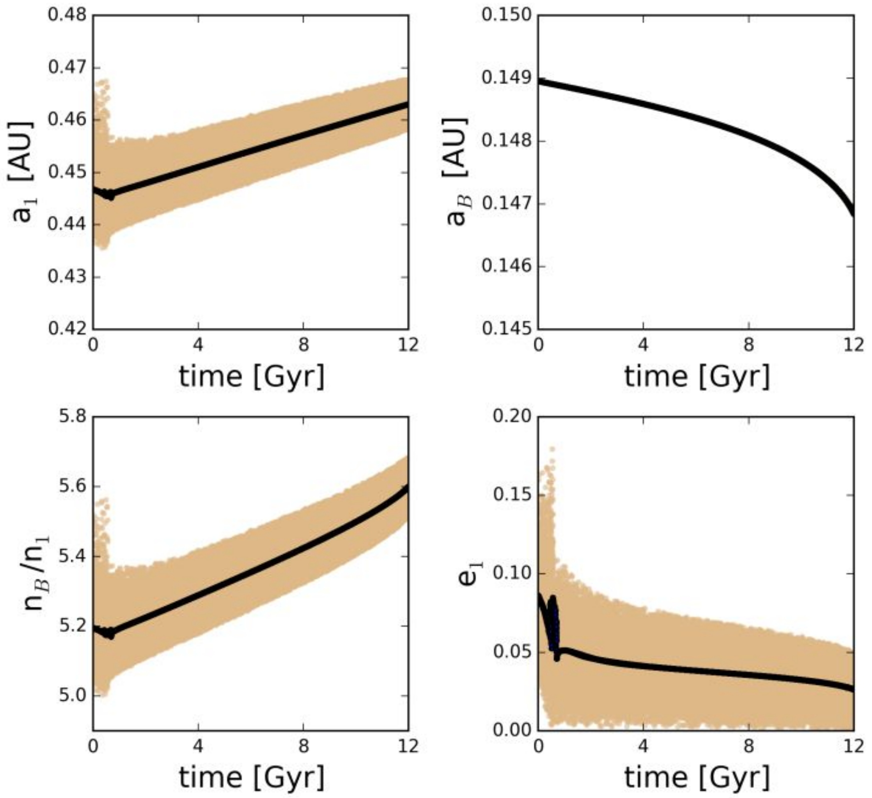
→ **WHY?**

In order to validate the hypothesis of **resonant capture** followed by **tidal evolution**:

- Restrictions on the primordial binary
- Q of the order of unity → 1 or 2 orders smaller than expected (e.g. Ferraz-Mello 2013)

TO IMPROVE:

- Non-self consistent model
- Model only valid for gaseous bodies

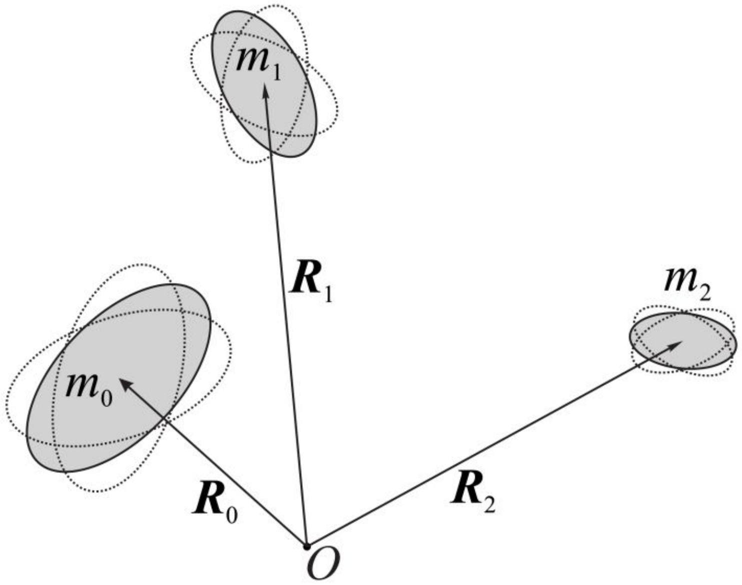


Self-Consistent **tides** in the **3BP** (Zoppetti+ 2019)

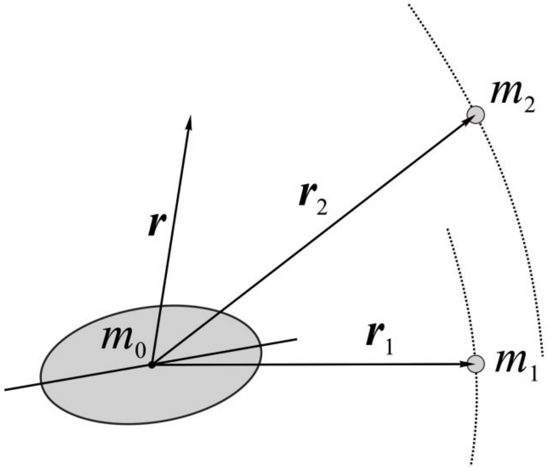
MOTIVATION

- Construct a self-consistent tidal (CTL) model in which all the bodies are considered extended and tidally interacting

Which tidal deformations have a net effect on the long-term dynamical evolution of the system?



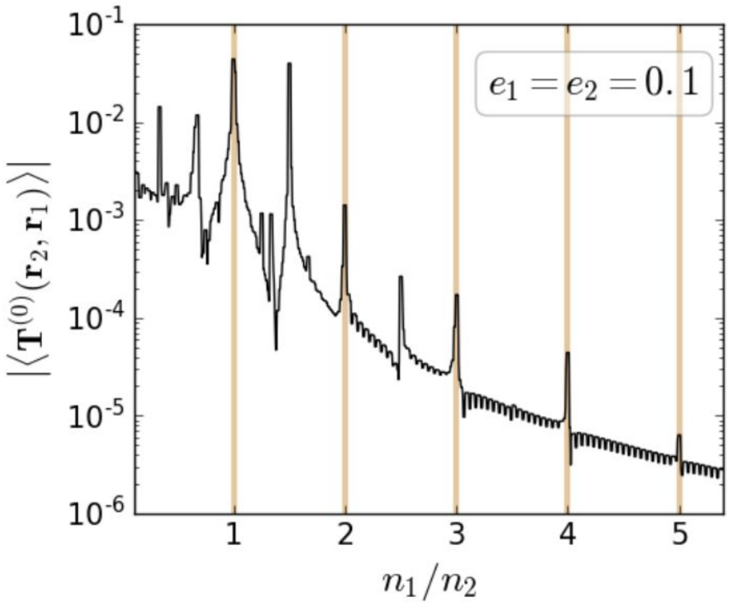
Cross tides



$$U(\mathbf{r}, \mathbf{r_1}) = U^{(0)}(\mathbf{r}, \mathbf{r_1}) + U^{(1)}(\mathbf{r}, \mathbf{r_1}) + \mathcal{O}(\Delta t_0^2),$$

$$\mathbf{f} = \nabla_{\mathbf{r}}(U^{(0)} + U^{(1)}) = \mathbf{f}^{(0)} + \mathbf{f}^{(1)},$$

$$\mathbf{T}(\mathbf{r}, \mathbf{r_1}) \simeq \mathbf{r} \times (\mathbf{f}^{(0)} + \mathbf{f}^{(1)}) = \mathbf{T}^{(0)}(\mathbf{r}, \mathbf{r_1}) + \mathbf{T}^{(1)}(\mathbf{r}, \mathbf{r_1}).$$



Cross tides are crucial in resonant configurations!
BUT have null net contribution outside

Equations of motion in the 3BP

From an inertial frame

$$m_i \ddot{\boldsymbol{R}}_i = \sum_{j=0,\ j\neq i}^{2} \left(\frac{\mathcal{G} m_i m_j}{|\boldsymbol{\Delta}_{ji}|^3} \boldsymbol{\Delta}_{ji} + (\boldsymbol{F}_{ij} - \boldsymbol{F}_{ji}) \right)$$

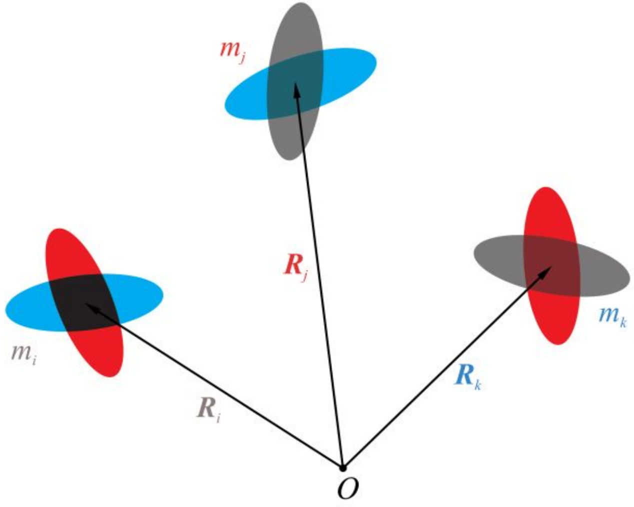
The **CTL** tidal force *(e.g. Mignard 1979)*

$$\boldsymbol{F}_{ij} = -\frac{\mathcal{K}_{ij}}{|\boldsymbol{\Delta}_{ij}|^{10}} \left[2(\boldsymbol{\Delta}_{ij} \cdot \dot{\boldsymbol{\Delta}}_{ij}) \boldsymbol{\Delta}_{ij} + \boldsymbol{\Delta}_{ij}^2 (\boldsymbol{\Delta}_{ij} \times \boldsymbol{\Omega}_j + \dot{\boldsymbol{\Delta}}_{ij}) \right]$$

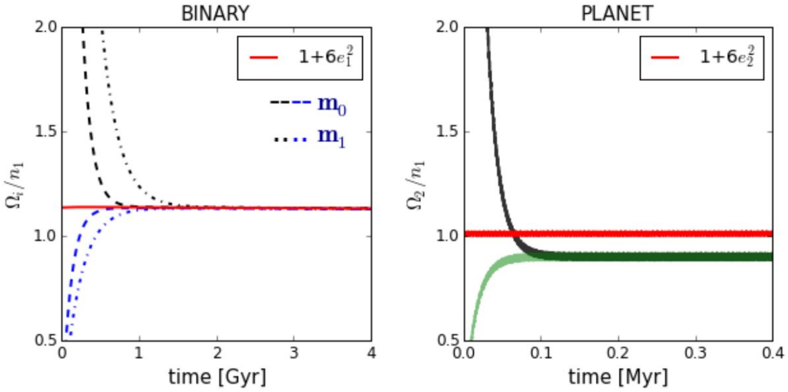
with $\mathcal{K}_{i,j} = 3\mathcal{G} m_i^2 \mathcal{R}_j^5 k_{2,j} \Delta t_j .$

From the conservation of the angular momentum

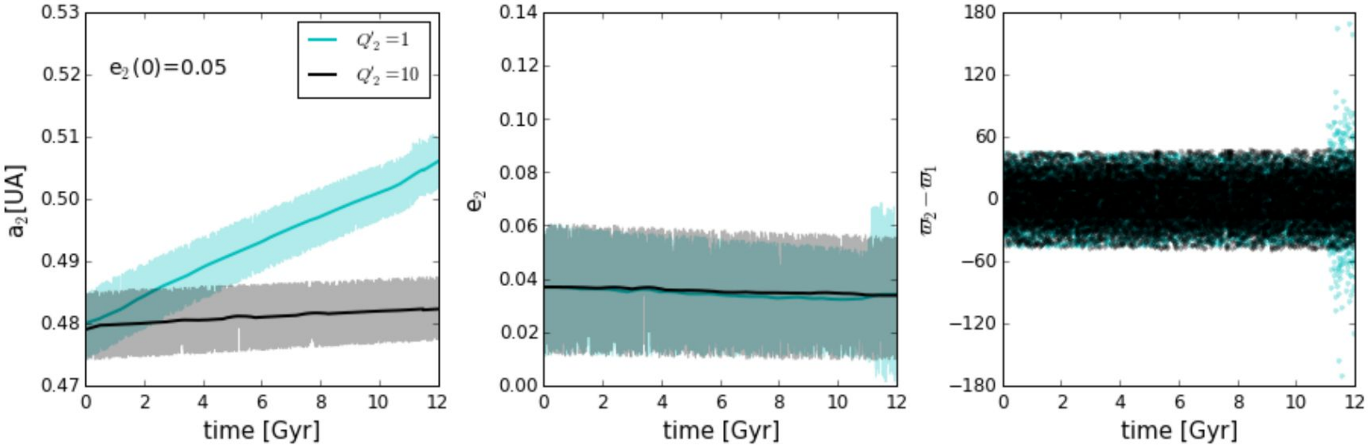
$$\frac{\mathrm{d}\boldsymbol{\Omega}_\mathbf{i}}{\mathrm{d}t} = \frac{1}{C_i} \sum_{j\neq i} \frac{\mathcal{K}_{ji}}{|\boldsymbol{\Delta}_{ji}|^6} \left[\frac{\boldsymbol{\Delta}_{ji} \times \dot{\boldsymbol{\Delta}}_{ji}}{|\boldsymbol{\Delta}_{ji}|^2} - \boldsymbol{\Omega}_i \right],$$



Numerical simulations



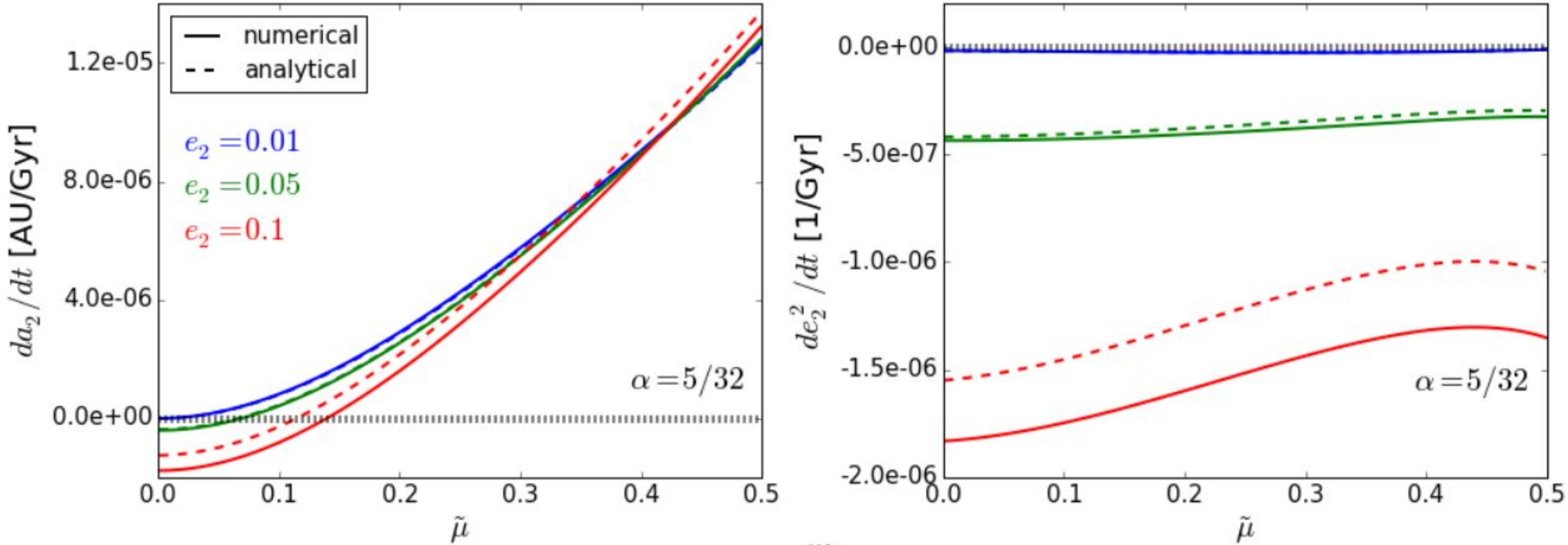
- shorter timescales
- sub-synchronous rotation



- longer timescales
- outward migration

Analytical secular solution

- We performed elliptical expansions of the CB planetary spin, semimajor axis and eccentricity up to 4th order in α=a_1/a_2 and up to 2nd order in e_1 and e_2, and average over the mean motions



$$\tilde{\mu} = \frac{m_1}{m_0 + m_1}$$

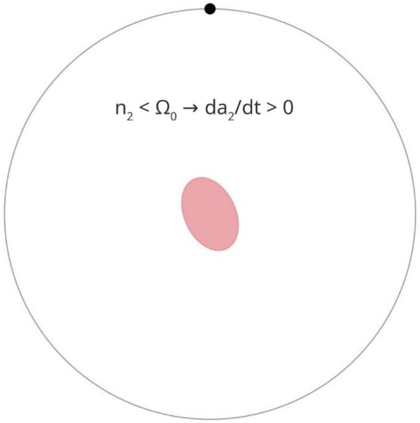
Geometrical **interpretation** of the outward migration

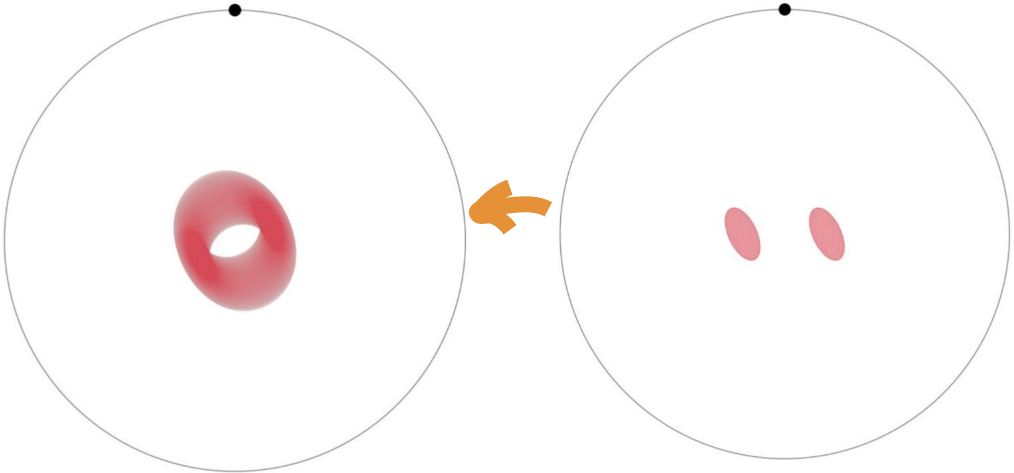
In the **2-body problem**, the case in which m_0 is the only extended mass with spin Ω_0 and m_2 is orbiting with e_2=0

In the **3-body problem**, the case in which m0 and m1 are extended bodies and synchronous e_1=0 and the planet is far enough for the binary ($\alpha \to 0$) with e_2=0

$$\frac{1}{a_2}\frac{da_2}{dt} = \frac{6n_2 m_2}{m_0}\left[k_{2,0}\Delta t_0\left(\frac{\mathcal{R}_0}{a_2}\right)^5\right](\Omega_0 - n_2),$$

$$\frac{1}{a_2}\left\langle\frac{da_2}{dt}\right\rangle = \frac{6n_2\, m_2}{m_0 + m_1}\left[k_{2,0}\Delta t_0\left(\frac{\mathcal{R}_0}{a_2}\right)^5 + k_{2,1}\Delta t_1\left(\frac{\mathcal{R}_1}{a_2}\right)^5\right](n_1 - n_2).$$

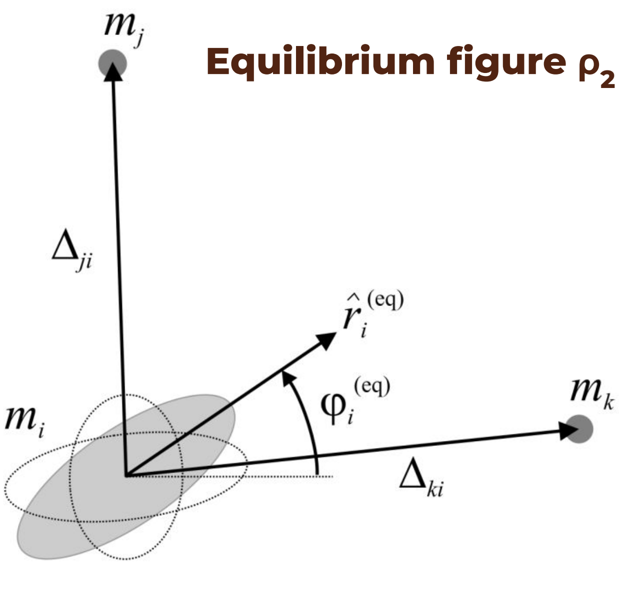


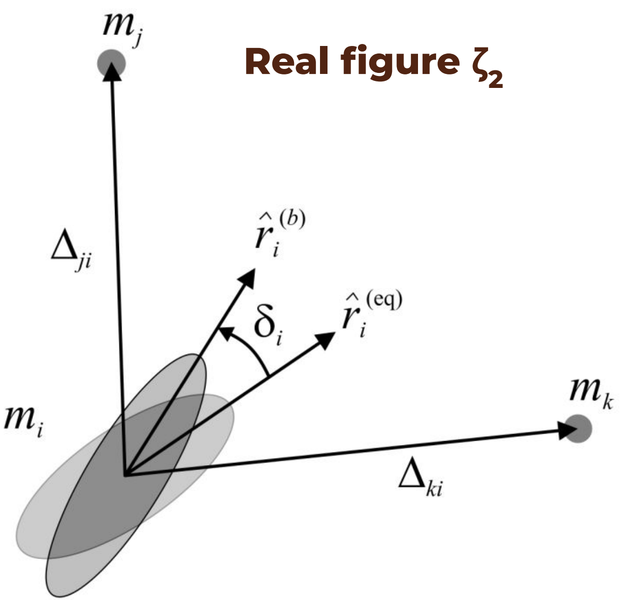


- Are our results (**sub-synchronous rotational state + outward migration**) dependent on the tidal model?

- Can we extend these results to the case of **stiff bodies**?

Creep tide model for CB planets





Creep equation

$$\dot{\zeta} = \gamma(\rho - \zeta).$$

relaxation factor ~ 1/viscosity

Shape (equatorial flattening $\mathcal{E}^\rho$ and polar flatenning $\mathcal{E}^z$ of an ellipsoid) + **orientation** (lagged δ respect to the equilibrium figure)of each body

Rotational evolution of CB planets (Zoppetti+ 2021)

- For the planet, we have

$$\dot{\delta}_2 = \Omega_2 - \varphi^{eq} - \frac{\gamma_2 \varepsilon_2^{\rho}}{2\mathscr{E}_2^{\rho}} \sin(2\delta_2)$$

$$\dot{\mathscr{E}}_2^{\rho} = \gamma_2(\varepsilon_2^{\rho} \cos(2\delta_2) - \mathscr{E}_2^{\rho})$$

$$\dot{\mathscr{E}}_2^{z} = \gamma_2(\varepsilon_2^{z} - \mathscr{E}_2^{z}),$$

orientation + **shape** evolution equations

- For the conservation of total angular momentum

$$\dot{\Omega}_2 = -\frac{2\mathscr{G} m_2}{5\mathscr{R}_2^3} \varepsilon_2^{\rho} \mathscr{E}_2^{\rho} \sin(2\delta_2).$$

spin evolution equation

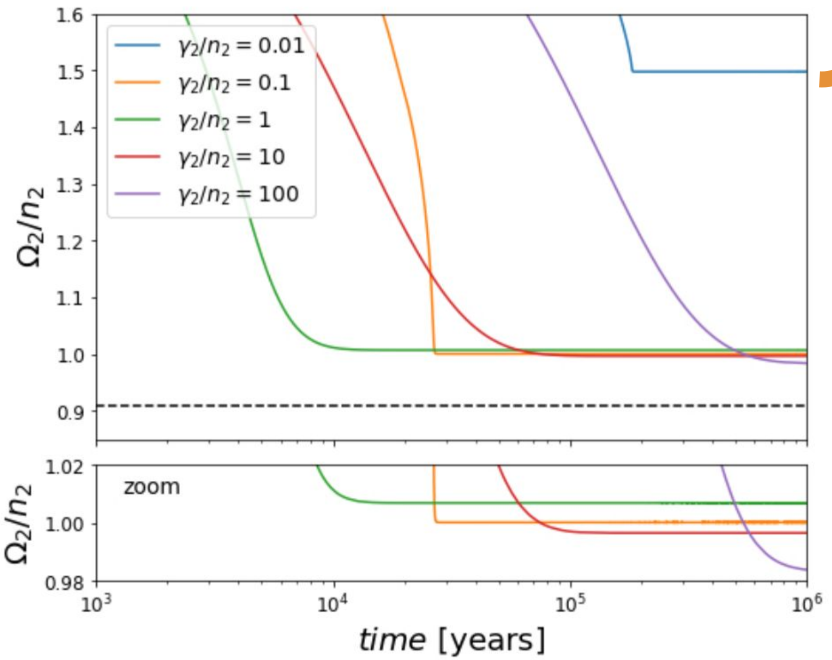
→ **rotational evolution**

Numerical simulation

Orbital elements expected to have secular variations on much longer timescales → assumed fixed in the integration *(except the mean anomalies)*

gaseous regime
$\gamma_2 >> n_2$

stiff regime
$\gamma_2 << n_2$



capture in the 3:2 spin-orbit resonance

- Stiff bodies captured in perfect synchronous state
- Gaseous bodies captured in sub-synchronous state

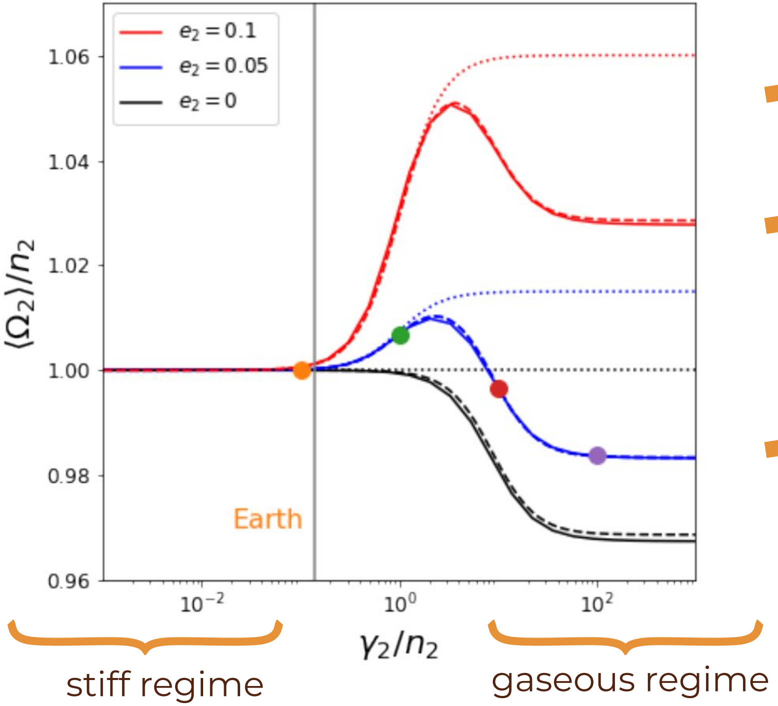
Stationary **rotational** solution

We obtained analytical secular expressions for the rotational evolution quantities
(4th order in α and 2nd order in e_1 and e_2)

__ **numerical**

--- **analytical**

.... **2BP**



- Far from the transition regime, solution independent of gamma
- **Stiff regime**: solution tends to perfect sincronism with n_2, ➔ not dependent on the masses and orbital parameters
- **Gaseous regime**: solution dependent of the masses and orbital parameters ➔ competition between the secondary mass and the planetary eccentricity e_2

- Assuming the rotational state is the pseudo-synchronous ➔ we use the solution of Zoppetti+ (2021) ➔ knowing the real shape and orientation of bodies

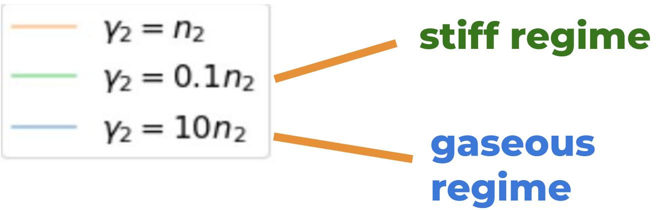
$$m_2\ddot{\mathbf{R}}_2 = \sum_{j=0}^{1}\left(\frac{\mathcal{G}m_j m_2}{|\mathbf{\Delta}_{j2}|^3}\mathbf{\Delta}_{j2} + (\mathbf{F}_{ij} - \mathbf{F}_{ji})\right)$$

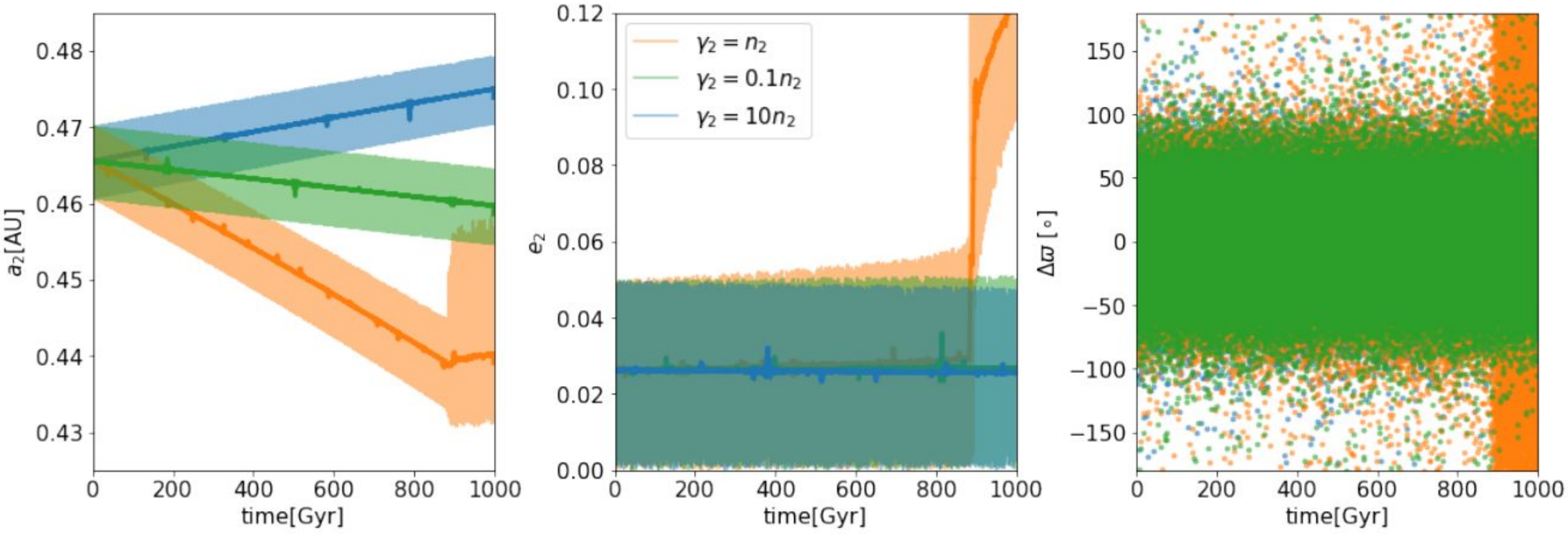
where

$$\mathbf{F}_{ji} = \frac{3\mathcal{G}m_j\overline{C}_i}{2\Delta_{ji}^5}\mathcal{E}_i^{\rho}\sin\left(2(\varphi_i^{\mathrm{eq}} + \delta_i) - 2\varphi_{ji}\right)(\hat{\mathbf{k}}\times\mathbf{\Delta}_{ji}) - \frac{3\mathcal{G}m_j\overline{C}_i}{2\Delta_{ji}^5}\left(\frac{3}{2}\mathcal{E}_i^{\rho}\cos\left(2(\varphi_i^{\mathrm{eq}} + \delta_i) - 2\varphi_{ji}\right) + \mathcal{E}_i^{z}\right)\mathbf{\Delta}_{ji},$$

Numerical simulations

- N-body integration + creep tides on the CB
- *Ad-hoc* pseudo-synchronous solution for the rotational state



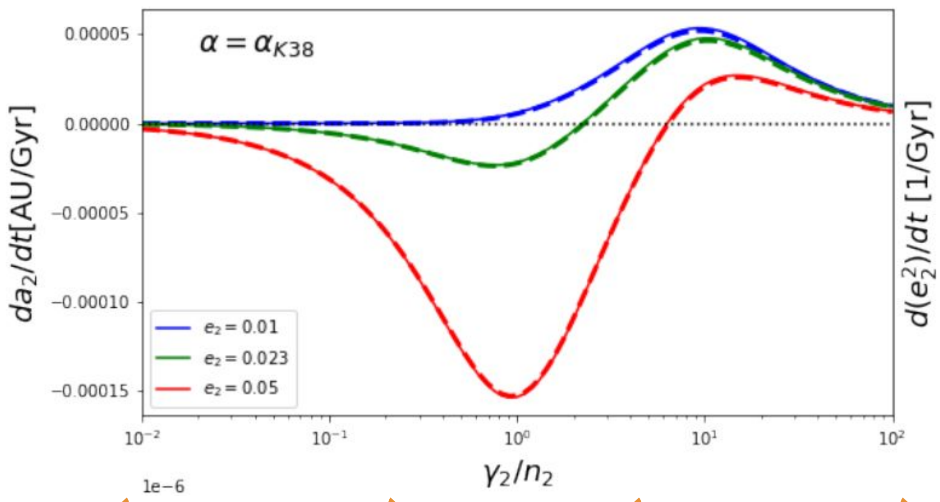


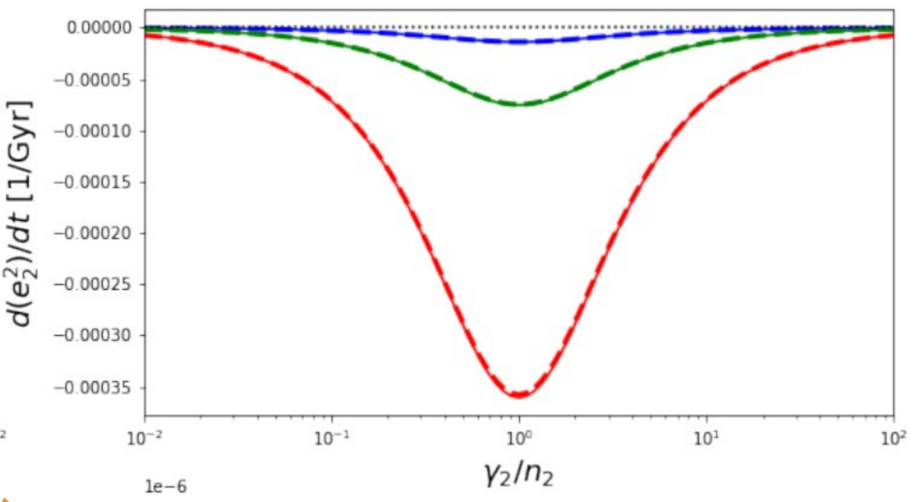
Orbital evolution

We obtained analytical secular expressions for the variational orbital equations

(4th order in α and 2nd order in e_1 and e_2)

___ numerical   - - - analytical





stiff regime   gaseous regime

Migration **direction** and **timescales**

for low-eccentric close planets

- **Criterion for determining the migration direction**

$$\langle \frac{da_2}{dt} \rangle = \frac{n_2(m_0+m_1)\mathcal{R}_2^5}{m_2 a_2^4} \sum_{i=0}^{4} A_i^{(a)} \alpha^i$$

$\langle \frac{da_2}{dt} \rangle = 0$ implies

$$A_0^{(a)} + A_4^{(a)} \alpha_{crit}^4 = 0,$$

$$\alpha_{crit}^4 = \frac{28}{173} \frac{n_2}{n_1 - n_2} \left(\frac{\gamma_2^2 + (2n_1 - 2n_2)^2}{\gamma_2^2 + n_2^2} \right) \frac{e_2^2}{\mathcal{M}_2^2}$$

Gaseous limit

$$\alpha_{crit,g}^4 = \frac{28}{173} \frac{n_2}{n_1 - n_2} \frac{e_2^2}{\mathcal{M}_2^2}$$

Stiff limit

$$\alpha_{crit,s}^4 = \frac{112}{173} \frac{n_1 - n_2}{n_2} \frac{e_2^2}{\mathcal{M}_2^2}$$

- **Timescales for the semimajor evolution**

$$\tau_a = \frac{a_2}{\langle \frac{da_2}{dt} \rangle} \simeq \frac{1}{n_2} \left(\frac{a_2}{\mathcal{R}_2} \right)^5 \left(\frac{m_2}{m_0 + m_1} \right) \frac{1}{\left| -\frac{63}{2} \frac{\gamma_2 n_2}{\gamma_2^2 + n_2^2} e_2^2 + \frac{1557}{16} \mathcal{M}_2^2 \frac{\gamma_2 (2n_1 - 2n_2)}{\gamma_2^2 + (2n_1 - 2n_2)^2} \alpha^4 \right|}$$

Application: **confirmed** CB systems

	α	e_2	M_2	viscosity regime	$\alpha_{crit,g}$	$\alpha_{crit,s}$	migration direction	$\tau_{a,gas}^{min}$[yr]	$\tau_{a,sti}^{min}$[yr]
Kepler-16[1]	0.318	0.0069	0.175	gaseous	**0.086**	0.260	outward	$\mathbf{3 \times 10^{13}}$	$6{\times}10^{14}$
Kepler-34[2]	0.210	0.182	0.250	gaseous	**0.309**	1.340	inward	$\mathbf{7 \times 10^{13}}$	$3{\times}10^{12}$
Kepler-35[2]	0.292	0.042	0.249	gaseous	**0.171**	0.559	outward	$\mathbf{2 \times 10^{12}}$	$1{\times}10^{12}$
Kepler-38[3]	0.316	0.023	0.165	?	0.162	0.492	?	$5{\times}10^{12}$	$8{\times}10^{12}$
Kepler-47b[4]	0.283	0.021	0.194	gaseous	**0.135**	0.455	outward	$\mathbf{3 \times 10^{12}}$	$6{\times}10^{12}$
Kepler-47d[5]	0.116	0.024	0.194	gaseous	**0.100**	0.701	outward	$\mathbf{2 \times 10^{15}}$	$3{\times}10^{13}$
Kepler-47c[4]	0.085	0.044	0.194	?	0.121	1.070	inward	$7{\times}10^{15}$	$7{\times}10^{13}$
Kepler-64 [6]	0.267	0.0702	0.166	gaseous	**0.261**	0.921	outward	$\mathbf{1 \times 10^{15}}$	$1{\times}10^{13}$
Kepler-413 [7]	0.286	0.1181	0.240	gaseous	**0.290**	0.966	inward	$\mathbf{5 \times 10^{13}}$	$3{\times}10^{11}$
Kepler-453 [8]	0.235	0.0359	0.142	gaseous	**0.191**	0.753	outward	$\mathbf{3 \times 10^{14}}$	$3{\times}10^{13}$
Kepler-1647 [9]	0.047	0.0581	0.247	gaseous	**0.098**	1.367	inward	$\mathbf{3 \times 10^{18}}$	$1{\times}10^{16}$
Kepler-1661 [10]	0.295	0.057	0.181	gaseous	**0.235**	0.761	outward	$\mathbf{2 \times 10^{14}}$	$3{\times}10^{13}$
TOI-1338 [11]	0.287	0.093	0.173	gaseous	**0.303**	1.007	inward	$\mathbf{5 \times 10^{12}}$	1×10^{11}

Direction of migration depends on the system

very long tidal timescales → low orbital evolution

Summary

- CB planets discovered close to the central binary →tidal forces.
- The binary evolves like in the tidal 2BP.
- The CB planet have a peculiar dynamical evolution → two perturbing bodies with comparable masses (instead of one).
- Using CTL tidal model in which all the bodies are considered extended

1) Planetary rotational evolution takes place in shorter timescales (Myrs) →different stationary (pseudo-synchronous) solution respect to the 2BP →possibility of sub-synchronous state →competition between the secondary mass and the planetary eccentricity
2) Planetary orbital evolution occurs in longer timescales (Gyrs) →eccentricity is always damped but the semimajor axis can increase →typically outward migration expected for “gaseous” CB planets in pseudo-synchronous rotation →competition between the m1 and the e2

- Using Creep tides model in which only the planet is an extended body (accurate approximation)

1) In the limit of gaseous bodies →recover the results of the CTL model
2) In the limit of stiff bodies:

2a) Rotational solution tends to perfect synchronous state →independent of the physical parameters

2b) Orbital evolution is always inward like in the 2BP

Discussion

- CTL model easy to apply to the all-extended 3BP in free rotation
  - When is exactly valid in exoplanetary systems? (in the CB context) Which is the value Deltat?
  - Tidal torques on resonant bodies →expected to have a strong 0th order contribution →effects on CB planets captured in high order MMRs?

- Creep tide model harder to apply to the all-extended 3BP →the case in which only the planet is extended represents an accurate approximation
  - Tends to the CTL model in the gaseous limit
  - The case of free rotations needs to be considered in a separate way from the synchronous case
  - Effect of the missing elastic tide?
  - Real bodies (e.g. the Earth) are not maxwellian →more complex models for exoplanetary systems?

- CB objects in the Solar System: small satellites of Pluto-Charon
  - Pluto-Charon are in double-synchronous state →final state of tidal evolution
  - Small moons very close to high-order MMRs →very oblate bodies with high obliquity spins

Muito
Obrigado